Fixed Length Differential Encoding for Real-Time Status Updates

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***Abstract*— We consider the status updates of a physical process over an unreliable channel. In this setting, one may not be able to reliably transmit the current state at all times. Instead, one is interested in the timeliness of the accurately received information. This is a setting for several cyber-physical system applications that require real-time monitoring and control. In this paper, we study periodic data transmission schemes at a single source which exploit the temporal correlation in the source messages. When the source has no feedback, it can periodically send the actual information, interspersed with differential messages. On the availability of receiver's feedback at the source, it can decide to send either the differential or the actual information at each transmission opportunity. For a fixed length coding, we show that the differential encoding improves the timeliness performance only if the receiver's feedback is available.**

***Index Terms*— Age of information, renewal theory, erasure channel, differential encoding, block codes.**

I. Introduction

RECENT advances in semiconductor devices and communication technologies have enabled wide scale adoption of digital devices. Wide variety of physical phenomena can be monitored and controlled in an automated fashion by increasingly cheap actuator units. A typical sensor has limited data processing and storage capability, and a local view of the phenomena under consideration. Hence the sensed information is relayed from the sensors to a monitor which processes the aggregate information. This is an example of many-to-one communication, with each sensor being a source sending observations to the common receiver at the monitor. For many real-time sensing and control applications, the latest observation renders previous observations obsolete. Examples of such applications are health and environment monitoring. A main objective in such scenarios is to improve the timeliness of the received information. Information timeliness can be quantified by a performance metric called *age of information* defined in [2] and [3]. This is the performance metric we adopt, to compare proposed data transmission schemes in this article.

For a renewal process, the age is defined as the time since the last renewal. This is a well studied metric in renewal theory. Age of information refers to the time elapsed at the receiver since the generation time of last information reception. Utilizing age of information for measuring timeliness of information symbols is a relatively new concept. This metric has been employed to understand real-time communication in [3]–[8]. We emphasize that the information age is different from the traditional performance metrics for reliable communication such as rate or queueing delay. Information queueing delay is a source metric, that measures limiting average of the amount of time spent by individual information packets in the transmit queue until reception. In contrast, the information age is a receiver metric, that measures the staleness of the latest information at the receiver. In particular, it is the aggregate of the time elapsed since last successful reception, and the delay between generation and reception instants of that information packet. In queueing systems, any dropped information packet at the source leads to an infinite queueing delay[1] for that packet. While the sole effect of packet dropping on the information age at the receiver, is in the linear increase of age until a successful reception.

In the articles referenced above, the uncertainty in the communication channel from the sensor to the monitor is modeled by a random service time. This is a somewhat idealized communication channel model in the sense that information is delayed by a random service time, but is always received reliably at the monitor, by the end of the service time. In this setting, age and queueing delay are related. In this article, we study real-time communication over *unreliable* channels that could corrupt the transmitted message, and render it unrecognizable at the receiver. In particular, we model the channel unreliability using an information theoretic bit-wise binary symmetric erasure channel. The binary erasure channel (BEC) is the simplest non-trivial channel model, and was introduced by Elias [9]. Binary symmetric erasure channel has been at the forefront of the theoretical development for unreliable communication channels due to its ease of understanding. For example, many coding and decoding algorithms were first developed for the erasure channel [10], [11]. Though, very popular theoretically, erasure channel can ably model several

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[1]in the sense that this particular packet is never received.

practical communication channels. For example, in digital baseband communication, one can find a partition such that the received waveform is declared 0 or 1 if the waveform is high or low, and declared erased when it is difficult to distinguish between high and low [12]. Erasure channels can also be used to model data networks [13], where packets either arrive correctly or are lost due to buffer overflows or excessive delays. In this channel, one can send one bit per channel use, that may get successfully transmitted or erased. Transmission reliability over this channel can be improved by encoding the source status message to a larger codeword.

For typical information theoretic channels, finite length transmissions over an unreliable channel results in non-zero probability of error. Hence, the reliability is achieved by encoding and sending an asymptotically large collection of messages over proportionally large number of channel uses, leading to vanishing probability of decoding failure. The performance metric for reliable communication is the limiting empirical average number of information bits that can be reliably transmitted over an unreliable channel. The maximum of the limiting information rate is called channel capacity, and can be achieved using sophisticated coding techniques that temporally spread the message over multiple usages of the channel. Capacity achieving codewords have very large decoding latency for typical information theoretic channels, since one has to wait until the reception of the entire codeword.

Real-time communication applications are sensitive to information latency, and hence information timeliness is a more appropriate performance metric. For such applications, having the latest information is more important than obtaining reliable but delayed information, and hence we adopt fixed finite length transmission schemes for each sensor message. As discussed above, finite length transmissions can be received reliably only with a non-zero probability. In our model, we send new information at each transmission opportunity, whether or not the previous transmission was received reliably. This is congruent to our goal of having fresh information at the receiver. There is an implicit assumption that the source always has some information to be transmitted, which holds true for physical processes.

We consider the case where the time-variation of the physical process being sensed, is slower than the update frequency. That is, between two sensor updates there is little change in the information being sent to the monitor. This setup is reasonable for monitoring physical processes such as temperature, humidity, traffic, and pollution that change gradually. This assumption would also hold if the sensing rate is faster than the rate of change of the process being sensed. For such processes, the change in source state between two transmissions is very small. Hence the number of bits needed to represent the difference between states at successive transmissions is much smaller than the number of bits needed to represent the actual state itself. Exploiting this property of the physical process under surveillance, one can send incremental change in states instead of the current state, at each transmission instant. This idea is already utilized by applications such as rsync [14] and http [15], which send encoded difference between the current and the previous state for traffic reduction.

We are interested in developing a framework for performance evaluation of various data transmission schemes for sending sensor information over this unreliable channel. We utilize this framework to compare three status update schemes that are described below.

1) The source encodes its current state, and transmits the encoded message to the monitor at each transmission instant. We will refer this transmission scheme as *true update*.
2) When there is no receiver feedback, the source encodes and transmits its actual state periodically, and sends the encoded incremental change in states between two true updates. This scheme is referred to as *incremental update without feedback*.
3) For *incremental updates with feedback*, the source has the knowledge of decoding success or failure of the last transmission. Hence the source transmits encoded incremental change in states or actual state, depending on the decoding success or failure of the last codeword.

True update is a simple scheme which acts as our benchmark for the two incremental update schemes. Differential encoding correlates the messages temporally, and the failure of an incremental update adversely affects the subsequent updates, if there is no feedback from the receiver. Hence the source periodically sends its actual state to monitor, as a correction measure in the event of an incremental update failure, for systems with no receiver feedback. In the incremental updates with feedback, a true update follows a decoding failure leading to a quick correction at the receiver.

We are using the following two metrics that capture information timeliness. Mean age of information is our primary performance metric. Since the updates are sent over an unreliable channel, there is a possibility that the update codeword is not decodable at the monitor. Thus, another metric of interest is the *probability of update failure* at the monitor. Higher probability of update failure indicates age buildup at the receiver. We use these metrics to compare the performance of the above described update schemes.

We review related literature in Section I-A, and outline our main contributions in I-B. We describe the system model in Section II, update schemes in Section III, and performance metrics in Section IV. For the update schemes considered, the performance metrics are computed in Section V. We make analytical comparisons in Section VI and propose optimal code-lengths in Section VII. We conclude with future directions in Section VIII.

A. Literature Review

Consider the following queueing system for status updates, where new status messages arrive at random time instants, and are buffered until their successful reception. For an M/M/1 update queueing system with *first come first serve* (FCFS) and *last come first serve* (LCFS) service disciplines, the mean age was computed in [3] and [5] respectively. Updates with multiple sources was considered in [4].

In contrast with the typical queueing systems with arbitrarily large buffers, one doesn't have to transmit each arriving status in the update systems. To reduce the information age, one should transmit the latest status update. In this case, one can choose to retain only finitely many recent arrivals, implemented by a finite buffer size. This model was studied for a single source in [6], for multiple sources in [7], and with status deadlines in [16]. Generalization of the service time distribution to a Gamma distribution is considered in [17]. Status update transmissions over a network is considered for multiple parallel links in [8], and scheduling problem for multiple source updates over a single wireless channel is considered in [18].

In many unreliable systems, there is a finite probability of transmission failure. This affords another decision dimension of whether to continue transmission of the old status update, or to transmit a new update. Retransmission of latest updates for random arrivals is considered in [19]. Latest update transmission for a source with continuous deterministic arrivals, is considered in [20]. Multiple sources with parallel unreliable links are considered in [21]. Channel-aware source update is considered in [22]. Channel-state update is considered in [23]. Optimal update policy for energy harvesting source is considered in [24] and [25].

B. Main Contributions

1) We consider a discrete information theoretic model for age-limited communication, where the correlated source takes values in a finite alphabet communicating over a bit-wise erasure channel. This model allows for packet drops, in contrast to the previous queueing-theoretic studies which model channel uncertainty in the random packet communication time.
2) We consider three source-correlation-aware status update schemes for monitoring slowly varying sources over unreliable channels, and compare their impact on information timeliness at the receiver.
3) We compute the distribution of the sampled age for the proposed model and demonstrate a stochastic ordering on the information age of these update schemes for fixed finite length coding. Hence, we obtain an order on any performance metric that is a non-decreasing function of the age.
4) We conclude that for a fixed finite length code, the source correlation can be effectively exploited with receiver feedback. We find the threshold on the feedback cost, below which it is beneficial to encode differentially.
5) We utilize an information theoretic approximation for the update failure probability under random codes, to find tight approximations for the optimal codeword length for the above update schemes.

First two points were first considered in [1]. In this article, we have considered an alternative approach of studying the sampled age that provides generalized proofs for the results in [1]. This approach also leads to the last three contributions that appear only in this version.

II. System Model

We consider a physical process with state $M(t)$ at time t, being sensed by the communication source at discrete instants. We assume that the source encodes each sampled state into a

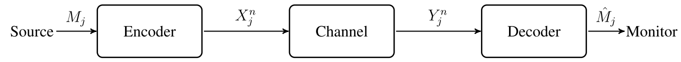


Fig. 1. We show an abstract discrete time communication model for a source with m-bit information M_j at time $t = (j-1)n+1$. The message M_j is encoded to an n-length codeword $X_j^n = (X_{(j-1)n+1}, \dots, X_{jn})$ and transmitted over an unreliable bit-wise *i.i.d.* binary symmetric erasure channel that outputs $Y_j^n = (Y_{(j-1)n+1}, \dots, Y_{jn})$. Each bit requires single channel use, and hence the output Y_j^n is obtained after n channel uses. From the output at time $t = jn+1$, the decoder finds an estimate $\hat{M}_j$ of the message transmitted n channel uses ago.

n-length update codeword for transmission over an unreliable channel to a remote monitor. We assume that only a single bit can be transmitted per channel use. Hence, the source can only transmit at periodic instants $\{(j-1)n+1 : j \in \mathbb{N}\}$, where $\mathbb{N}$ denotes the set of positive integers. Receiver receives the jth codeword at time $jn+1$ and estimates the source state. We illustrate the abstract system model in Fig. 1, and discuss each component in detail below.

A. Source

We assume that the physical process under consideration belongs to a finite alphabet, and hence the source message can be represented by finitely many bits, say $m \leqslant n$. Therefore, we can assume without any loss of generality that the physical process $M(t) \in \{0,1\}^m$ at any time t. We assume that the source samples the physical process at discrete instants $\{(j-1)n+1 : j \in \mathbb{N}\}$. The discrete time sampled process at the source is denoted by $\{M_j = M(n(j-1)+1) : j \in \mathbb{N}\}$. We assume that the physical process under consideration is highly temporally correlated, such that the difference in the sampled states at times $t-n$ and t is small for any time t. In particular, this difference $\delta M(t) \triangleq M(t) - M(t-n)$ can be represented by k bits where $1 \leq k \leq m$. Notice that, the number of bits needed to represent the difference would depend on the temporal correlation of the process, and the time-interval n.

B. Encoding

At any transmission instant t, the source message can either be the m-bit current state $M(t)$ or the k-bit difference $\delta M(t)$ of the current state $M(t)$ from the previously transmitted state $M(t-n)$. Depending on the update protocol, the source message at jth transmission instant $t = (j-1)n+1$ is encoded to an n length codeword $X_j^n = (X_{(j-1)n+1}, \dots, X_{jn})$ for each $j \in \mathbb{N}$. The encoded codeword corresponding to the true state $M(t)$ and the difference $\delta M(t)$ are called *true update* and *incremental update* respectively. Our analysis applies for any permutation invariant coding scheme, where the probability of decoding failure depends solely on the number of erasures in a codeword, and not their location.

C. Channel Model

We consider a bit-wise independent and identically distributed (*i.i.d.*) binary symmetric erasure channel, such that the channel output $Y_i \in \{0,1,e\}$ corresponding to the ith input bit $X_i \in \{0,1\}$ is given by

$$Y_i = e1_{\{Y_i \neq X_i\}} + X_i 1_{\{Y_i = X_i\}},$$

where e denotes an erasure symbol. Further, each bit of the update packet can be erased independently and identically with probability $\epsilon = \Pr\{Y_i \neq X_i\}$. Since each bit erasure $1_{\{Y_i \neq X_i\}}$ is an *i.i.d.* Bernoulli random variable, the number of bit erasures $E_j = \sum_{i=(j-1)n+1}^{jn} 1_{\{Y_i \neq X_i\}}$ in the jth update packet of length n has a binomial distribution,

$$\Pr\{E_j = \ell\} = \binom{n}{\ell} \epsilon^{\ell}(1-\epsilon)^{n-\ell}, \quad \text{for } \ell \in \{0, 1, \cdots, n\}.$$

D. Monitor

We measure the time in terms of channel use. We assume that each bit requires single channel use to reach from the source to the monitor. Hence, an n-length update packet is completely received at the monitor after n channel uses since its transmission, this corresponds to an update reception. That is, the jth update codeword sent at time $n(j-1)+1$ is received at instant $nj+1$.

E. Decoding

From the received channel output $Y_j^n = (Y_{(j-1)n+1}, \ldots, Y_{jn})$ at time $jn+1$, one can form an estimate $\hat{X}_j^n$. If the received update codeword gets decoded successfully, we call this reception as a successful update or successful reception. The corresponding update packet is referred to as a successful update. The event of decoding failure for the jth received codeword is denoted by

$$\xi_j \triangleq 1_{\{X_j^n \neq \hat{X}_j^n\}} \quad \text{for each } j \in \mathbb{N}.$$

Due to the *i.i.d.* nature of the erasure channel and the permutation invariant coding, the decoding failure events at the receiver are independent Bernoulli random variables. Conditioned on the number of erasures E_j in the jth codeword with r information bits and $n-r$ parity bits, the probability of decoding failure for a permutation invariant code is denoted by

$$P(n, n-r, E_j) \triangleq \mathbb{E}[\xi_j | n-r, E_j].$$

The number of information bits encoded to an n length update packet is m and k, for the true and incremental update respectively. Hence, the respective unconditional probabilities of decoding failure for true and incremental updates are

$$p_a = P(n, n-m) \triangleq \mathbb{E}P(n, n-m, E_j), \quad \text{and}$$
$$p_d = P(n, n-k) \triangleq \mathbb{E}P(n, n-k, E_j),$$

where, expectation is taken over the random variable E_j. In this article, we consider the erasure probability $\epsilon \in (0, 1)$. Since $k \leq m$, using coupling argument one can show that $0 < p_d \leq p_a < 1$.

Example 1 (Random coding): Even though our proposed analysis is valid for any symmetric coding scheme, we use a random coding scheme [26] for illustration throughout this work. For the random coding scheme, conditioned on the number of erasures E in a n-length codeword with r parity bits, the probability of decoding failure [26] is

$$P(n, r, E) = (1 - \prod_{i=0}^{E-1} (1 - 2^{i-r})) 1_{\{1 \leq E \leq r\}} + 1_{\{E > r\}}.$$

We are interested in the timeliness performance for small codewords used in real-time communication. Therefore, we have taken codeword length $n = 120$ inspired by the system parameters used in GSM based wireless links [27]. We take the information bits for the true and the incremental updates to be $m = 105$ and $k = 90$, respectively. For this choice of codeword lengths and information bits for random coding scheme and erasure probability $\epsilon = 0.1$, we get $p_d = 8.36 \times 10^{-5}, p_a = 0.289$. Unless specified otherwise, we will be using these values in the numerical results. For illustration, we have chosen the periodicity of true updates $q \in \{2, 6\}$, for the differential encoding scheme without feedback.

III. Update Transmission Schemes

We are interested in understanding the performance of the three update transmission schemes as discussed in Section I. To this end, we define γ_j be the indicator of the jth received update being a true update. Given an update is true or incremental, the update decoding failure events are independent with mean p_a and p_d respectively. Hence, it follows that $\mathbb{E}[\xi_j|\gamma_j] = p_a\gamma_j + p_d(1-\gamma_j)$. We describe the update schemes and their impact on update decoding failure events in detail, in the following.

A. True Updates

First we consider our benchmark scheme, where the source sends the true update at every transmission opportunity. That is, the source encodes m-bit state $M(t)$ to jth codeword X_j^n at jth transmission instant $t = (j-1)n+1$ for each $j \in N$. It follows that $\gamma_j = 1$ for each $j \in \mathbb{N}$. Hence for the true update scheme, the number of parity bits in each codeword is $n-m$, and the sequence of decoding failure indicators $(\xi_j : j \in \mathbb{N})$ is *iid* Bernoulli with common mean $\mathbb{E}[\xi_j] = p_a$ for each $j \in \mathbb{N}$.

B. Incremental Updates Without Feedback

In the second scheme, the source periodically sends the true update at times $\{(j-1)qn+1 : j \in \mathbb{N}\}$ for a fixed integral period $q \geq 2$. At the $q-1$ codeword transmission opportunities between the transmissions of two true updates, the source encodes the differential information to send incremental updates. That is, a transmitted update is true update if and only if $j-1$ is a multiple of q, that is $\gamma_j = 1_{\{j \bmod q = 1\}}$ for each $j \in \mathbb{N}$. Hence for the incremental updates without feedback, the collection of indicators $(\xi_j : j \in \mathbb{N})$ is an independent sequence with the non-stationary mean

$$\mathbb{E}[\xi_j|\gamma_j] = p_a 1_{\{j \bmod q = 1\}} + p_d 1_{\{j \bmod q \neq 1\}} \quad \text{for each } j \in \mathbb{N}.$$

C. Incremental Updates With Feedback

In the third scheme, we assume the availability of an immediate and accurate feedback from the monitor. In this scheme, the source begins transmission with a true update, i.e. $\gamma_0 = 1$. Subsequently, a true update is transmitted if and only if the source receives a negative feedback from the monitor, indicating the decoding failure of the last update. Source continues to send incremental updates if there is no negative feedback, indicating decoding success of last update. At the jth update transmission, ξ_{j-1} indicates the decoding failure of the $(j-1)$th update, and hence $\gamma_j = \xi_{j-1}$ for each $j \in \mathbb{N}$.

IV. Performance Metrics

For an erasure channel, there is no erroneous decoding at the receiver. Hence, there is either a decoding success or a decoding failure of the received message. The latest information available at the monitor is the last correctly decoded update. At time t, we denote the generation time of the last successfully decoded source state by $U(t)$. The generation time $U(t)$ remains constant until the reception of an update packet. Since it takes n time units for an update transmission, $U(t) = U(jn+1)$ for each $t \in \{jn+2, \dots, (j+1)n\}$. At an update packet reception instant $t = jn+1$, we have

$$U(t) = (t-n)(1-\xi_j) + U(t-n)\xi_j.$$

That is, the generation time $U(t)$ resets to $t-n$ at the successful decoding of an update codeword, and remains constant otherwise.

We quantify the timeliness of the update using the *information age* [3] denoted by the age $A(t)$ at time t as

$$A(t) \triangleq t - U(t) \tag{1}$$

at time t. From the above discussion, it follows that the age $A(t)$ resets to value n at the successful reception of any update, and is linearly increasing at all other instants. We denote the age process sampled at jth update codeword reception instant $jn+1$ by $A_j \triangleq A(jn+1)$. For simplicity, we assume to start from a successful reception. That is, the sampled age at the reception instant of the 0th update is $A_0 = n$. From the evolution of generation times, it follows that the age at the jth reception is

$$A_j = n(1-\xi_j) + (n + A_{j-1})\xi_j = n + A_{j-1}\xi_j.$$

The sampled age resets to n if the codeword is successfully decoded, otherwise it increases by n from the last sampled age. This implies $A_j \in \{n, 2n, 3n, \dots\}$ and $A_j = n$ if and only if $\xi_j = 0$. Further, the information age at any discrete instant is completely determined by the sample age at the last codeword reception instant. In particular for $t \in \{jn+1, \dots, (j+1)n\}$, we have

$$A(t) = A_j + (t - jn - 1).$$

Therefore, we only focus on the sampled age process denoted by $(A_j : j \in \mathbb{N})$. We consider the following performance metrics for the evaluation of update schemes, derived from the limiting distribution of the sampled information age.

A. Mean Information Age

First performance metric of interest is the limiting value of empirical average age defined as $\lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s)$. The cumulative sum of age between the two reception instants $jn+1$ and $(j+1)n+1$ can be expressed in terms of sampled age at the jth reception, as

$$\sum_{s=jn+1}^{(j+1)n} A(s) = n\left(A_j + \frac{(n-1)}{2}\right).$$

Summing over all $s \in \{1, \dots, t\}$ and choosing p such that $np+1 \le t \le n(p+1)$, we get

$$n\sum_{j=0}^{p-1}\left(A_j + \frac{(n-1)}{2}\right) \le \sum_{s=1}^{t} A(s) \le n\sum_{j=0}^{p}\left(A_j + \frac{(n-1)}{2}\right).$$

Dividing by t and taking limits, it follows that the difference in limiting averages of the age and the sampled age is a constant,

$$\lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t} A(s) = \lim_{N\to\infty}\frac{1}{N}\sum_{j=0}^{N-1} A_j + \frac{(n-1)}{2}.$$

B. Probability of Update Failure

Our second metric of interest is limiting empirical average of number of decoding failures. Recall that the indicator $\xi_j = 0$ if and only if the sampled age $A_j = n$, and hence $\xi_j = 1$ if and only if $A_j \ge 2n$. Using this fact, we can write the limiting average of decoding failures as

$$\lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N}\xi_j = \lim_{N\to\infty}\frac{1}{N}\sum_{j=1}^{N} 1_{\{A_j \ge 2n\}}.$$

It turns out that the decoding failure process is ergodic, and hence the limiting average of decoding failure events is the unconditional probability of an *update failure*.

C. Mean Delay

The capacity of the erasure channel under consideration is $1-\epsilon$, while we transmit one bit per channel use. That is, the arrival rate of bits is higher than the service rate, and we are transmitting at a rate higher than the channel capacity. This results in the average delay of the packets to be infinite. Indeed, let D_i be the delay of the ith information codeword. Then, we see that

$$D_i = n(1-\xi_i) + \infty\xi_i \quad \text{for each } i \in \mathbb{N}.$$

We will see in the following sections, that for the same system the limiting average age is finite. Furthermore, for the argument's sake if we only consider the delay of successfully received information codewords, then $D_i = n$ for each $i \in \mathbb{N}$ such that $\xi_i = 0$. This implies that average delay of each successful codeword is n, and hence this metric doesn't capture the impact of update schemes. Since, we have completely characterized the per packet delay, we will not consider this metric any further in this article. We see that age is a receiver metric, whereas delay is a per packet metric [22], [24].

V. Regenerative Process for the Update Schemes

Applying the renewal reward theorem [28] to suitable renewal reward processes, one can compute the limiting values of sampled age distribution for the different update schemes. From limiting distributions, one can derive several performance evaluation metrics for the update schemes.

We will consider only the sampled process at the reception instants $\{jn+1 : j \in \mathbb{N}\}$. Note that monitor can receive j updates by the time $t = jn+1$. Let $R_0 = 0$ and R_i be the number of update receptions until the ith successfully decoded true update. Note that $nR_i + 1$ is the time of ith successful reception of a true update, where

$$R_i = \inf\{j > R_{i-1} : \xi_j = 0, \gamma_j = 1\}, \quad i \in \mathbb{N}.$$

Let $T_i = R_i - R_{i-1}$ denote the number of update codeword receptions between two successfully decoded true update codewords. The sequence $(T_i : i \in \mathbb{N})$ is *i.i.d.* with $\mathbb{E}T_i < \infty$, for the individual schemes. Since $R_i = \sum_{\ell=1}^{i} T_\ell$, it follows that the random sequence $(R_i : i \in \mathbb{N}_0)$ is a discrete renewal process, where $\mathbb{N}_0 \triangleq \mathbb{N} \cup \{0\}$ denotes the set of non-negative integers. The ith renewal interval is denoted by $I_i \triangleq \{R_{i-1}, \dots, R_i - 1\}$, and has length T_i. Since the ith renewal instant R_i is sum of finitely many independent almost surely finite random variables $(T_1, \dots, T_i)$, it is also almost surely finite.

At the reception of a successful update packet, the age resets to n since the successfully received codeword was sent n channel uses ago. Hence the sampled age at ith renewal instant $A_{R_i} = n$ for each $i \in \mathbb{N}$. It follows that the sampled age sequence $(A_j : j \in \mathbb{N})$ is a regenerative process with the associated renewal sequence $(R_i : i \in \mathbb{N})$. Conditioned on the length of the ith renewal period T_i, we can find the number of times the sampled age A_j exceeds a threshold ln in the ith renewal interval I_i as $\sum_{j \in I_i} 1_{\{A_j \geq ln\}}$. The sampled age process A_j over a renewal interval I_i does not depend on the age evolution in other renewal intervals. In addition, the sum $\sum_{j \in I_i} 1_{\{A_j \geq ln\}}$ is bounded by the length of the renewal interval T_i. Hence, this sum is also *i.i.d.* and has finite mean. Therefore, this sum can be thought of as the reward for the renewal process $(R_i : i \in \mathbb{N}_0)$. We denote the limiting sampled age by A, where $\lim_{j\to\infty} A_j = A$ in distribution. Applying the renewal reward theorem [28] to the renewal process $(R_i : i \in \mathbb{N}_0)$ and the reward process $(1_{\{A_j \geq jn\}} : j \in \mathbb{N})$, we can compute the limiting distribution of the sampled age process for the three update schemes,

$$\Pr\{A \geq ln\} \triangleq \lim_{j\to\infty} \Pr\{A_j \geq ln\} = \frac{\mathbb{E}\sum_{j\in I_i} 1_{\{A_j \geq ln\}}}{\mathbb{E}T_i}. \quad (2)$$

From this limiting distribution, we can compute the limiting average age in terms of the limiting average sampled age as

$$A_{avg} = \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s) = \mathbb{E}A + \frac{(n-1)}{2}, \quad (3)$$

and the limiting average of update failure as

$$P_f = \lim_{N\to\infty} \frac{1}{N} \sum_{j=1}^{N} \xi_j = \Pr\{A \geq 2n\}. \quad (4)$$

We will show that there is a stochastic ordering on the limiting sampled age for the three schemes. Since above performance metrics are both increasing functions of the limiting sampled age, the stochastic ordering is preserved for these metrics.

Lemma 2: Let Z_i be the number of true updates received by the monitor in the ith renewal interval I_i. The sequence $(Z_i : i \in \mathbb{N})$ is i.i.d. *geometric with the success parameter $(1 - p_a)$ for all three update schemes under consideration.*

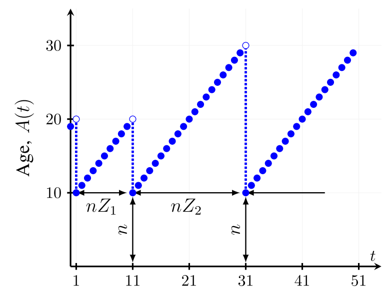


Fig. 2. This plot shows one sample path of the age process for the true updates scheme. We have taken the codeword length $n = 10$. First update sent at time 1 is decoded successfully at time $n + 1$, and hence $Z_1 = 1$. Second update sent at time $n + 1$, fails to get decoded at time $2n + 1$, leading to one decoding failure. The following update sent at time $2n + 1$ is received successfully at time $3n + 1$, and hence $Z_2 = 2$. Here, since only true updates are sent, number of receptions in interval I_i is $T_i = Z_i$. Consequently, $R_1 = 1$ and $R_2 = 3$.

Proof: Independence of $(Z_i : i \in \mathbb{N})$ follows from the independence of decoding failure events for true updates. In each renewal interval I_i, only a single true update is successfully decoded. Hence $Z_i - 1$ is the number of consecutive true update decoding failures before a success. Since the decoding failure events for true updates are *i.i.d.* Bernoulli with probability p_a, the result follows. ■

A. True Updates

For the true updates scheme, each update codeword contains the true state information. Hence, the number of receptions T_i in the interval I_i is equal to the number of true updates Z_i in this interval. It follows that the sequence of inter-renewal lengths $(T_i : i \in \mathbb{N})$ are *i.i.d.*.

In the ith renewal interval I_i, the age starts at value n from the R_{i-1}th reception and increases linearly until R_ith reception. We illustrate a sample path evolution of the information age for this scheme in Fig. 2. In particular,

$$A_{R_{i-1}+j} = jn, \text{ for } j \in \{0, \dots, T_i - 1\}.$$

Theorem 3: Let $(A)_+ \triangleq \max\{A, 0\}$. The limiting distribution of sampled age for the true updates scheme is

$$\Pr\{A \geq ln\} = \lim_{j\to\infty} \Pr\{A_j \geq ln\} = \frac{\mathbb{E}(Z_i - l + 1)_+}{\mathbb{E}Z_i}.$$

Proof: In each renewal interval I_i for a true update scheme, sampled age is n at the beginning of the renewal interval I_i, i.e. $A_{R_{i-1}} = n$. Further the samples age increases linearly in steps of n for other reception instants in this interval, i.e. $A_j = (j - R_{i-1} + 1)n$ for all $j \in I_i$. Hence, we can write the number of times the sampled age exceeds a threshold nl in the ith renewal interval as

$$\sum_{j\in I_i} 1_{\{A_j \geq nl\}} = \sum_{j=R_{i-1}}^{R_i - 1} 1_{\{j - R_{i-1} + 1 \geq l\}} = \sum_{j=0}^{Z_i - 1} 1_{\{j \geq l-1\}} = (Z_i - l + 1)_+.$$

Result follows from taking expectations and dividing it by the mean length of one renewal period. ■

Corollary 4: The limiting average age for the true updates scheme is

$$A_{avg} = \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s) = \tfrac{(n-1)}{2} + \tfrac{n}{(1-p_a)}.$$

Proof: Since the set of values each sampled age takes is $\{n, 2n, 3n, \dots\}$. This is also the range of values assumed by the limiting sampled age. Hence,

$$\mathbb{E}A = n\sum_{l\in\mathbb{N}} l \Pr\{A = ln\} = n\sum_{l\in\mathbb{N}} \Pr\{A \geq ln\}.$$

From the distribution of limiting sampled age defined in Theorem 3 and using monotone convergence theorem [29] to exchange summation and expectation, we get

$$\sum_{l\in\mathbb{N}} \Pr\{A \geq ln\} = \sum_{l\in\mathbb{N}} \frac{\mathbb{E}(Z_i - l + 1)_+}{\mathbb{E}Z_i} = \frac{\mathbb{E}\sum_{l=1}^{Z_i}(Z_i - l + 1)}{\mathbb{E}Z_i}.$$

Since the sum of first n positive integer is $\frac{n(n+1)}{2}$, we get

$$\mathbb{E}A = n\sum_{l\in\mathbb{N}} \Pr\{A \geq ln\} = \frac{n\mathbb{E}Z_i(Z_i+1)}{2\mathbb{E}Z_i}.$$

Result follows from (3), and the geometric distribution of Z_i. ∎

Corollary 5: The unconditional probability of decoding failure for true updates is p_a.

Proof: This follows directly from the observation that the probability of true update failure is p_a. Alternatively, since $Z_i \geq 1$, we have $\mathbb{E}(Z_i - 1)_+ = \mathbb{E}(Z_i - 1)$. Therefore, we can also deduce this corollary from (4), $P_f = \frac{\mathbb{E}(Z_i-1)_+}{\mathbb{E}Z_i} = \frac{\mathbb{E}(Z_i-1)}{\mathbb{E}Z_i} = 1 - (1-p_a) = p_a$. ∎

B. Incremental Updates Without Feedback

In this scheme, the encoder sends $q-1$ incremental updates between two true updates. Since Z_i true updates are transmitted between $(i-1)$th and ith successful reception of true update, the total number of update transmissions is $T_i = qZ_i$ in this interval I_i. From the *i.i.d.* geometric nature of the sequence $(Z_i : i \in \mathbb{N})$, it follows that the sequence $(T_i : i \in \mathbb{N})$ is also *i.i.d.* with finite mean. We observe that, after the first incremental update decoding failure, receiver cannot successfully decode the source state until the next successful reception of the true state. We define $\bar{W}_i$ to be the number of successful source state receptions in the ith renewal interval.

Lemma 6: For each renewal interval I_i, the number of successful receptions $\bar{W}_i \in \{1, \dots, q\}$ is independent of the number of true updates Z_i. Further, the sequence $(\bar{W}_i : i \in \mathbb{N})$ is i.i.d. *with truncated geometric distribution*

$$\Pr\{\bar{W}_i = k\} = (1-p_d)^{k-1}\left(p_d 1_{\{1\leq k<q\}} + 1_{\{k=q\}}\right).$$

Proof: We note that $\bar{W}_i - 1$ is the number of contiguous incremental updates, decoded successfully in the ith interval. Since in each renewal interval, at least one update is successfully received, $\bar{W}_i \geq 1$. Further, if $\bar{W}_i - 1 = q - 1$, then the next update contains the true state information. If this update is decoded successfully, then $Z_i = 1$ and the ith renewal occurs. Otherwise $Z_i > 1$, and all the subsequent incremental updates in this renewal period are useless at the monitor. From the independence of the channel realizations, it follows that $(\bar{W}_i : i \in \mathbb{N})$ is an *i.i.d.* sequence, and that $\bar{W}_i$ and Z_i are independent. Since the decoding success events of incremental updates are *i.i.d.* Bernoulli with probability $1-p_d$ and $\bar{W}_i \leq q$, the distribution of $\bar{W}_i$ is truncated geometric. ∎

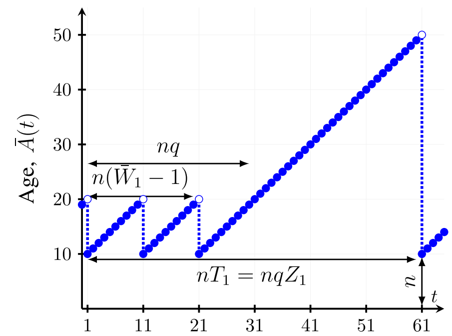


Fig. 3. This plot shows a sample path for the age process in one renewal interval, for the incremental updates and no feedback. We have taken the codeword length $n = 10$, and the period after which the true update is sent as $n(q-1) = 20$. In this example, the first $q-1$ incremental updates are successfully decoded, i.e. $\bar{W}_1 - 1 = 2$. The source sends the true update at the qth transmission opportunity, which fails to get decoded. Since the source has no feedback, it starts sending incremental updates for next $q-1$ transmission opportunities. Finally, the following true update is decoded successfully at the monitor, and hence $Z_1 = 2$.

In the ith renewal interval, the age is reset to n at discrete instants $nR_{i-1} + 1 + ln$ for $l \in \{0, 1, \dots, \bar{W}_i - 1\}$ because of the successful reception of the source state. The age grows linearly at all other points in the interval. We illustrate a sample path evolution of the age process for this scheme in Fig. 3. The sampled age at jth reception for this scheme is denoted $\bar{A}_j$, and hence we can write the sampled age $\bar{A}_{R_{i-1}+j}$ for $j \in \{0, \dots, T_i - 1\}$ in the ith renewal interval as

$$\bar{A}_{R_{i-1}+j} = n1_{\{j\leq \bar{W}_i - 1\}} + n(j + 2 - \bar{W}_i)1_{\{\bar{W}_i \leq j \leq qZ_i - 1\}}. \tag{5}$$

Theorem 7: The limiting distribution of sampled age for the incremental updates without feedback is $\Pr\{\bar{A} \geq ln\} = \lim_{j\to\infty}\Pr\{\bar{A}_j \geq ln\} = 1_{\{l=1\}} + \frac{\mathbb{E}(qZ_i - \bar{W}_i - l + 2)_+}{q\mathbb{E}Z_i}1_{\{l\geq 2\}}$.

Proof: In each renewal interval I_i for incremental update without feedback scheme, sampled age is n at the beginning of the renewal interval I_i, i.e., $\bar{A}_{R_{i-1}} = n$. The sampled age resets to n at the reception of each successful differential update, i.e. $A_{R_{i-1}+j} = n$ for $j \in \{1, \dots, \bar{W}_i - 1\}$. Further the sampled age increases linearly in steps of n at all other reception instants in this interval as given by (5). Since $\bar{A}_j \in \{n, 2n, 3n, \dots\}$, it follows that $1_{\{\bar{A}_j \geq n\}} = 1$ for all $j \in \mathbb{N}$. Therefore, we only consider the indicators $1_{\{\bar{A}_j \geq ln\}}$ for $l \geq 2$. We can write the number of times the sampled age exceeds a threshold ln for $l \geq 2$ in the ith renewal interval as

$$\sum_{j\in I_i} 1_{\{\bar{A}_j \geq ln\}} = \sum_{\ell=0}^{qZ_i-1} 1_{\{\bar{A}_{R_{i-1}+\ell} \geq nl\}} = \sum_{\ell=\bar{W}_i}^{qZ_i-1} 1_{\{\ell+2-\bar{W}_i \geq l\}}$$
$$= (qZ_i - \bar{W}_i - l + 2)_+.$$

Result follows from taking expectations and dividing it by the mean length of one renewal period. ∎

Corollary 8: The limiting empirical average age for incremental updates without feedback is

$$\bar{A}_{avg} = \frac{(n-1)}{2} - n\left(\mathbb{E}\bar{W}_i - \frac{3}{2}\right) + \frac{nq\mathbb{E}Z_i^2}{2\mathbb{E}Z_i} + \frac{n\mathbb{E}\bar{W}_i(\bar{W}_i-1)}{2q\mathbb{E}Z_i}.$$

Proof: Since the limiting sampled age assumes values in the set $\{n, 2n, 3n, \dots\}$, we have $\mathbb{E}\bar{A} = n\sum_{l\in\mathbb{N}} \Pr\{\bar{A} \geq ln\}$. From the distribution of limiting sampled age defined in Theorem 7, and application of monotone convergence theorem for exchanging summation and expectation,

$$\sum_{l\in\mathbb{N}} \Pr\{\bar{A} \geq ln\} = n + \sum_{l\geq 2} \frac{\mathbb{E}(qZ_i - \bar{W}_i - l + 2)_+}{q\mathbb{E}Z_i}.$$

Using the formula for summation of first n positive integer, we get

$$\mathbb{E}\bar{A} = n + \frac{n\mathbb{E}(qZ_i - \bar{W}_i)(qZ_i - \bar{W}_i + 1)}{2q\mathbb{E}Z_i}.$$

Result follows from (3) and independence of Z_i and $\bar{W}_i$. ■

Corollary 9: The unconditional probability of update failure for incremental updates without feedback is $\bar{P}_f = \frac{\mathbb{E}(qZ_i - \bar{W}_i)}{q\mathbb{E}Z_i}$.

Proof: Since $\bar{W}_i \leq q$ and $Z_i \geq 1$, it follows that $(qZ_i - \bar{W}_i)_+ = (qZ_i - \bar{W}_i)$. Hence, we get the unconditional probability of update failure from (4) and the limiting sampled age distribution for incremental updates without feedback. ■

C. Incremental Updates With Feedback

In this scheme, the source receives an immediate and accurate feedback from the monitor, at the instants of decoding failure. The source transmits incremental updates until it gets a decoding failure feedback from the monitor. From the next transmission opportunity onwards, the source keeps sending true state updates until a successful reception. We let Z_i and W_i respectively denote the number of true and incremental updates sent in the interval I_i, then $T_i = Z_i + W_i$. The process $(Z_i : i \in \mathbb{N})$ is *i.i.d.* geometric with success parameter $(1-p_a)$ as before. From the independence of erasure channel, it follows that Z_i and W_i are independent for each $i \in \mathbb{N}$. We have the following lemma for the number of incremental updates.

Lemma 10: v The number of incremental updates $(W_i : i \in \mathbb{N})$ are *i.i.d.* geometric with the success parameter p_d.

Proof: Independence of W_i's follows from the independence of the erasure channel. Further, $W_i - 1$ is the number of incremental updates before the first decoding failure in the ith renewal interval. Since the decoding failure events are *i.i.d.* Bernoulli with probability p_d for incremental updates, the result follows. ■

In the ith renewal interval, the age is reset to n at times $nR_{i-1} + 1 + ln$ for $l \in \{0, 1, \dots, W_i - 1\}$, corresponding to the instants when the source state can be successfully decoded from the incremental updates. Age grows linearly otherwise at all other points in the interval. We illustrate a sample path evolution of the age process for this scheme in Fig. 4. Denoting the age for this scheme by $\hat{A}_j$ at the jth update reception instant, we can write the sampled age $\hat{A}_{R_{i-1}+j}$ for $j \in \{0, \dots, T_i - 1\}$ in the ith renewal interval as

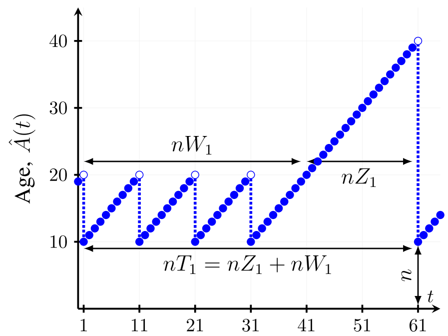


Fig. 4. This plot shows a sample path of the age process in one renewal interval, for the incremental updates with feedback. We have taken the codeword length $n = 10$. In this example, first $W_1 - 1 = 3$ incremental updates are successfully decoded. Fourth incremental update sent at time $3n + 1$ fails to get decoded. The source starts sending the true updates from the next transmission opportunity at $4n+1$. True updates sent at instant $4n+1$ fails to get decoded, with the first success after $Z_1 = 2$ transmissions.

$$\hat{A}_{R_{i-1}+j} = n1_{\{j\leq W_i - 1\}} + n(j + 2 - W_i)1_{\{W_i \leq j\}}. \quad (6)$$

Theorem 11: The limiting distribution of sampled age for incremental updates with feedback is $\Pr\{\hat{A} \geq ln\} = \lim_{j\to\infty} \Pr\{\hat{A}_j \geq ln\} = 1_{\{l=1\}} + \frac{\mathbb{E}(Z_i - l + 2)_+}{\mathbb{E}W_i + \mathbb{E}Z_i} 1_{\{l\geq 2\}}$.

Proof: In each renewal interval I_i for incremental update with feedback scheme, sampled age is n at the beginning of the renewal interval I_i, i.e. $\hat{A}_{R_{i-1}} = n$. The sampled age resets to n at the reception of each successful differential update, i.e. $A_{R_{i-1}+j} = n$ for $j \in \{0, \dots, W_i - 1\}$. Further the sampled age increases linearly in steps of n at all other reception instants in this interval as given by (6). Since $\hat{A}_j \in \{n, 2n, 3n, \dots\}$, it follows that $1_{\{\hat{A}_j \geq n\}} = 1$ for all $j \in \mathbb{N}$. Therefore, we only consider the indicators $1_{\{\hat{A}_j \geq ln\}}$ for $l \geq 2$. We can write the number of times the sampled age exceeds a threshold nl for $l \geq 2$ in the ith renewal interval as

$$\sum_{j\in I_i} 1_{\{\hat{A}_j \geq nl\}} = \sum_{\ell=0}^{Z_i+W_i-1} 1_{\{\hat{A}_{R_{i-1}+\ell} \geq nl\}} = \sum_{\ell=W_i}^{Z_i+W_i-1} 1_{\{\ell+2-W_i \geq l\}} = (Z_i - l + 2)_+.$$

Result follows from taking expectations and dividing by $\mathbb{E}Z_i$. ■

Corollary 12: The limiting empirical average age for incremental updates with feedback is

$$\hat{A}_{avg} = \frac{(n-1)}{2} + n + \frac{n\mathbb{E}Z_i(Z_i+1)}{2\mathbb{E}(Z_i + W_i)}.$$

Proof: Since the set of values limiting sampled age takes is $\{n, 2n, 3n, \dots\}$, we have $\mathbb{E}\hat{A} = n\sum_{l\in\mathbb{N}} \Pr\{\hat{A} \geq nl\}$. From the distribution of limiting sampled age defined in Theorem 11 and application of monotone convergence theorem, we get

$$\sum_{l\in\mathbb{N}} \Pr\{\hat{A} \geq nl\} = n + \sum_{l\geq 2} \frac{\mathbb{E}(Z_i - l + 2)_+}{\mathbb{E}W_i + \mathbb{E}Z_i}.$$

Since the summation of first n positive integers is $\frac{n(n+1)}{2}$, we get

$$\mathbb{E}\hat{A} = n + \frac{n\mathbb{E}Z_i(Z_i+1)}{2\mathbb{E}(Z_i+W_i)}.$$

Result follows from (3) and independence of Z_i and W_i. ∎

Corollary 13: The unconditional probability of update failure for incremental updates with feedback is $\hat{P}_f = \frac{\mathbb{E}Z_i}{\mathbb{E}(Z_i+W_i)}$.

Proof: Since $Z_i \geq 1$, it follows that $(Z_i)_+ = Z_i$. Hence, we get the unconditional probability of update failure from (4) and the limiting sampled age distribution for incremental updates without feedback. ∎

VI. Comparison of the Update Schemes

We now analytically compare the performance of the three source state encoding schemes, under the two metrics of interest. Since the number of parity bits in an incremental update is higher than the true update, the probability of update failure is lower for the incremental update. Using $\mathbb{E}W_i = \frac{1}{p_d}$ and $\mathbb{E}Z_i = \frac{1}{1-p_a}$, we can rewrite the fact $p_d \leq p_a$, as the inequality

$$(\mathbb{E}W_i)^{-1} + (\mathbb{E}Z_i)^{-1} \leq 1. \tag{7}$$

Further, we observe that $\bar{W}_i \leq q$. This can be re-written for any $l \geq 2$ as

$$\bar{W}_i - 1 \leq (q-1)(l-1). \tag{8}$$

We also observe that due to memoryless property of the geometric random variable Z_i, for any $l \geq 2$

$$1 - (\mathbb{E}Z_i)^{-1} = \Pr\{Z_i > 1\} = \Pr\{Z_i \geq l | Z_i \geq l-1\}. \tag{9}$$

We aggregate the above results to prove a stochastic ordering on the sampled age process for the three update schemes.

A. Stochastic Ordering of Limiting Age

We show that for any arbitrary streaming update source, the limiting age for the three schemes can be ordered. A random variable X is said to be stochastically larger [30] than another random variable Y, and denoted by $X \geq_{\text{st}} Y$, if $\Pr\{X > x\} \geq \Pr\{Y > x\}$ for all $x \in \mathbb{R}$.

Theorem 14: The limiting sampled age for the true, the incremental without feedback, and the incremental with feedback schemes are denoted by $A, \bar{A}, \hat{A}$ respectively, and have the following stochastic ordering $\hat{A} \leq_{st} A \leq_{st} \bar{A}$.

Proof: Let's fix $l \in \mathbb{N}$. When $l = 1$, the complementary distribution functions for each of the three update schemes are equal to unity. Therefore, we consider $l \geq 2$, without any loss of generality. We first compare the true and the incremental updates with feedback. From the memoryless property (9) of the number of true updates Z_i in the ith renewal interval, it follows that $\Pr\{Z_i \geq l-1\} = \Pr\{Z_i \geq l-1 | Z_i \geq l-2\}\Pr\{Z_i \geq l-2\} = \left(1 - \frac{1}{\mathbb{E}Z_i}\right)\Pr\{Z_i \geq l-2\}$ for $l \geq 2$. We note that the equality in the above expression holds trivially for $l = 2$. Hence, we can rewrite this result as $\Pr\{Z_i - l + 1 \geq k\} = \left(1 - \frac{1}{\mathbb{E}Z_i}\right)\Pr\{Z_i - l + 2 \geq k\}$ for any $k \geq 0$. Summing both sides over $k \in \mathbb{N}$, and consequently dividing both sides by $\mathbb{E}Z_i$, we get

$$\frac{\mathbb{E}(Z_i - l + 1)_+}{\mathbb{E}Z_i} = \left(1 - \frac{1}{\mathbb{E}Z_i}\right)\frac{\mathbb{E}(Z_i - l + 2)_+}{\mathbb{E}Z_i}.$$

To show the stochastic ordering for incremental and true updates, it suffices to show that $(1 - \frac{1}{\mathbb{E}Z_i})\frac{1}{\mathbb{E}Z_i} \geq \frac{1}{\mathbb{E}Z_i + \mathbb{E}W_i}$. This is equivalent to showing $1 - \frac{1}{\mathbb{E}Z_i} \geq 1 - \frac{\mathbb{E}W_i}{\mathbb{E}W_i + \mathbb{E}Z_i}$. However, this holds from the failure probability inequality (7) for true and incremental updates, which implies $\mathbb{E}Z_i + \mathbb{E}W_i \leq \mathbb{E}Z_i\mathbb{E}W_i$. Therefore, we get the first stochastic ordering.

Next we consider the true and the incremental updates without feedback. The number of incremental updates in a renewal interval is truncated geometric for the no feedback scheme. From (8), we obtain $(Z_i - l + 1)_+ \leq \frac{1}{q}\left(qZ_i - \bar{W}_i - l + 2\right)_+$. The second stochastic ordering follows from taking expectation on both sides, and dividing by $\mathbb{E}Z_i$. ∎

This theorem tells us that for any metric that is an increasing function of sampled age, the three schemes maintain their performance order. In particular, we have the following corollary.

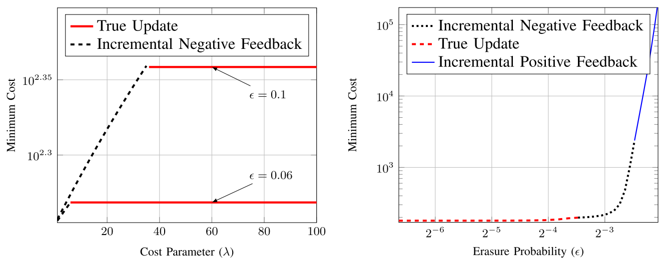
Corollary 15: The limiting average ages and probability of the update failures have the same order for the three update schemes

$$\lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t}\hat{A}(s) \leq \lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t}A(s) \leq \lim_{t\to\infty}\frac{1}{t}\sum_{s=1}^{t}\bar{A}(s),$$
$$\hat{P}_f \leq P_f \leq \bar{P}_f.$$

Proof: Cumulative sum of age in the time duration for jth update reception is $\sum_{t=jn+1}^{(j+1)n} A(s) = nA_j + \frac{n(n-1)}{2}$, and the event of decoding failure for jth update is $\{\xi_j = 1\} = \{A_j \geq 2n\}$. Therefore, the empirical average age and the number of update failures in any renewal interval, are increasing functions of the sampled age. From the Theorem 14 the limiting sampled age of all three update schemes are stochastically ordered. Further, it follows from [30, Proposition 9.1.2] that the mean value of the non-decreasing function of stochastically ordered random variables retains the same order. From the ergodicity of decoding events, it follows that the limiting empirical average converges to the mean, and the result follows. ∎

B. Feedback Overhead

The above analysis suggests that to minimize both age metrics, one should transmit true updates when there is no feedback, and resort to incremental updates when feedback is available. However, even when available, the feedback may be expensive [31]–[33]. In an incremental update scheme with negative feedback, the number of negative feedback messages equal the number of true updates in a renewal interval. Hence, the mean number of negative feedback messages in a renewal interval is $\frac{\mathbb{E}Z_i}{\mathbb{E}Z_i + \mathbb{E}W_i}$. The incremental update scheme can also be supported by sending positive feedback, with the number of messages equaling the number of incremental updates in a renewal interval. The mean number of positive feedback messages in a renewal interval is $\frac{\mathbb{E}W_i}{\mathbb{E}Z_i + \mathbb{E}W_i}$. It follows that the receiver should send "rare" feedback messages, to minimize the feedback overhead. This translates to sending negative feedback when the channel is good, and positive feedback



(a) Values of erasure probability $\epsilon \in \{0.06, 0.1\}$.

(b) Feedback cost $\lambda = 25$.

Fig. 5. Variation of the minimum cost with the feedback cost λ and erasure probability ϵ, for $m = 105, k = 90, n = 120$ bits.

when the channel is bad. That is, to reduce feedback overhead one can select positive feedback if $\mathbb{E}W_i \leq \mathbb{E}Z_i$, and negative feedback otherwise.

To consider the impact of feedback overhead, we consider minimization of the cost function which is a linear combination of the age and the feedback overhead. We let a fixed scalar λ represent the cost of feedback overhead in terms of per unit average age. Hence, one chooses to send incremental update with feedback if

$$\hat{A}_{avg} + \lambda \frac{\min\{\mathbb{E}Z, \mathbb{E}W\}}{\mathbb{E}Z + \mathbb{E}W} \leq A_{avg}.$$

Otherwise, one sends true updates. We note that, the limiting average of number of feedback messages is given by renewal reward theorem as the ratio of mean number of feedback messages $\min\{\mathbb{E}Z, \mathbb{E}W\}$ and the mean number of updates $\mathbb{E}Z + \mathbb{E}W$ in a renewal interval. It follows that the minimum cost of update transmission is

$$\min\left\{A_{avg}, \hat{A}_{avg} + \lambda \frac{\mathbb{E}Z}{\mathbb{E}Z + \mathbb{E}W}, \hat{A}_{avg} + \lambda \frac{\mathbb{E}W}{\mathbb{E}Z + \mathbb{E}W}\right\}.$$

We have plotted this minimum cost with respect to feedback cost λ in Fig. 5(a), which illustrates that it is cheaper to send the true updates, if the feedback cost λ is higher than a certain threshold. Beyond this threshold, the feedback cost parameter λ has no impact on the minimum cost since the true update scheme doesn't employ any feedback.

We also plot the minimum update transmission cost (aggregate of age and feedback cost) with respect to the bit-erasure probability ϵ for feedback cost parameter $\lambda = 25$ in Fig. 5(b). There are three different possible regimes for this curve. Red regime identifies the values of erasure probability such that transmission of true update is the most cost effective solution. This regime corresponds to $\epsilon \leq 0.09$ in Fig. 5(b). The blue and the dotted black regimes identify the differential update schemes, where the receiver sends positive feedback (for differential update successfully decoded) and negative feedback (for update failure) respectively. When the channel is not very poor, it is cheaper to send negative feedback, and this corresponds to the values of channel erasure probability $\epsilon \in (0.09, 0.18)$. When the channel deteriorates, differential update with positive feedback is employed, since there are fewer positive than negative feedbacks for such channels. In general, the selection of negative/positive/no feedback depends on the channel quality and the feedback cost.

VII. Optimal Code Length

The optimal code-length depends on the coding and the update scheme. We first derive an approximation for update decoding failure as a function of codeword length for random codes. Thereafter, we find the approximate optimal code length for random coding in all three update schemes.

Lemma 16: For sufficiently small erasure probability ϵ and sufficiently large parity $n - m$ in an n-length codeword with random coding, we can approximate the probability of decoding failure for the codeword in terms of $\bar{\epsilon} = 1 - \epsilon$ as

$$P(n, n-m) \approx 2^{-n\bar{\epsilon}+m} 1_{\{n\bar{\epsilon} > m\}} + 1_{\{n\bar{\epsilon} \leq m\}}. \quad (10)$$

Proof: The number of erasures E in an n realizations of an *i.i.d.* erasure channel with erasure probability ϵ is a Binomial random variable with parameters (n, ϵ) and mean $n\epsilon$. Ignoring the fact that mean number of erasures may not be an integer, we call it to be the effective number of erasures in a codeword, and denote it by $e^* = n\epsilon$. For sufficiently large n and small ϵ, we can approximate the weighted binomial coefficient $\binom{n}{k}\epsilon^k(1-\epsilon)^{n-k} \approx 2^{-nD(\frac{k}{n}||\epsilon)}$, where $D(p||q) = p\log\frac{p}{q} + \bar{p}\log\frac{\bar{p}}{\bar{q}}$ is the KL distance between two distributions $\{p, \bar{p}\}$ and $\{q, \bar{q}\}$ on binary random variables. Hence, we can write the unconditional probability of decoding failure for the n length code with $n - r$ information bits as

$$P(n, r) = \mathbb{E}P(n, r, E) \approx \sum_{e=0}^{n} P(n, r, e) 2^{-nD(\frac{e}{n}||\epsilon)}. \quad (11)$$

Since $D(p||q) \geq 0$ with equality at $p = q$, we know that all the terms in the above summation have non-positive exponents. For sufficiently large n, the above sum of exponentials is dominated by the slowest decaying exponential. The slowest decaying exponential corresponds to the number of effective erasures $e = e^*$, and hence $P(n, n-m) \approx P(n, n-m, e^*)$. When the number of erasures e exceeds the number of parity bits r, the probability of error $P(n, r, e) = 1$. Error probability $P(n, r, e)$ is less than unity in the case when $e < r$. In this case, we approximate $1 - \prod_{i=0}^{e-1}(1-\alpha_i)$ by its first order terms $\sum_{i=0}^{e-1}\alpha_i$ where $\alpha_i << 1$ for each i. When r is sufficiently large, we have $2^{i-r} << 1$ for each $i \in \{0, \dots, e-1\}$. Hence, we can approximate the probability of decoding failure given e erasures for parity bits $r > e$ as

$$P(n, r, e) = 1 - \prod_{i=0}^{e-1}(1 - 2^{i-r}) \approx \sum_{i=0}^{e-1} 2^{i-r} = (2^{e-r} - 2^{-r}).$$

For $r = n - m$, the condition $n\bar{\epsilon} > m$ is equivalent to $n\epsilon > r$. In this case, we use $P(n, n-m) \approx P(n, n-m, e^*)$ and the fact that $2^{n\epsilon - r} >> 2^{-r}$ for large r to get the last approximation. ∎

For a fixed number of actual information bits m and differential information bits k, the decoding failure probabilities for actual and differential updates are given by $p_a = P(n, n-m)$ and $p_d = P(n, n-k)$ respectively. By Lemma 16, we know that $p_a \approx 1$ when $n\bar{\epsilon} \leq m$, and in this case average age is arbitrarily large for all three update schemes. We will show that there exists an optimal code-length for actual updates and the incremental updates with feedback when the code-length is sufficiently large, that is $n\bar{\epsilon} > m$. In the region $n\bar{\epsilon} > m$, we can use the approximation $p_a \approx 2^{m-n\bar{\epsilon}}$ and $p_d \approx 2^{k-n\bar{\epsilon}}$ from Lemma 16. In the following theorem, we would treat these approximations as equalities for the sake of clarity, and write $\ln p_a = (m - n\bar{\epsilon}) \ln 2$ and $\ln p_d = (k - n\bar{\epsilon}) \ln 2$. We observe that $p_d < p_a \in (0,1)$ and are both monotonically decreasing in n for fixed $m > k$. In terms of $\alpha = 1 - 2^{k-m}$, we can write $p_d = (1-\alpha)p_a$ where $\alpha \in (0,1)$.

Theorem 17: Let $m > \max\{k, \frac{2}{\ln 2}\}$*, and for incremental updates without feedback* $p_d \approx 0$*. Then for all three update schemes, there exists a unique code-length* $n^* \in \{n \in \mathbb{N} : n \geq \frac{m+1}{\bar{\epsilon}}\}$ *that minimizes the average age.*

Proof: Treating codeword-length n as a real variable, we will show for all three update schemes under consideration, the average age is a convex function of the code-length n for the prescribed region and hence has a unique minimum. This is achieved by showing the non-negativity of the second derivative of average age with respect to code-length. The unique optimal code-length n^* corresponds to the point where the average age has derivative zero.

From Corollary 4, we know the limiting average age for true update scheme is $A_{avg} + \frac{1}{2} = n(\frac{1}{2} + \frac{1}{1-p_a})$. We can write the first two derivatives of this average age with respect to n as $\frac{dA_{avg}}{dn} = \frac{1}{2} + \frac{1}{(1-p_a)} + \frac{n}{(1-p_a)^2}\frac{dp_a}{dn}$, and $\frac{d^2A_{avg}}{dn^2} = \frac{2p_a\bar{\epsilon}\ln 2}{(1-p_a)^3}\left(\left(\frac{n\bar{\epsilon}}{2/\ln 2} - 1\right) + p_a\left(1 + \frac{n\bar{\epsilon}}{2/\ln 2}\right)\right)$. Since $n\bar{\epsilon} > m > \frac{2}{\ln 2}$, the second derivative $\frac{d^2A_{avg}}{dn^2}$ is always positive, and the optimal n^* is the unique solution to the following implicit equation $(2-p_a)^2 = 1 + 2p_a n\bar{\epsilon}\ln 2$.

For the incremental updates without feedback, we would approximate the decoding failure probability of incremental update by $p_d \approx 0$. This is a good approximation for $k < m$, and this implies $\bar{W}_i \approx q$ for each renewal cycle i. Utilizing this approximation in Corollary 8, we can write the limiting average age for this scheme as $\bar{A}_{avg} + \frac{1}{2} \approx n\left[\frac{5-3q}{2} + \frac{q}{\bar{p}_a} + \frac{(q-1)\bar{p}_a}{2}\right]$. We can write the first two derivatives as $\frac{d\bar{A}_{avg}}{dn} = \frac{(5-3q)}{2} + \frac{q}{\bar{p}_a} + \frac{(q-1)\bar{p}_a}{2} + n\left[\frac{q}{\bar{p}_a^2} - \frac{(q-1)}{2}\right]\frac{dp_a}{dn}$ and $\frac{d^2\bar{A}_{avg}}{dn^2} = -2\frac{dp_a}{dn}\left(\frac{n\bar{\epsilon}}{\frac{2}{\ln 2}} - 1\right)\left(\frac{q}{\bar{p}_a^2} - \frac{(q-1)}{2}\right) + \frac{2nq}{\bar{p}_a^3}\left(\frac{dp_a}{dn}\right)^2$. For $n\bar{\epsilon} > m > \frac{2}{\ln 2}$, we see that $\frac{d^2\bar{A}_{avg}}{dn^2} > 0$ and the average age is convex. We conclude that the approximate age-optimal codeword-length n^* is the unique solution of $\frac{(5-3q)}{2} + \frac{q}{\bar{p}_a} + \frac{(q-1)\bar{p}_a}{2} = \left(\frac{q}{\bar{p}_a^2} - \frac{(q-1)}{2}\right) n p_a \bar{\epsilon}\ln 2$.

From Corollary 12, we write the limiting average age of incremental update with feedback scheme as $\hat{A}_{avg} + \frac{1}{2} = n(\frac{3}{2} + \frac{1}{(1-p_a)} - \frac{1}{(p_d+1-p_a)})$. We observe that we can write the first derivative of limiting average age in this case as $\frac{d\hat{A}_{avg}}{dn} = \frac{3}{2} + \frac{d}{dn}\left(\frac{n}{1-p_a} - \frac{n}{(1-p_a\alpha)}\right)$. It follows that

$$-\frac{\frac{d^2\hat{A}_{avg}}{dn^2}}{\frac{dp_a}{dn}} = \frac{2}{(1-p_a)^3}\left(\left(\frac{n\bar{\epsilon}}{\frac{2}{\ln 2}} - 1\right) + p_a\left(\frac{n\bar{\epsilon}}{\frac{2}{\ln 2}} + 1\right)\right) - \frac{2\alpha}{(1-\alpha p_a)^3}\left(\left(\frac{n\bar{\epsilon}}{\frac{2}{\ln 2}} - 1\right) + \alpha p_a\left(\frac{n\bar{\epsilon}}{\frac{2}{\ln 2}} + 1\right)\right).$$

Since $n\bar{\epsilon} \geq m+1$, we have $p_a^2 \leq \frac{1}{4}$. In particular, it implies that $1 - 3p_a(p_a - p_d) > 0$. We further notice that the second derivative is positive for $n\bar{\epsilon} > m > \frac{2}{\ln 2}$ if $\frac{p_a}{(1-p_a)^3} \geq \frac{\alpha p_a}{(1-\alpha p_a)^3}$, which holds true since $\frac{(1-\alpha p_a)^3}{(1-p_a)^3} > 1 > \alpha$. Hence, the optimal code-length n^* for this case is given by the unique solution to the implicit equation

$$\frac{3}{2} + \frac{1}{1-p_a} - \frac{1}{1-p_a+p_d} = \frac{np_a\bar{\epsilon}\ln 2}{(1-p_a)^2} - \frac{n(p_a-p_d)\bar{\epsilon}\ln 2}{(1-p_a+p_d)^2}.$$

■

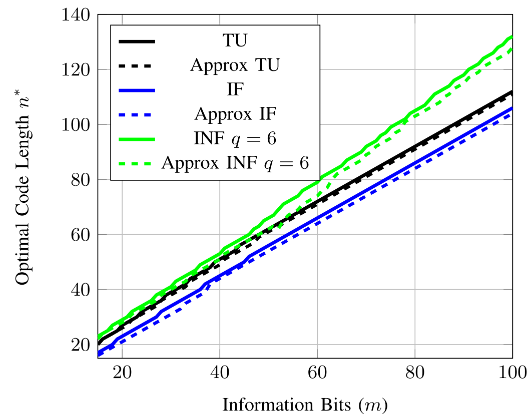


Fig. 6. We plot the variation of average-age minimizing code length n^* with respect to information bits $m \in \{15, \ldots, 100\}$ for the three update schemes when the differential information is fixed as $k = 10$ bits and the erasure probability $\epsilon = 0.1$. TU, IF, and INF stand for true updates, incremental updates with feedback, and incremental updates without feedback respectively.

We verify the tightness of our proposed approximation in Fig. 6, for the system parameters used in this paper. We have plotted the numerically obtained optimal code length n^* for all three update transmission schemes, together with the corresponding approximate value proposed in Theorem 17, as a function of information bits m. We observe that the approximations remain tight for all three schemes, even with the increase in the number of information bits m.

VIII. Conclusion and Future Work

We considered a slowly varying source sending real-time updates over a single unreliable link, modeled by an *i.i.d.* bit-wise binary symmetric erasure channel. We compared the timeliness performance of the differential encoding with and without feedback, to the true state encoding. We found that for a fixed finite length code the differential encoding is better than the actual state encoding, if there is an accurate and immediate feedback, and worse when there is no feedback. Our work can be generalized to accurate but delayed feedback, when the source remains slowly varying during the feedback duration. If the feedback delay is a multiple of codeword transmission duration, then one can split transmission into multiple parallel streams each following the proposed update protocol. In addition, the proposed model can also be utilized for the erasure feedback channels. In this case, one can continue to send a single bit feedback per transmission, and all the update protocols send a true update when the feedback is erased. It would also be interesting to explore joint source channel codes that optimize the timeliness performance for the structured sources.

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