Differential Encoding for Real-Time Status Updates

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***Abstract*—For many applications in sensor networks and cyber-physical systems, receiving timely information is of utmost importance. In this article, we study data transmission schemes for a single source, sending periodic updates to a receiver through an unreliable channel. We consider two schemes that exploit the temporal correlation in the source messages, to send differential information to the receiver. Taking advantage of the receiver feedback in the first scheme, the source can decide between the differential and the actual information, to be sent at each transmission opportunity. Contrastingly, in the second scheme without any feedback, the source periodically sends the actual information, interspersed with differential messages. We observe that the differential encoding improves the timeliness performance, only if the receiver feedback is available.**

***Index Terms*—age of information, renewal theory, erasure channel, differential encoding, block codes.**

I. Introduction

With reduced costs of sensor units, we are witnessing an increasing number of sensor deployments for monitoring a wide range of physical phenomena. A typical sensor has limited data processing capability, and the sensed information is relayed from the sensors to a monitor which processes this information. In sensing applications such as health and environment monitoring, the latest observation makes the older information obsolete, and hence the timeliness of information is a metric of paramount interest. Information timeliness can be quantified by a metric called *age of information* defined in [1]. This is the the performance metric we adopt in this article.

We would like to differentiate this metric from the traditional communication performance metric of reliability. Reliable communication focuses on the maximum rate of information that can be transmitted over an unreliable channel with vanishing probability of message decoding failure. These rates can be achieved utilizing sophisticated coding techniques using the channel multiple times. However, timeliness is an appropriate performance metric for real-time communication applications, where having the latest information is more important than obtaining reliable but delayed information.

Timeliness is a relatively new metric that has been employed to understand real-time communication in [1]–[6]. In the referenced articles, the communication channel model from the sensor to the monitor is somewhat idealized. In the channel models considered in these articles, the transmission is always reliable and the channel uncertainty is only reflected in the randomness of the service time. In this article, we study real-time communication over *unreliable* channels that corrupt the transmitted message, and render it unrecognizable at the receiver. In particular, we model the channel unreliability using an information theoretic bit-wise binary symmetric erasure channel. One can improve the probability of successful information transmission across this channel by sending additional redundant information. Additional redundancies result in higher number of channel uses. Hence, even though the transmitted message is more robust to the channel unreliability, it takes longer to be received. Therefore, the improved reliability comes at a cost of freshness.

We are interested in finding data transmission schemes for sending sensor information over this unreliable channel. One obvious scheme at the sensor is to encode its current state, and transmit the encoded message to the monitor at each transmission opportunity. We consider the case where the time-variation of the physical process being sensed, is slower than the update frequency. That is, between two sensor updates there is little change in the information being sent to the monitor. This setup is reasonable for monitoring physical processes such as temperature, humidity, traffic, and pollution that change gradually. This assumption would also hold if the sensing rate is high enough, so that the process being sensed does not change much during the inter-sensing interval.

For slowly varying processes, the change in state can be represented by a smaller number of bits, when compared to the actual state. Exploiting this property of the physical process under surveillance, one can send incremental change in states instead of the current state, at each transmission opportunity. This idea is already utilized by applications such as rsync [7] and http [8], that send encoded difference between the current and the previous state for traffic reduction. Analogously, one reasonable scheme at the sensors would be to encode and transmit the difference in states, rather than the true state. In this situation, one can afford to send larger number of redundant bits in the same length codeword, thereby reducing the probability of decoding failure. However, differential encoding correlates the messages temporally, and the failure of an incremental update adversely affects the subsequent updates, if there is no receiver feedback. For systems with no feedback from the receiver, the source can periodically send

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the true states. This enables the monitor to correct itself in the event of an incremental update failure. For systems that can get the receiver feedback indicating decoding failures, the source can send the actual state in the next transmission opportunity.

In this article, we analyze the performance of the above mentioned data transmission schemes using the timeliness metric as the performance criterion. This provides us insights into the trade-off between timeliness and the additional protection offered by the differential encoding.

A. Main Contributions

We consider a source that is sampling a temporally correlated physical process. The source encodes this sampled data and transmits it to a monitor over an unreliable channel. We consider information timeliness as a performance metric. While earlier studies analyzed the effect of unreliability due to random service times of the updates, we study the effect of unreliability due to channel erasures. Further, we are also interested in the selection of suitable data transmission schemes for maximizing freshness of the received information. We analyze two schemes that use differential encoding, and compare it to the scheme that sends true updates at every transmission opportunity. We find that the differential encoding only helps if there is a feedback from the receiver to the source.

II. Background

We describe the system model, performance metric, and the update transmission schemes in this section.

A. System Model

We consider a source that is sensing the state of a physical process denoted by $X(t)$. The sensor samples the state $X(t)$ and sends the encoded state X^n to a remote monitor through an unreliable channel. We assume that the source encodes each message into a n-length codeword for transmission. Hence, the source can only transmit opportunistically at periodic times $\{n(j-1)+1 : j \in \mathbb{N}\}$. We illustrate the abstract system model in Figure 1, and discuss each component in detail below.

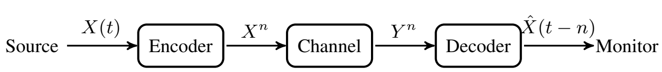

Figure 1. We show an abstract discrete time communication model for a source with m-bit information $X(t)$ at time t. The message $X(t)$ is encoded to an n-length codeword X^n and transmitted over an unreliable bit-wise *iid* binary symmetric erasure channel that outputs Y^n. Each bit requires single channel use, and hence the output Y^n is obtained after n channel uses. From the output, the decoder finds an estimate $\hat{X}(t-n)$ of the message transmitted n channel uses ago.

1) Source: We assume that the source samples the physical process at discrete times. We also assume that the source message can be represented by finitely many bits, say m. We capture the temporal correlation of the physical process $X(t) \in \{0,1\}^m$ by the following assumption. The state of the physical process sampled at times $t-n$ and t do not differ much, and this difference $X(t)-X(t-n)$ can be represented by $1 \le k \le m$ bits. Notice, that the number of bits to represent the difference would depend on the mixing time of the process, and the time-interval n.

2) Monitor: We measure the time in terms of channel use. An n-length update packet is received at the monitor after n channel uses since its generation. That is, an update codeword sent at time $n(j-1)+1$ is decoded at instant $nj+1$.

3) Channel Model: We consider a bit-wise *iid* binary symmetric erasure channel, such that for each $i \in \{1,\dots,n\}$, the channel output $Y_i \in \{0,1,e\}$ corresponding to the input $X_i \in \{0,1\}$ is given by

$$Y_i = e1_{\{Y_i \neq X_i\}} + X_i 1_{\{Y_i = X_i\}},$$

where e denotes an erasure symbol. Further, each bit of the update packet can be erased independently and identically with probability $\epsilon = \Pr\{Y_i \neq X_i\}$. Since each bit erasure is *iid* Bernoulli random variable, we have the following lemma.

Lemma 1. *The number of bit erasures E in an n-length update packet has a binomial distribution,*

$$\Pr\{E=j\} = \binom{n}{j}\epsilon^j(1-\epsilon)^{n-j}, \text{ for } j \in \{0,1,\cdots,n\}.$$

4) Encoding: The source message can either be the m-bit current state or the k-bit difference from the previously transmitted state. The encoded message corresponding to the true state $X(t)$ and the difference $X(t)-X(t-n)$ are called *true update* and *incremental update* respectively. Our analysis applies for any permutation invariant coding scheme, where the probability of decoding failure depends solely on the number of erasures in a codeword, and not its location.

5) Decoding: Conditioned on the number of erasures E in an n-length codeword with $n-r$ parity bits, the probability of decoding failure for a permutation invariant code is denoted $P(n, n-r, E)$. The unconditioned probability of decoding failure for true and incremental updates are respectively

$$p_1 = \mathbb{E}P(n, n-m, E) \quad \text{and} \quad p_2 = \mathbb{E}P(n, n-k, E).$$

Due to the nature of the erasure channel and permutation invariant coding, the decoding failures at the receiver are independent Bernoulli random variables. In this article, we consider the erasure probability $\epsilon \in (0,1)$. Together with the fact that $k \le m$, it implies that $0 < p_2 \le p_1 < 1$.

B. Performance Metric

The latest information available at the monitor is the last correctly decoded update. Let $U(t)$ denote the generation time of the last successfully decoded source state at time t. We quantify the timeliness of the update using the *information age* [1] or age $A(t)$ at time t as

$$A(t) = t - U(t). \tag{1}$$

The performance metric we are interested in, is the limiting value of average age defined as $\lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s)$.

C. Update Transmission Schemes

We are interested in understanding the impact of the following update transmission schemes on the information age at the monitor.

1) True Updates: First, we consider a scheme where the source sends the encoded current state at every transmission opportunity, i.e. at times $\{(j-1)n+1 : j \in \mathbb{N}\}$. This acts as our baseline scheme for comparison.

2) Incremental Updates without Feedback: Second, we consider the scheme where the source periodically sends the encoded current state at times $\{(j-1)qn+1 : j \in \mathbb{N}\}$ for a fixed $q \in \mathbb{N}$. At the $q-1$ transmission opportunities between the transmission of two true updates, the source encodes the differential information to send incremental updates.

3) Incremental Updates with Feedback: In the third scheme, we consider the availability of an immediate and accurate feedback from the monitor. In this scheme, the source begins transmission with a true update and sends a true update each time it receives monitor feedback of a decoding failure. At all other times, the source sends incremental updates.

III. Computation of Limiting Average Age

We will compute the limiting values of average age for the different update schemes using renewal theory. The main technique we would utilize is the renewal reward theorem [9]. Let $N(t)$ be a process that counts the number of successful receptions of the true update till time t. We denote $S_0 = 0$ and let S_i be the time instant of the i^{th} successful reception of the true update. For all three schemes, we would show that the i^{th} inter-arrival time $T_i = S_i - S_{i-1}$ is *iid* and has finite mean. Hence, the counting process $N(t)$ is a renewal process. From the independence of channel realizations, it is clear that the renewal periods T_i's are independent. Therefore, it suffices to show that T_i's are also identically distributed and have a finite mean. Conditioned on the length of the i^{th} renewal period T_i, we will find the accumulated age $S(T_i) = \sum_{t=S_{i-1}}^{S_i-1} A(t)$ in the i^{th} renewal interval $[S_{i-1}, S_i - 1]$. Since $A(S_i) = n$, the accumulated age $S(T_i)$ depends solely on renewal interval length T_i, and is also *iid* with finite mean. We denote the limiting average age by $\mathbb{E}A$. The following almost sure equality follows from the application of renewal reward theorem [9] to renewal process $N(t)$ and the reward process $A(t)$,

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s) = \mathbb{E}S(T_i)/\mathbb{E}T_i.$$

Since the length of an update codeword is fixed to be n, hence each codeword is sent every n channel uses, $\frac{T_i}{n}$ is the number of updates codewords sent during the i^{th} renewal interval.

A. True Updates

Let Z_i be the number of true updates transmitted by the source in the interval $[S_{i-1}, S_i - 1]$ before the i^{th} successful reception at time S_i. For the true updates scheme, each update codeword contains the true state information, and hence $T_i = nZ_i$. We can show that the number of true updates $\{Z_i : i \in \mathbb{N}\}$ are *iid* and have finite mean.

Lemma 2. *The number of true updates $\{Z_i : i \in \mathbb{N}\}$ are* iid *geometric with the success parameter $(1-p_1)$.*

Proof. Independence of $\{Z_i : i \in \mathbb{N}\}$ follows from the independence of erasure channels. We observe that $Z_i - 1$ is the number of true updates sent before a successful reception. Since the update decoding failures are *iid* Bernoulli with probability p_1, the result follows. ■

That is, we have shown that the i^{th} renewal occurs for the counting process $N(t)$ at time $S_i = nZ_i$, for this scheme. Further, in the i^{th} renewal interval, the age starts at n from the instant S_{i-1} and increases linearly till time $S_i - 1$. We illustrate the evolution of the age of information for this scheme using an example in Figure 2.

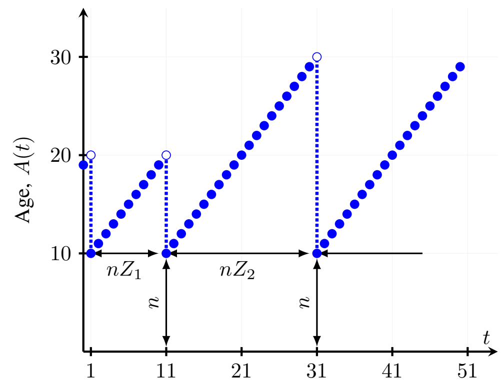

Figure 2. This plot shows one sample path of the age process for the true update scheme. We have taken the codeword length $n = 10$. First update sent at time 1 is decoded successfully at time $n+1$, and hence $Z_1 = 1$. Second update sent at time $n+1$, fails to get decoded at time $2n+1$, leading to one decoding failure. The following update sent at time $2n+1$ is received successfully at time $3n+1$, and hence $Z_2 = 2$.

Theorem 3. *The limiting average age for the true update scheme almost surely equals*

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s) = \frac{(n-1)}{2} + \frac{n}{(1-p_1)}.$$

Proof. The cumulative sum of ages for i^{th} renewal interval is

$$S(nZ_i) = \sum_{j=0}^{nZ_i-1} (n+j) = n^2 Z_i + \frac{nZ_i(nZ_i-1)}{2}.$$

Since $\{Z_i\}$ is an *iid* sequence, so is the sequence $\{S(nZ_i)\}$. From the finiteness of the first two moments of the geometric random variable Z_i, it follows that the cumulative age $S(nZ_i)$ has finite mean. From the application of renewal reward theorem [9], we get

$$\mathbb{E}A = \mathbb{E}S(nZ_i)/\mathbb{E}nZ_i = n - 1/2 + n\mathbb{E}Z_i^2/2\mathbb{E}Z_i. \quad (2)$$

The result follows from substituting the first and second moment of the geometric random variable Z_i. ■

B. Incremental Updates without Feedback

Let Z_i be the total number of true updates transmitted by the source in the interval $[S_{i-1}, S_i-1]$. In this scheme, the encoder sends $q-1$ incremental updates between two true updates, and hence $T_i = nqZ_i$. Using similar arguments as in Section III-A, we can show that $\{Z_i : i \in \mathbb{N}\}$ is an *iid* process distributed geometrically with success parameter $(1-p_1)$. That is, we have shown that the counting process $N(t)$ is a renewal process for this scheme as well. We observe that, after the failure of the first incremental update, receiver can't successfully decode the source state till the next successful reception of the true state. We define $\bar{W}_i$ to be the number of successful source state receptions in the i^{th} renewal interval.

Lemma 4. *For each renewal interval, the number of successful receptions $1 \leq \bar{W}_i \leq q$, and is independent of the number of true updates Z_i. Further, the sequence $\{\bar{W}_i : i \in \mathbb{N}\}$ is* iid *and distributed as truncated geometric*

$$\Pr\{\bar{W}_i = k\} = (1-p_2)^{k-1}\left(p_2 1_{\{1\leq k<q\}} + 1_{\{k=q\}}\right).$$

Proof. We note that $\bar{W}_i - 1$ is the number of contiguous incremental updates, decoded successfully in the i^{th} interval. Since in each renewal interval, at least one update is successfully received, $1 \leq \bar{W}_i$. Further, if $\bar{W}_i - 1 = q-1$, then the next update contains the true state information. If this update is decoded successfully, then $Z_i = 1$ and i^{th} renewal occurs. Otherwise, $Z_i > 1$ and all the subsequent incremental updates in this renewal period, are useless at the monitor. From the independence of the channel realizations, it follows that $\{\bar{W}_i : i \in \mathbb{N}\}$ is an *iid* sequence, and that $\bar{W}_i$ and Z_i are independent. Since the decoding success of incremental updates are *iid* Bernoulli with probability $1-p_2$ and $\bar{W}_i \leq q$, the distribution of $\bar{W}_i$ is truncated geometric. ∎

In the i^{th} renewal interval, the age is reset to n at instants $S_{i-1} + jn$ for $j \in \{0, 1, \dots, \bar{W}_i - 1\}$ corresponding to the successful reception of the source state. The age grows linearly at all other points in the interval. We illustrate the evolution of the age process for this scheme, with an example in Figure 3.

Theorem 5. *The limiting average age of information for the incremental updates without feedback is given by*

$$\mathbb{E}\bar{A} \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} \bar{A}(s) = \frac{\mathbb{E}T_i^2}{2\mathbb{E}T_i} + \frac{n^2\mathbb{E}\bar{W}_i(\bar{W}_i-1)}{2\mathbb{E}T_i} - \left(n\mathbb{E}(\bar{W}_i-2) + \frac{1}{2}\right).$$

Proof. The cumulative sum of ages $S(T_i)$ in the i^{th} renewal interval can be written as

$$S(T_i) = \sum_{j=1}^{\bar{W}_i-1}\sum_{k=0}^{n-1}(n+k) + \sum_{j=n(\bar{W}_i-1)}^{T_i-1}(n+j-n(\bar{W}_i-1)),$$
$$= \frac{n^2\bar{W}_i(\bar{W}_i-1)}{2} + \frac{T_i^2}{2} - \left(n(\bar{W}_i-2)+\frac{1}{2}\right)T_i.$$

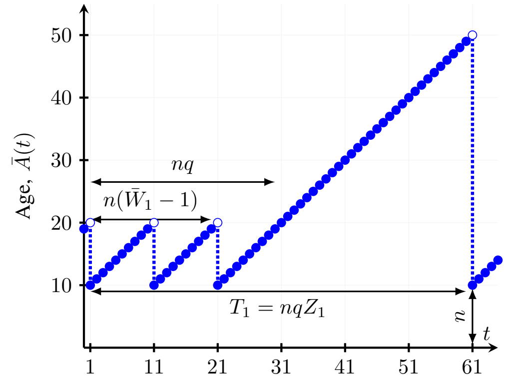

Figure 3. This plot shows a sample path for the age process in one renewal interval, for the incremental updates and no feedback. We have taken the codeword length $n = 10$, and the period after which the true update is sent as $n(q-1) = 20$. In this example, the first $q-1$ incremental updates are successfully decoded, i.e. $\bar{W}_1 - 1 = 2$. The source sends the true update at the q^{th} transmission opportunity, which fails to get decoded. Since the source has no feedback, it starts sending incremental update for next $q-1$ transmission opportunities. Finally, the following true update is decoded successfully at the monitor, and hence $Z_1 = 2$.

The result follows from taking expectations and independence of T_i and $\bar{W}_i$, and applying renewal reward theorem. ∎

C. Incremental Updates With Feedback

We let Z_i and W_i respectively denote the number of true and incremental updates sent in the interval $[S_{i-1}, S_i-1]$, then

$$T_i = nZ_i + nW_i.$$

The process $\{Z_i : i \in \mathbb{N}\}$ is *iid* geometric with success parameter $(1-p_1)$ as before. From the independence of erasure channel, it follows that Z_i and W_i are independent for each $i \in \mathbb{N}$. We have the following lemma for the number of incremental updates.

Lemma 6. *The number of incremental updates $\{W_i : i \in \mathbb{N}\}$ are* iid *geometric with the success parameter p_2.*

Proof. Independence of W_i's follows from the independence of the erasure channel. Further, $W_i - 1$ is the number of incremental updates before the first decoding failure in the i^{th} inter-arrival interval of the counting process $N(t)$. Since the decoding failure events are *iid* Bernoulli with probability p_2 for incremental updates, the result follows. ∎

It follows that the periods T_i are *iid* and have finite mean, and hence the counting process $N(t)$ is a renewal process for this scheme as well. In the renewal interval $[S_{i-1}, S_i - 1]$, the age is reset to n at times $S_{i-1} + jn$ for $j \in \{0, 1, \dots, W_i - 1\}$, corresponding to the instants when the source state can be successfully decoded from the incremental updates. Age grows linearly otherwise at all other points in the interval. We illustrate the evolution of the age process for this scheme with an example in Figure 4.

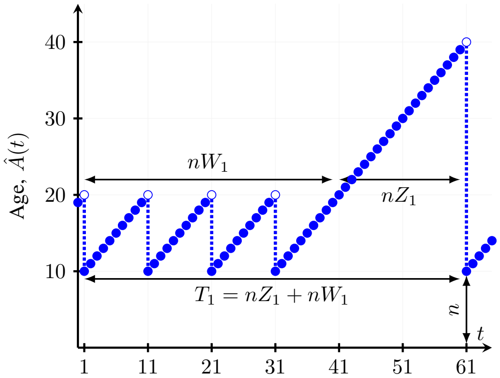

Figure 4. This plot shows a sample path of the age process in one renewal interval, for the incremental updates with feedback. We have taken the codeword length $n = 10$. In this example, first $W_1 - 1 = 3$ incremental updates are successfully decoded. Fourth incremental update sent at time $3n + 1$ fails to get decoded. The source starts sending the true updates from the next transmission opportunity at $4n+1$. True updates sent at instant $4n+1$ fails to get decoded, with the first success after $Z_1 = 2$ transmissions.

Theorem 7. *The limiting average age of information for the incremental updates with feedback is given by*

$$\mathbb{E}\hat{A} \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} \hat{A}(s) = \frac{(3n-1)}{2} + \frac{n(\mathbb{E}Z_i^2 + \mathbb{E}Z_i)}{2(\mathbb{E}W_i + \mathbb{E}Z_i)}. \quad (3)$$

Proof. The cumulative age $S(T_i)$ over the i^{th} renewal period T_i, is the sum of the cumulative age due to incremental and true updates. During this interval, all but the last incremental update are decoded correctly. Therefore, we can write

$$\begin{aligned} S(T_i) &= \sum_{j=1}^{W_i-1}\sum_{k=0}^{n-1}(n+k) + \sum_{k=0}^{T_i-n(W_i-1)-1}(n+k) \\ &= (3n-1)T_i/2 + n^2(Z_i+1)Z_i/2. \end{aligned}$$

Since $\{T_i\}$ and $\{Z_i\}$ are *iid* and mutually independent sequences, it follows that $\{S(T_i)\}$ is also an *iid* sequence. Further, $S(T_i)$ has a finite mean from the finiteness of the mean of the renewal period T_i and the first two moment of random variable Z_i. The result follows from taking expectations and applying the renewal reward theorem. ∎

IV. Analytical and Numerical Comparisons

We now compare the timeliness performance of three source state encoding schemes.

A. Analytical Comparison

We show that for any arbitrary streaming update source, the limiting average age for the three schemes can be ordered.

Theorem 8. *The mean age of information for the three schemes satisfy,*

$$\mathbb{E}\hat{A} \leq \mathbb{E}A \leq \mathbb{E}\bar{A}. \quad (4)$$

Proof. We define $\lambda = \frac{\mathbb{E}W_i}{\mathbb{E}W_i + \mathbb{E}Z_i} \in (0,1)$. Since $\mathbb{E}W_i = p_2^{-1}$ and $\mathbb{E}Z_i = (1-p_1)^{-1}$, and $p_2 \leq p_1$, it follows that

$$\mathbb{E}A - \frac{(n-1)}{2} - \frac{n}{\lambda} = n\left(\frac{1-p_2}{1-p_1} - 1\right) \geq 0.$$

The first inequality in (4) follows by writing the mean age $\mathbb{E}\hat{A}$ for the incremental update scheme with feedback as

$$\mathbb{E}\hat{A} = \mathbb{E}A - n\lambda\left(\frac{1-p_2}{1-p_1} - 1\right).$$

For the incremental updates without feedback, the minimum cumulative age over a renewal cycle T_i is attained when the first $q-1$ incremental updates succeed. That is,

$$S(T_i) \geq \frac{1}{2}(T_i - nq)^2 - \frac{n^2 q}{2} + \left(2n - \frac{1}{2}\right)T_i.$$

Substituting $T_i = nqZ_i$ above, and applying the renewal reward theorem, we get

$$\mathbb{E}\bar{A} \geq n(q-1)\mathbb{E}(Z_i-1)^2/2\mathbb{E}Z_i + \mathbb{E}A.$$

The result follows from the non-negativity of $(Z_i - 1)^2$. ∎

B. Numerical Comparison

Even though our proposed analysis is valid for any symmetric coding scheme, we use a random coding scheme [10] for the illustration purposes. For the random coding scheme, conditioned on the number of erasures E in a n-length codeword with $n-r$ parity bits, the probability of decoding failure [10] is

$$P(n, n-r, E) = 1 - \prod_{i=0}^{E-1}\left(1 - 2^{i-(n-r)}\right).$$

We are interested in the timeliness performance for small codewords used in real-time communication. Therefore, we have taken codeword length $n = 120$ inspired by the system parameters used in GSM based wireless links [11]. We have also taken the periodicity of true updates $q \in \{2, 6\}$, for the differential encoding scheme without feedback.

In Figure 5, we plot the limiting average age for the three schemes as the number of information bits m increases in $\{110, \cdots, 120\}$. The erasure probability is set to be 0.001, and we assume that the differential information can be represent by $k = 10$ bits, irrespective of m. From Theorem 8, we know the order on the performance of the three schemes. We observe that as the code-rate m/n of the true update increases, the average age remains invariant for the incremental updates with feedback, while it increases for the other two schemes. We know that the incremental update scheme without feedback is identical to the true update scheme when $q = 1$. Further, we see that the performance of this scheme gets worse as the periodicity q increases.

We compare the limiting average age of information for all three schemes in Figure 6, as the erasure probability ϵ increases in $[0.01, 0.1]$. We take a fixed number of information bits $m = 105$ in each codeword of length $n = 120$, and assume that the differential information can be represented by $k = 10$ bits. The qualitative behavior of limiting average age with increasing erasure probability is similar to that of with increasing number of information bits.

Finally, we compare the limiting average age of information for all three schemes in Figure 7, as the incremental

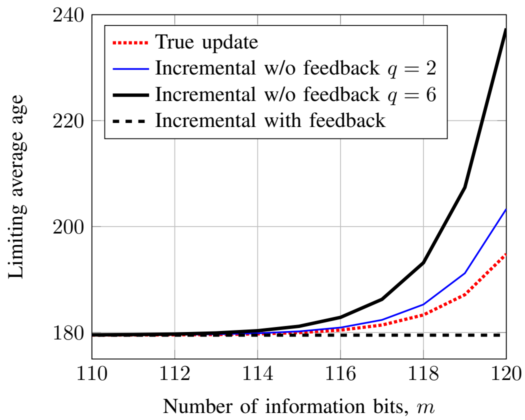

Figure 5. This plot shows the variation of limiting average age as the number of information bits m grows. We have chosen the codeword length $n = 120$ and the differential information is assumed to be represented by $k = 10$ bits. The erasure probability of the *iid* bit-wise binary symmetric channel is taken as $\epsilon = 0.001$.

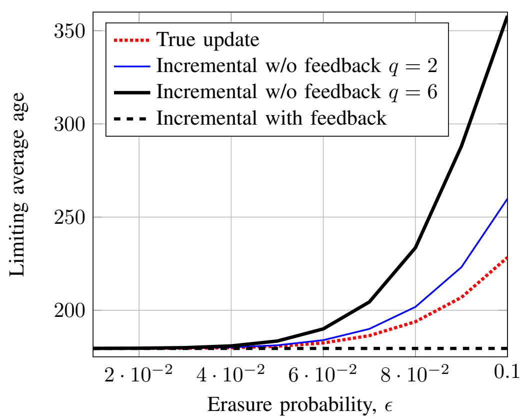

Figure 6. This plot shows the variation of limiting average age as erasure probability ϵ increases. We have chosen the number of information bits $m = 105$, in each codeword of length $n = 120$. The number of bits needed to represent the differential information is taken as $k = 10$.

information bits k grows in $\{1, \dots, m\}$. We have selected the erasure probability $\epsilon = 0.1$ and the number of information bits $m = 105$ in each codeword of length $n = 120$ bits. We observe that the limiting average age is constant for a wide range of k, and shoots up as k approaches m. As expected, the performance gain of the differential encoding with feedback when compared to the true state encoding, diminishes as k approaches m.

V. Conclusion and Future Work

We considered a slowly varying source sending real-time updates over a single unreliable link, modeled by an *iid* bit-wise binary symmetric erasure channel. We compared the timeliness performance of the differential encoding with and without feedback, to the true state encoding. We found that the differential encoding is better than the actual state encoding, if there is a receiver feedback, and worse otherwise. Even though the differential encoding offers increased protection for the

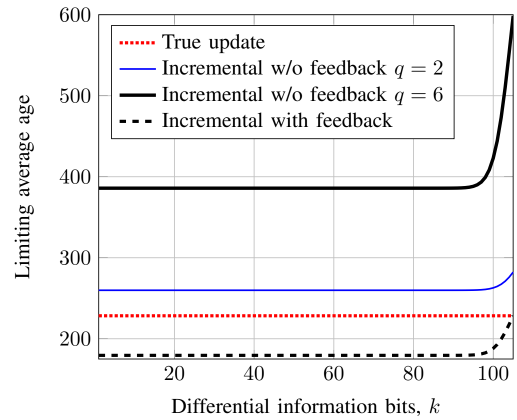

Figure 7. This plot shows the variation of the limiting average age as the number of bits needed to represent incremental information bits, k, grows. We have chosen the number of information bits $m = 105$, in each codeword of length $n = 120$ bits. The erasure probability of the *iid* bit-wise binary symmetric channel is taken as $\epsilon = 0.1$.

message bits, a single decoding failure can corrupt multiple future messages, if there is no receiver feedback. Thus, we infer that the feedback is crucial in exploiting the advantages of differential encoding, for the reduction of the information age. We have considered a general source constrained by the assumptions on the temporal correlation of the states. An extension would be to consider source messages with structured temporal correlation, e.g. mixing times of Markov sources. It would also be interesting to explore joint source channel codes that optimize the timeliness performance for the structured sources.

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