Variable Length Differential Encoding for Real-Time Status Updates

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***Abstract*— Timely reception of information is of utmost importance in many cyber-physical systems. We study timely status update schemes for a single source. We consider a scheme which exploits the temporal correlation in source messages to send differential information to the receiver, over an unreliable channel. In this scheme, the source sends the true state information, interleaved with differential messages. We observe that the differential encoding improves the timeliness performance even in the absence of feedback, if the codeword lengths for both differential and true messages are chosen judiciously.**

***Index Terms*— Age of information, renewal theory, erasure channel, differential encoding, block codes.**

I. Introduction

TIMELY reception of information is critical for many cyber-physical systems that require real-time estimation or control. A typical physical source of information always has some information to transmit, and we refer to it as *always on* source. If the source information rate is larger than the channel capacity, there is no possibility of reliable communication of all source messages over such channels. In such a setting, one can potentially measure performance of different transmission schemes by delay of successively received packets and loss rates. An alternative metric that well captures the timeliness performance of transmission schemes is called *age of information* [1]. At time t, the information age at the receiver is denoted by

$$A(t) \triangleq t - U(t), \tag{1}$$

where $U(t)$ denotes the generation time of last successfully decoded update. This is related to age metric in renewal theory [2]. In fact, if each successful information reception event is a renewal instant, the age of information is the current age of last renewal. The study of timeliness in real-time systems has garnered a lot of attention in the past decade. In most early works, scheduling of status updates under various queueing models was considered in [3]–[5], which aims to minimize the average age. Status updates for multiple sources are considered in [6]–[9], and for multiple sources with memory in [10]–[12]. Variants of automatic repeat request (ARQ) protocols to reduce information age for single server queues are considered in [13]–[15].

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We consider an unreliable bit-wise channel model popular in information theory literature. Specifically, we focus on the simplest non-trivial unreliable channel, the binary erasure channel. This channel model has been considered for timely status updates in [16]–[21]. Forward error correction with hybrid ARQ for age reduction is considered in [16], [18], [21], for minimum age without feedback in [19], for differential encoding of sources with memory in [17], [20], and with source sampling in [22], [23]. We assume that the source sends coded update messages. We notice that high redundancy implies a lower chance of update failure and hence higher likelihood of age reduction after a single codeword reception. However, if the redundancy is very high, the message itself is not very timely. We also consider a source with memory, where successive updates can exploit the temporal correlation in source messages. The source can either send a *true update* codeword corresponding to its true state, or a *differential update* codeword corresponding to the difference between the current state and the previous one.

When there is an immediate and perfect feedback from the receiver, opportunistic differential updates are shown to outperform a scheme that only sends true updates, for highly correlated sources in [17] and Markov sources in [20]. In practice, receiver feedback may be imperfect, or expensive to obtain, or may not be available at all. We are interested in the case when there is no receiver feedback. In this case, we consider a scheme in which a true update is sent periodically interspersed with differential updates. This is motivated by the fact that the decoding failure of a single differential update leads to the source state decoding failure for all subsequent consecutive differential updates even if they are correctly decoded, due to desynchronization. Hence, true updates are sent periodically to ensure that the source state can be reacquired in case of failures. It was shown in [17] that the average age for a transmission scheme employing such a periodic differential update policy without feedback, is worse than sending true updates. In this work, we show that the previous result was due to a poor choice of codeword length. We show that the average age of this periodic transmission scheme without feedback can be smaller than that of the scheme that always sends true updates. In the rest of the letter, we denote the set of non-negative integers by $\mathbb{N}_0$, the set of positive integers by $\mathbb{N}$, and the set of first n positive integers by $[n]$.

II. System Model

We consider a physical process whose state at time t is denoted by $M(t)$, sampled at discrete time instants. We assume that each sampled source state is encoded and transmitted immediately. We denote the jth sampling instant by t_{j-1}, and the jth sampled message by $M_j \triangleq M(t_{j-1})$. The system model is illustrated in Fig. 1.

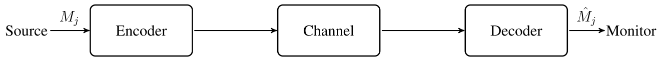

Fig. 1. We show a discrete time communication model for a source with information M_j at jth sampling time t_{j-1}, which is immediately encoded to an n_j-length codeword, and transmitted over an unreliable bit-wise independent and identically distributed (*i.i.d.*) binary symmetric erasure channel with unit delay. The output codeword is obtained after n_j channel uses at time $t_j = t_{j-1} + n_j$, and decoded as $\hat{M}_j$.

In this letter, we restrict our attention to source messages of finitely many bits. When the sampled state M_j is encoded using n_a bits, the codeword is called a *true update*. We assume that the physical process under consideration is highly temporally correlated [17], such that the difference in states of the sampled process at instants t_{j-1} and t_{j-2} can be represented by a binary sequence of finite length. When the difference in sampled states $\delta_j \triangleq M_j - M_{j-1}$ is encoded using n_d bits, the codeword is called a *differential update*. We assume that $n_a \geqslant n_d$, and that the encoding is forward-error correcting[1] and permutation invariant.[2]

We consider a *q-periodic incremental update scheme* without feedback. In this scheme, every qth source sample is encoded as a true update, and the remaining source samples are encoded as differential updates. For $q = 1$, this scheme reduces to sending true updates at all times, referred to as *true update scheme*, and serves as our benchmark. We assume that the transmission starts with a true update at time $t_{-1} = 0$, followed by $(q-1)$ differential updates. It follows that the length of jth update codeword is $n_j = n_a \mathbb{1}_{\{j=kq\}} + n_d \mathbb{1}_{\{j \neq kq\}}$, where $k \in \mathbb{N}_0$ and $\mathbb{1}_{\{\cdot\}}$ is the indicator function.

We consider a bit-wise *i.i.d.* binary symmetric erasure channel with bit erasure probability ϵ and unit delay per channel use. Each transmitted bit in $\{0, 1\}$ is received after one time unit, either correctly or erased. Therefore, the jth binary codeword of length n_j transmitted at time t_{j-1} is received at time $t_j = t_{j-1} + n_j$. The number of bit erasures in the jth received codeword is denoted by E_j. Note that t_j also indicates the instant for sampling the source for $(j+1)$th message. That is, we assume that the source is sampled as soon as the channel is available. We define the transmission duration of q consecutive updates as $\bar{n} \triangleq n_a + (q-1)n_d$. We can write $j = kq + \ell$ uniquely for $k = \lfloor \frac{j}{q} \rfloor$ and $\ell = j - kq \in \{0, \dots, q-1\}$, and write the reception time of jth update as $t_j = k\bar{n} + n_a + \ell n_d, \quad \ell \in \{0, \dots, q-1\}$. The decoder first decodes the update message, and subsequently decodes the source message.

Decoding Update Message: Update codeword is decoded as $\hat{M}_j$ when $j = kq$, and $\hat{\delta}_j$ otherwise. The event of decoding failure for the jth received codeword is denoted by

$$\xi_j \triangleq \mathbb{1}_{\{M_j \neq \hat{M}_j\}} \mathbb{1}_{\{j=kq\}} + \mathbb{1}_{\{\delta_j \neq \hat{\delta}_j\}} \mathbb{1}_{\{j \neq kq\}}. \tag{2}$$

Since the erasure indicators are *i.i.d.* Bernoulli random variables, it follows that the joint random sequence of number of erasures and update decoding failure indicators $((E_j, \xi_j) : j \in \mathbb{N}_0)$ is independent, where the marginal distribution of E_j is Binomial with parameters (n_j, ϵ) and the conditional expectation $\mathbb{E}[\xi_j | E_j]$ depends on the chosen forward error correcting code.[3] We can write the unconditional probability of decoding failure for jth codeword as $p_j \triangleq \mathbb{E}\xi_j = \mathbb{E}[\mathbb{E}[\xi_j | E_j]]$. The unconditional probabilities of decoding failure for true and incremental updates are denoted by p_a and p_d respectively.

Decoding Source Message: For $j = kq$, the source state is decoded directly from the received codeword. When $j \neq kq$, the source state is successfully decoded as $\hat{M}_j = \hat{M}_{kq} + \sum_{i=kq+1}^{j} \hat{\delta}_i$ only if all updates $\{kq, \dots, j\}$ have been successfully received. We denote the decoding failure of the source state M_j at the reception instant t_j, by the indicator $\mathbb{1}_{\{M_j \neq \hat{M}_j\}}$.

III. Incremental Update Without Feedback

To evaluate the proposed q-periodic incremental update scheme, we consider the limiting time average of information age as the performance metric, defined as

$$\bar{A} \triangleq \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} A(t). \tag{3}$$

We show that to compute the above metric, it suffices to focus on the average of the sampled age. As such, we study the evolution of sampled age for the q-periodic incremental update scheme, and compute its average.

Since it takes n_j time units for the jth update transmission from the source to the monitor, it follows that the age $A(t)$ resets to value n_j at the successful reception of any update, and is linearly increasing at all other instants. We denote the age process sampled at the jth update codeword reception instant t_j by $A_j \triangleq A(t_j)$. We denote the sampled age at the reception instant $t_{-1} = 0$ of the (-1)th update as $A_{-1} = 0$. It follows that the age at the jth update reception instant is

$$A_j = n_j + A_{j-1} \mathbb{1}_{\{M_j \neq \hat{M}_j\}}. \tag{4}$$

The sampled age resets to n_j if the codeword is successfully decoded, otherwise it increases by n_j from the last sampled age. Further, the information age at any discrete instant is completely determined by the sample age at the last codeword reception instant. In particular, we have

$$A(t) = A_j + (t - t_j), \quad t \in \{t_j, \dots, t_{j+1} - 1\}. \tag{5}$$

A. Renewal Process

Recall that kth true update is received at time $t_{kq} = k\bar{n} + n_a$. Let N_i denote the number of true updates transmitted until successful reception of i true updates. That is, let $N_0 \triangleq -1$ and define $N_i \triangleq \inf\{k > N_{i-1} : \xi_{kq} = 0\}$. By definition of N_i and (4) on sampled age evolution, we get $A_{N_i} = n_a$. Let R_i denote the time instant of ith successful reception of true update and define $R_0 \triangleq 0$. Since each true update reception is followed by $(q-1)$ differential updates, we have $R_i = N_i \bar{n} + n_a$. We define the time interval between two successful receptions of a true update as $T_i \triangleq R_i - R_{i-1}$, and the number of true updates sent in this interval as $Z_i \triangleq (N_i - N_{i-1}) = \frac{T_i}{\bar{n}}$. Sample path of the age process $A(t)$ for a 3-periodic incremental update scheme is illustrated in Fig. 2 for two renewal intervals.

[1]In the absence of feedback, we can only send fixed-length codewords.

[2]The decoding failure of a permutation invariant code depends only on the number of erasures in a codeword, rather than their locations.

[3]This follows from the permutation invariance property of the code.

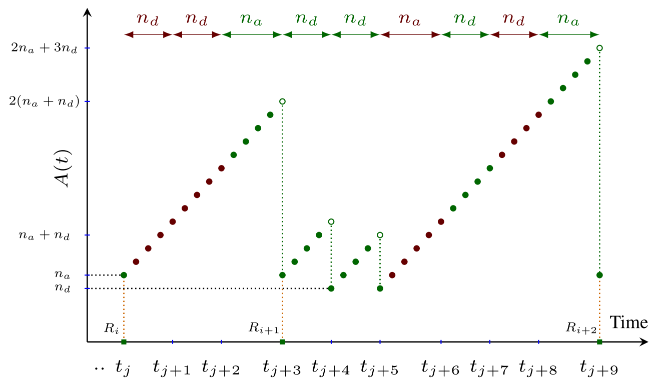

Fig. 2. In the ith renewal interval, the first differential update fails and the age increases linearly until the next renewal instant $R_{i+1} = t_{j+3}$, when a true update is decoded successfully. In the $(i+1)$th renewal interval, there is an actual update failure, and hence all the subsequent differential updates are rejected.

Lemma 1: The random sequence $(Z_i : i \in \mathbb{N})$ *is* i.i.d. *geometric with success probability* $(1 - p_a)$.

Proof: From the *i.i.d.* nature of the channel erasures, it follows that successful reception of each true update is an *i.i.d.* random variable with success probability $1 - p_a$. ■

Corollary 2: The random variable R_i *is the ith renewal instant for the* q*-periodic incremental update scheme.*

Since the age resets to n_a after successful reception of a true update, and the erasure channel realizations are independent, it follows that the age is a regenerative process and the cumulative age in each renewal interval is independent. We will show that the cumulative age in each renewal interval has a finite mean, and hence can be thought of as a reward process. Applying the renewal reward theorem [2] to the age process $(A(t) : t \in \mathbb{N}_0)$, we get

$$\bar{A} = \frac{\mathbb{E}\sum_{s=0}^{T_i-1} A(R_{i-1}+s)}{\mathbb{E}T_i}. \tag{6}$$

Remark 1: From Lemma 1, we obtain $\mathbb{E}T_i = \frac{\bar{n}}{1-p_a}$.

In the following, we characterize the mean of cumulative age in one renewal period, to compute the limiting average age $\bar{A}(q)$ for the q-periodic update scheme.

B. Cumulative Age in a Renewal Interval

We focus on the ith renewal interval $\{R_{i-1}, \dots, R_i - 1\}$ for the proposed update scheme. This renewal period accounts for transmission of $Z_i q$ updates, out of which there are Z_i true updates and $Z_i(q-1)$ differential updates. By definition, there are $(Z_i - 1)$ failed true updates in the ith renewal interval. A failed true update implies a source state decoding failure for the subsequent $(Z_i - 1)(q - 1)$ differential updates in this interval. Therefore, we focus on the first $(q-1)$ differential updates after renewal instant R_{i-1} and define $\bar{W}_i$ as the number of source state decoding successes in the ith renewal interval. This implies that $\bar{W}_i - 1 \in \{0, \dots, q-1\}$ consecutive differential updates are successfully received in this renewal interval.

Lemma 3: [17, Lemma 6] The number of true updates Z_i *and the number of successful updates* $\bar{W}_i$ *in the ith renewal interval remain independent and the sequence* $(\bar{W}_i : i \in \mathbb{N})$ *is* i.i.d. *with the common distribution*

$$P\{\bar{W}_i = w\} = (1-p_d)^{w-1}(p_d \mathbb{1}_{\{w<q\}} + \mathbb{1}_{\{w=q\}}). \tag{7}$$

Proof: The independence of Z_i and $\bar{W}_i - 1$ follows from the independence of erasure channel. Since each incremental update fails with probability p_d, the result follows. ■

Since the source state can not be decoded after $\bar{W}_i - 1$ differential updates, it follows that the age linearly increases after the time instant $R_{i-1} + (\bar{W}_i - 1)n_d$. Therefore, we can partition the ith renewal interval into two subintervals. The first subinterval corresponds to the successful reception of consecutive differential updates, and is denoted by $J_i \triangleq \{R_{i-1}, \dots, R_{i-1} + (\bar{W}_i - 1)n_d - 1\}$. Age increases linearly in the second subinterval denoted by $K_i \triangleq \{R_{i-1} + (\bar{W}_i - 1)n_d, \dots, R_i - 1\}$. We observe that the sampled age resets at successful update receptions for $j \in \{0, \dots, \bar{W}_i - 1\}$, to

$$A(R_{i-1} + jn_d) = n_d \mathbb{1}_{\{j\neq 0\}} + n_a \mathbb{1}_{\{j=0\}}. \tag{8}$$

Theorem 4: The limiting average age for q-periodic incremental update scheme with codeword lengths n_a *and* n_d *for true and incremental updates is given by*

$$\begin{aligned}\bar{A}(q) = &\frac{\bar{n}\mathbb{E}Z_i^2}{2\mathbb{E}Z_i} - \frac{1}{2} - n_d\mathbb{E}(\bar{W}_i - 2) + (n_a - n_d)P\{\bar{W}_i = 1\} \\ &+ \frac{n_d^2\mathbb{E}(\bar{W}_i(\bar{W}_i - 1))}{2\bar{n}\mathbb{E}Z_i} + \frac{n_d(n_a - n_d)P\{\bar{W}_i > 1\}}{\bar{n}\mathbb{E}Z_i}.\end{aligned}$$

Proof: We can write the cumulative sum of information age in the ith renewal interval as sum of age over disjoint intervals J_i and K_i, such that $\sum_{s=R_{i-1}}^{R_i-1} A(s) = \sum_{s\in J_i} A(s) + \sum_{s\in K_i} A(s)$. We compute the two summations separately. We observe that $J_i \neq \emptyset$ only when $\bar{W}_i > 1$. From (8) for age at successful update receptions in the ith renewal interval, we obtain that the age at the $(i-1)$th successful reception of true update is $A(R_{i-1}) = n_a$ and the age at the successful reception of differential updates is $A(R_{i-1} + wn_d) = n_d$ for $w \in [\bar{W}_i - 1]$. From (5) for evolution of age after an update reception, we know that the age increases linearly between two successful receptions. Combining these two results, we can write the first summation as

$$\sum_{s\in J_i} A(s) = (n_a - n_d)n_d \mathbb{1}_{\{\bar{W}_i > 1\}} + (\bar{W}_i - 1)\frac{n_d(3n_d - 1)}{2}.$$

We recall that the age increases linearly from the instant $R_{i-1} + (\bar{W}_i - 1)n_d$ until the end of ith renewal interval $R_i - 1$, and the age $A(R_{i-1} + (\bar{W}_i - 1)n_d) = n_d + (n_a - n_d)\mathbb{1}_{\{\bar{W}_i = 1\}}$. Further, $R_i - R_{i-1} = T_i = \bar{n}Z_i$, and hence the second sum is

$$\begin{aligned}\sum_{s\in K_i} A(s) = &\bar{n}Z_i(n_d + (n_a - n_d)\mathbb{1}_{\{\bar{W}_i=1\}}) + \frac{\bar{n}Z_i(\bar{n}Z_i - 1)}{2} \\ &+ \frac{n_d^2\bar{W}_i(\bar{W}_i - 1)}{2} \\ &- \frac{n_d(\bar{W}_i - 1)}{2}(3n_d - 1 + 2\bar{n}Z_i).\end{aligned}$$

Summing both terms, we get the cumulative sum of age in the ith renewal interval. The result follows from (6) for the average age, independence of $\bar{W}_i$ and Z_i, and the fact that $T_i = \bar{n}Z_i$. ■

The following result follows by setting $q = 1$ in Theorem 4, which implies $\bar{W}_i = 1$ and $\bar{n} = n_a$.

Corollary 5: The average age for true update scheme with codeword length n_a is given by $\bar{A}(1) = n_a\left(\frac{\mathbb{E}Z_i^2}{2\mathbb{E}Z_i} + 1\right) - \frac{1}{2}$.

Consider the case when the codeword lengths are sufficiently large compared to the information bits, and the random coding is employed for the forward error correction. Under the following approximation for the probability of update decoding failure, we can show the existence of an optimal differential update codeword length $n_d^*(n_a)$ for a given true update codeword length n_a.

Lemma 6: [17, Lemma 16] Consider an n length codeword with l information bits, encoded using random coding scheme. The probability of decoding failure p for this codeword transmitted over an i.i.d. *erasure channel with erasure probability ϵ can be approximated as $p \approx 2^{-n(1-\epsilon)+l}$ when $n(1-\epsilon) \gg l$.*

In the following, we will assume that the codeword lengths n_a, n_d are positive real numbers, to understand the behavior of average age as a function of codeword lengths.

Theorem 7: If the update failure probabilities p_a, p_d are sufficiently small, then there exists a codeword length n_d^* that minimizes the average age. For this choice of $n_d = n_d^*$, we have $\bar{A}(q) \leqslant \bar{A}(1)$ for $q \geqslant 2$.

Proof: Since the number of true updates for a successful decoding obeys the geometric distribution, $\frac{\mathbb{E}Z_i^2}{\mathbb{E}Z_i} = \frac{1+p_a}{1-p_a} \approx 1+2\,p_a$. From the hypothesis that $p_d \approx 0$, we obtain that $\bar{W}_i \approx q$. Under these two approximations, we can write the approximate average age of q-periodic incremental update as

$$\bar{A}(q) \approx \frac{(1-p_a)}{2\bar{n}}(n_d^2\, q(q-1) + 2n_d(n_a - n_d)(1-p_a)) + \frac{\bar{n}(1+2p_a)}{2} - n_d(q-2) - \frac{1}{2}.$$

Recall that $\bar{n} = n_a + (q-1)n_d$, and hence we observe that the average age $\bar{A}(q)$ is a function of the differential update codeword length n_d. For $q \geqslant 2$, we have

$$\frac{\partial^2 \bar{A}(q)}{\partial n_d^2} = \frac{2(1-p_a)n_a^2 q(q-2)}{\bar{n}^3} \geqslant 0. \tag{9}$$

It follows that the average age of differential encoding is marginally convex in n_d, and hence there exists a unique codeword length n_d^* at which we obtain a minimum. The optimal codeword length n_d^* is the unique solution to the implicit equation $\frac{\partial \bar{A}}{\partial n_d} = 0$. That is, n_d^* is the root of the following implicit equation

$$\frac{q+1}{2(1-p_a)} = \frac{(q-2)n_a(n_a - n_d)}{\bar{n}^2} - \frac{n_d(q-2)}{\bar{n}}. \tag{10}$$

We can write the difference in approximate average age for the q-periodic incremental update scheme and the true update scheme, as

$$\bar{A}(q) - \bar{A}(1) = \frac{(q+1)n_d}{2} - \frac{(q-2)n_d(1-p_a)(n_a - n_d)}{\bar{n}} - n_a.$$

Substituting optimal differential update codeword length $n_d = n_d^*$ from the implicit equation (10) in the above equation,

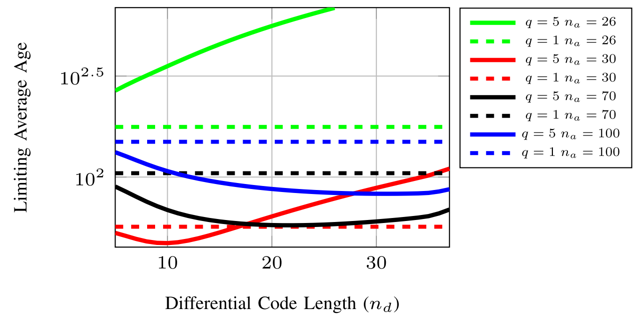

Fig. 3. This plot shows the variation of limiting average age for a q-periodic incremental update scheme in the differential update codeword length $n_d \in \{5, \cdots, 36\}$, for different values of true update codeword length n_a. The number of information bits for differential and true updates are 5 and 25 respectively, the periodicity for true updates is $q = 5$, and the erasure probability is $\epsilon = 0.1$.

we obtain the average age difference for $q \geqslant 2$ as

$$\bar{A}(q) - \bar{A}(1) = -n_a\left(\frac{(1-p_a)(n_d^*)^2 q(q-2)}{\bar{n}^2} + 1\right) \leqslant 0.$$

■

IV. Numerical Results and Approximations

There are three system parameters that we can select to achieve the optimal average age for the expression in Theorem 4. First, the choice of period q, which affects the number of interleaved messages between two true updates. The other two parameters are codeword lengths n_a and n_d for true and differential updates respectively. In this work, we restrict our attention solely to the selection of differential update codeword length n_d, while keeping the period q and the true update codeword length n_a fixed. We compare the performance of q-periodic incremental update scheme to that of the true update scheme. The probability of decoding failure for an update codeword depends on the codeword length, the number of information bits, the channel erasure probability ϵ, and the performance of the employed forward error correcting code. For the subsequent numerical studies, we employ random coding for forward error correction [17].

In Fig. 3, we have plotted the limiting average age for both update schemes as we vary the differential update code length $n_d \in \{b, \ldots, n_a\}$. The limiting average age for q-periodic incremental and true update schemes are denoted by solid and dashed curves respectively, with different colors for different values of $n_a \in \{26, 30, 70, 100\}$. We can observe that there exists average-age minimizing differential update codeword length $n_d^*(n_a)$ for each value of n_a. Further, we see that with a careful selection of codeword lengths for true and differential updates, q-periodic incremental updates can reduce the average age when compared to the true updates.

In Fig. 4, we have plotted the limiting average age of q-periodic incremental updates versus the true update codeword length n_a. The dotted red performance curve is obtained by choosing the differential update codeword length equal to that of true updates i.e. $n_d = n_a$. The solid purple performance curve is for the true update scheme. The densely dotted

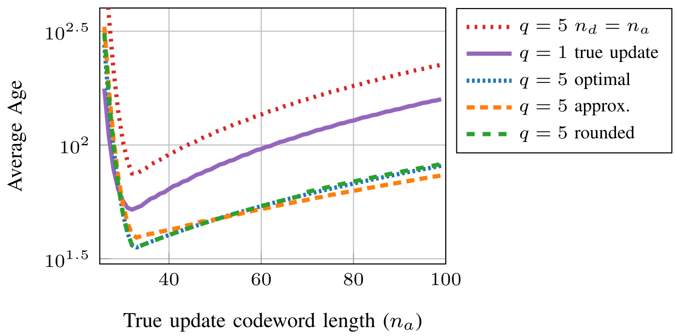

Fig. 4. The average age performance for q-periodic schemes with respect to the true update codeword lengths $n_a \in \{26, \dots, 100\}$ for the periods $q = 5$ and $q = 1$ (true updates). The information bits for differential and true updates are 5 and 25 respectively, and the erasure probability is $\epsilon = 0.1$.

blue performance curve is obtained for the optimal choice of differential update codeword length $n_d^*(n_a)$ found by an exhaustive numerical search for each true update codeword length n_a. We observe that the true update scheme outperforms q-periodic scheme when a fixed codeword length is used for both differential and true updates. However, with a careful selection of two codeword lengths, the q-periodic scheme can outperform the true update scheme.

The performance curves obtained by rounding off the real length $n_d^*(n_a)$ obtained in Theorem 7 are plotted as a densely dashed orange curve using the update decoding failure probability approximation in Lemma 6, and a dashed green curve without this approximation. We observe that the average age performance for the rounded codeword lengths n_d^* found by solving the implicit equation is very close to that of the optimal codeword lengths found by the exhaustive search. Finally, we would like to point out that there exists an optimal true update codeword length $n_a^*(n_d^*)$ which minimizes the average age among all q-periodic incremental update schemes with a fixed period q.

V. Conclusion and Future Work

We computed the average age performance of q-periodic incremental update scheme, which exploits the temporal source correlation in the absence of feedback. This scheme degenerates to always transmitting true updates for $q = 1$. We show that the average age can be reduced for the former, with a careful selection of codeword lengths for true and differential updates. We considered an *i.i.d.* erasure channel, which can be generalized to channels with error and memory. The age needs to be carefully defined for channels with error, since the received codeword may be erroneously decoded in this case. It is clear that the feedback would improve the average age performance for channels with memory. A possible extension would be to find transmission schemes that minimize the average age for channels with memory, with or without feedback.

References

[1] S. Kaul, R. Yates, and M. Gruteser, "Real-time status: How often should one update?" in *Proc. IEEE Int. Conf. Comp. Commun. (INFOCOM)*, Mar. 2012, pp. 2731–2735.

[2] S. M. Ross, *Stochastic Processes* (Wiley Series in Probability and Statistics: Probability and Statistics). Hoboken, NJ, USA: Wiley, 1996.

[3] C. Kam, S. Kompella, and A. Ephremides, "Age of information under random updates," in *Proc. IEEE Int. Symp. Inf. Theory*, Jul. 2013, pp. 66–70.

[4] M. Costa, M. Codreanu, and A. Ephremides, "On the age of information in status update systems with packet management," *IEEE Trans. Inf. Theory*, vol. 62, no. 4, pp. 1897–1910, Apr. 2016.

[5] A. M. Bedewy, Y. Sun, and N. B. Shroff, "The age of information in multihop networks," *IEEE/ACM Trans. Netw.*, vol. 27, no. 3, pp. 1248–1257, Jun. 2019.

[6] R. D. Yates and S. K. Kaul, "The age of information: Real-time status updating by multiple sources," *IEEE Trans. Inf. Theory*, vol. 65, no. 3, pp. 1807–1827, Mar. 2019.

[7] A. Kosta, N. Pappas, A. Ephremides, and V. Angelakis, "Age of information performance of multiaccess strategies with packet management," *J. Commun. Netw.*, vol. 21, no. 3, pp. 244–255, Jun. 2019.

[8] I. Kadota, A. Sinha, E. Uysal-Biyikoglu, R. Singh, and E. Modiano, "Scheduling policies for minimizing age of information in broadcast wireless networks," *IEEE/ACM Trans. Netw.*, vol. 26, no. 6, pp. 2637–2650, Dec. 2018.

[9] S. K. Kaul and R. D. Yates, "Timely updates by multiple sources: The M/M/1 queue revisited," in *Proc. 54th Annu. Conf. Inf. Sci. Syst. (CISS)*, Mar. 2020, pp. 1–6.

[10] J. Hribar, M. Costa, N. Kaminski, and L. A. DaSilva, "Using correlated information to extend device lifetime," *IEEE Internet Things J.*, vol. 6, no. 2, pp. 2439–2448, Apr. 2019.

[11] Q. He, G. Dan, and V. Fodor, "Joint assignment and scheduling for minimizing age of correlated information," *IEEE/ACM Trans. Netw.*, vol. 27, no. 5, pp. 1887–1900, Oct. 2019.

[12] A. E. Kalor and P. Popovski, "Minimizing the age of information from sensors with common observations," *IEEE Wireless Commun. Lett.*, vol. 8, no. 5, pp. 1390–1393, Oct. 2019.

[13] H. Sac, T. Bacinoglu, E. Uysal-Biyikoglu, and G. Durisi, "Age-optimal channel coding blocklength for an M/G/1 queue with HARQ," in *Proc. IEEE 19th Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Jun. 2018, pp. 1–5.

[14] R. D. Yates, E. Najm, E. Soljanin, and J. Zhong, "Timely updates over an erasure channel," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jun. 2017, pp. 316–320.

[15] D. Li, S. Wu, Y. Wang, J. Jiao, and Q. Zhang, "Age-optimal HARQ design for freshness-critical satellite-IoT systems," *IEEE Internet Things J.*, vol. 7, no. 3, pp. 2066–2076, Mar. 2020.

[16] P. Parag, A. Taghavi, and J.-F. Chamberland, "On real-time status updates over symbol erasure channels," in *Proc. IEEE Wireless Commun. Netw. Conf. (WCNC)*, Mar. 2017, pp. 1–6.

[17] S. Bhambay, S. Poojary, and P. Parag, "Fixed length differential encoding for real-time status updates," *IEEE Trans. Commun.*, vol. 67, no. 3, pp. 2381–2392, Mar. 2019.

[18] S. C. Bobbili, P. Parag, and J.-F. Chamberland, "Real-time status updates with perfect feedback over erasure channels," *IEEE Trans. Commun.*, vol. 68, no. 9, pp. 5363–5374, Sep. 2020.

[19] E. Najm, E. Telatar, and R. Nasser, "Optimal age over erasure channels," in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2019, pp. 335–339.

[20] S. Poojary, S. Bhambay, and P. Parag, "Real-time status updates for Markov source," *IEEE Trans. Inf. Theory*, vol. 65, no. 9, pp. 5737–5749, Sep. 2019.

[21] E. T. Ceran, D. Gunduz, and A. Gyorgy, "Average age of information with hybrid ARQ under a resource constraint," *IEEE Trans. Wireless Commun.*, vol. 18, no. 3, pp. 1900–1913, Mar. 2019.

[22] C. Kam, S. Kompella, G. D. Nguyen, J. E. Wieselthier, and A. Ephremides, "Towards an effective age of information: Remote estimation of a Markov source," in *Proc. IEEE INFOCOM-IEEE Conf. Comput. Commun. Workshops (INFOCOM WKSHPS)*, Apr. 2018, pp. 367–372.

[23] Z. Jiang and S. Zhou, "Status from a random field: How densely should one update?" in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Jul. 2019, pp. 1037–1041.