On Average Latency for File Access in Distributed Coded Storage

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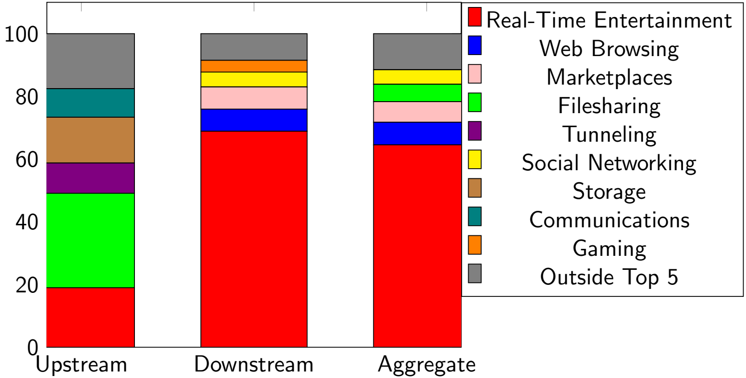
Dominant traffic on Internet

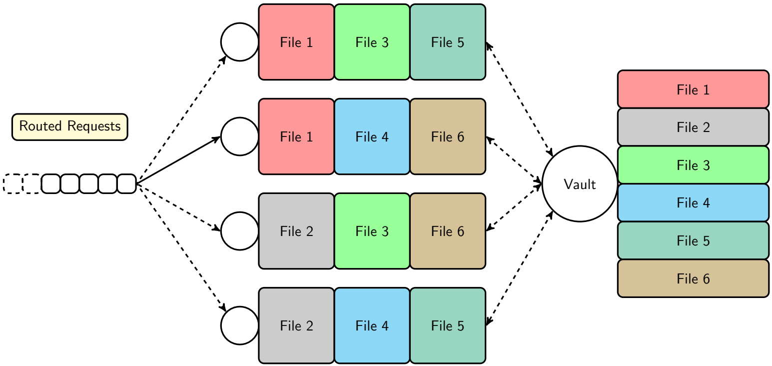
Peak Period Traffic Composition (North America)



- Real-Time Entertainment: 64.54% for downstream and 36.56 % for mobile access[1]

[1] https://www.sandvine.com/downloads/general/global-internet-phenomena/2015/global-internet-phenomena-report-latin-america-and-north-america.pdf

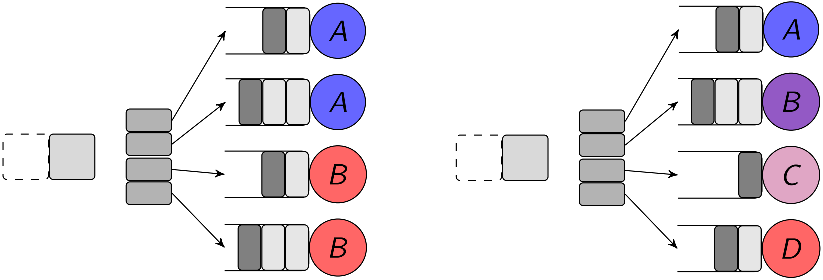
Established Solutions – Content Delivery Network



Congestion Prevention and Outage Protection

- Mirroring content with local servers
- Media file on multiple servers

Question: Duplication versus MDS Coding



Reduction of access time

- How many fragments for a single message?
- How to encode and store at the distributed storage nodes?

Pertinent References (very incomplete)

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System Model

File storage

- Each media file divided into k pieces
- Pieces encoded and stored on n servers

Arrival of requests

- Each request wants entire media file
- Poisson arrival of requests with rate λ

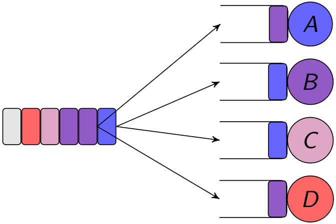
Time in the system

- Till the reception of whole file

Service at each server

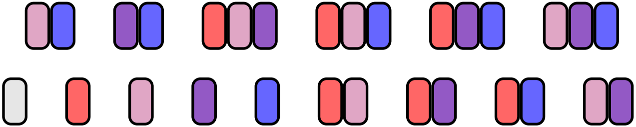
- IID exponential service time with rate k/n

Parallel Processing of Requests



- Service rate available to each request is proportional to number of servers processing the requests in parallel

State Space Structure



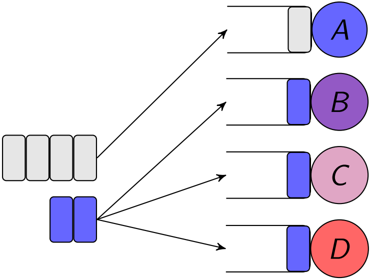
Keeping Track of Partially Fulfilled Requests

- Element of state vector $Y_S(t)$ is number of users with given subset S of pieces

Continuous-Time Markov Chain

- $\mathbf{Y}(t) = \{Y_S(t) : S \subset [n], |S| < k\}$ is a Markov process

Priority Scheduling



Mean shortest remaining time processing

- Priority to jobs with higher number of pieces
- FIFO scheduling among jobs with equal number of pieces

State Space Collapse

Theorem

For duplication and coding schemes under priority scheduling and parallel processing model, collection

$$\mathcal{S}(t) = \{S \subset [n] : Y_S(t) > 0, |S| < k\}$$

of information subsets is totally ordered in terms of set inclusion

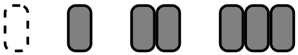
Corollary

Let $Y_i(t)$ be number of requests with i information symbols at time t, then

$$\mathbf{Y}(t) = (Y_0(t), Y_1(t), \ldots, Y_{k-1}(t))$$

is Markov process

State Transitions of Collapsed System



Arrival of Requests

- Unit increase in $Y_0(t) = Y_0(t-) + 1$ with rate λ

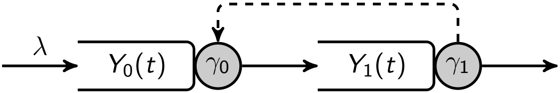
Getting Additional Symbol

- Unit increase in $Y_i(t) = Y_i(t-) + 1$
- Unit decrease in $Y_{i-1}(t) = Y_{i-1}(t-) - 1$

Getting Last Missing Symbol

- Unit decrease in $Y_{k-1}(t) = Y_{k-1}(t-) - 1$

Tandem Queue Interpretation



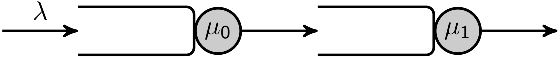
Tandem Queue with Pooled Resources

- Servers with empty buffers help upstream
- Aggregate service at level i becomes

$$\sum_{j=i}^{l_i(t)-1} \gamma_j \qquad \text{where} \qquad l_i(t) = k \wedge \{l > i : Y_l(t) > 0\}$$

- No explicit description of stationary distribution for multi-dimensional Markov process

Bounding and Separating



Theorem[†]

When $\lambda < \min \mu_i$, tandem queue has product form distribution

$$\pi(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\mu_i} \left(1 - \frac{\lambda}{\mu_i}\right)^{y_i}$$

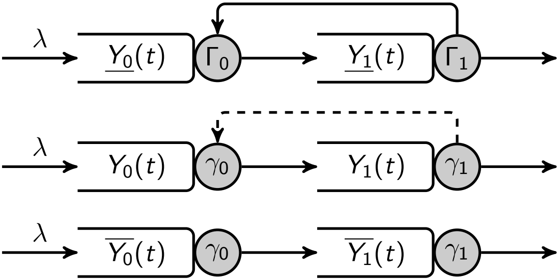
Uniform Bounds on Service Rate

Transition rates are uniformly bounded by

$$\gamma_i \leq \sum_{j=i}^{l_i(y)-1} \gamma_j \leq \sum_{j=i}^{k-1} \gamma_j \triangleq \Gamma_i$$

[†] F. P. Kelly, Reversibility and Stochastic Networks. New York, NY, USA: Cambridge University Press, 2011.

Bounds on Tandem Queue



Lower Bound

Higher values for service rates yield lower bound on queue distribution

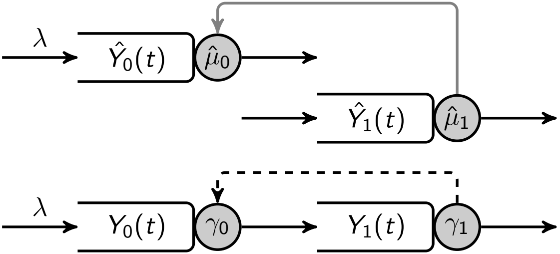
$$\underline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\Gamma_i}\left(1 - \frac{\lambda}{\Gamma_i}\right)^{y_i}$$

Upper Bound

Lower values for service rate yield upper bound on queue distribution

$$\overline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\gamma_i}\left(1 - \frac{\lambda}{\gamma_i}\right)^{y_i}$$

Approximating Pooled Tandem Queue



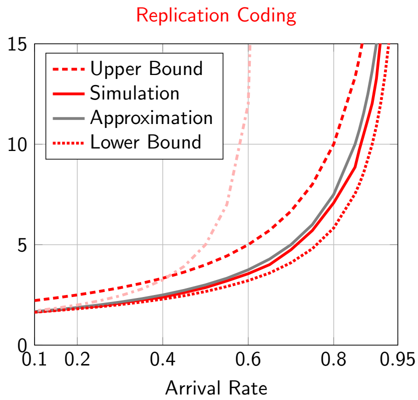
Independence Approximation with Statistical Averaging

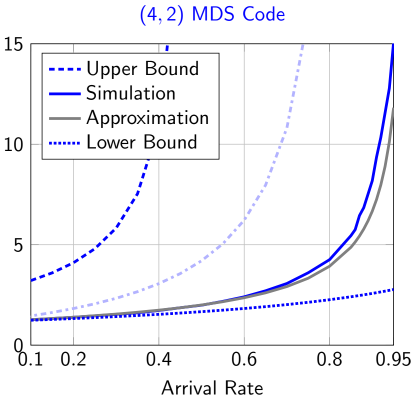
Service rate is equal to base service rate γ_i plus cascade effect, averaged over time

$$\hat{\mu}_{k-1} = \gamma_{k-1}$$
$$\hat{\mu}_i = \gamma_i + \hat{\mu}_{i+1}\hat{\pi}_{i+1}(0)$$

$$\hat{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\hat{\mu}_i}\left(1 - \frac{\lambda}{\hat{\mu}_i}\right)^{y_i}$$

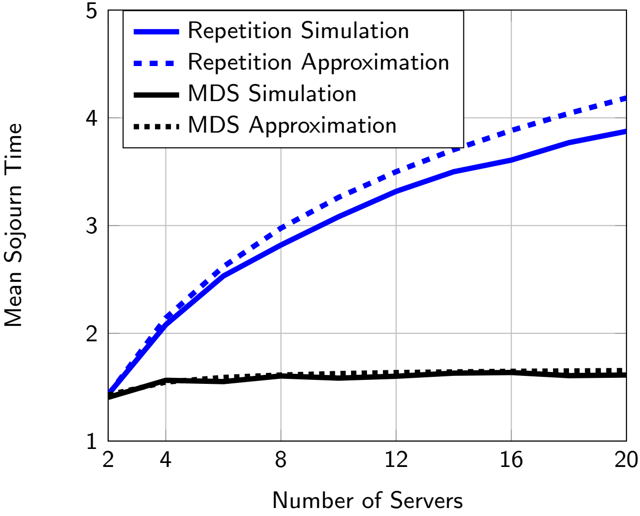
Mean Sojourn Time





- MDS coding significantly outperforms replication
- Bounding techniques are only meaningful under light loads
- Approximation is accurate over range of loads

Comparing Replication versus MDS Coding



Arrival rate 0.3 units and coding rate $n/k = 2$

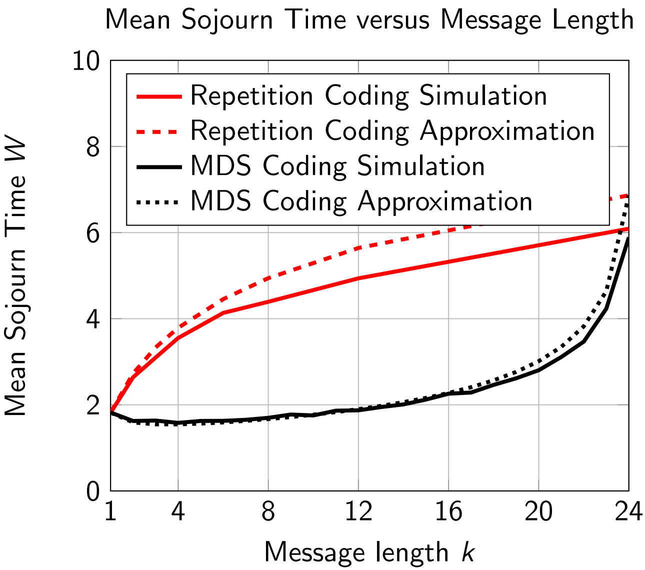


Figure: For rate $\lambda = 0.45$ and $n = 24$ servers.

Summary and Discussion

Main Contributions

- Analytical framework for study of distributed computation and storage systems
- Upper and lower bounds to analyze replication and MDS codes
- A tight closed-form approximation to study distributed storage codes
- MDS codes are better suited for large distributed systems
- Mean access time is better for MDS codes for all code-rates