Real-Time Status Updates for Correlated Source

Sudheer Poojary
Sanidhay Bhambay
Parimal Parag

Electrical and Communication Engineering
Indian Institute of Science, Bangalore

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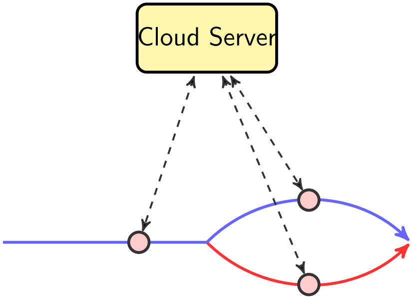
Why timely update?

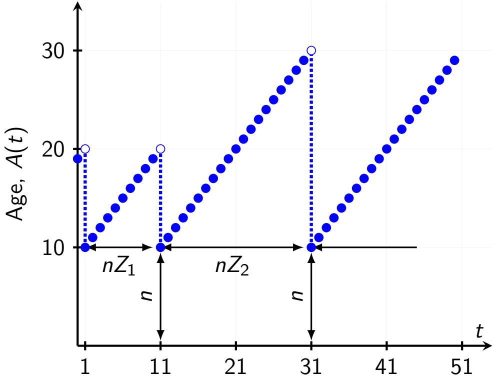
Real-Time Update



- Critical to know the status update before decision making
- Cyber-physical systems: Environmental/health monitoring
- Internet of Things: Real-time actuation/control

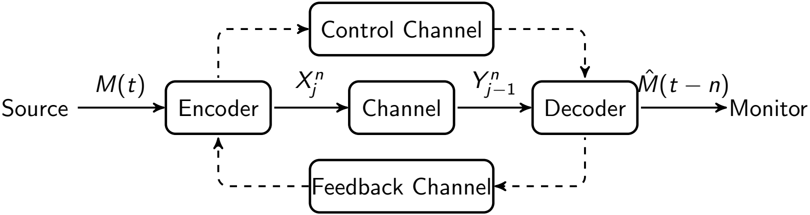
How to measure timeliness?

Real-Time Update



- Last correctly decoded message generated at $U(t)$
- Smaller the age $A(t) = t - U(t)$, more timely the message
- Goal: Minimize limiting average age $\lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s)$

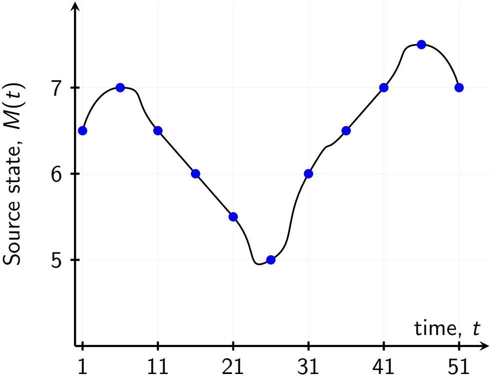
Link Model



Context

- Point-to-point communication with limited feedback
- Reliability through finite block-length coding
- Control channel with information about coding scheme

Source Model



- Sampled source $M_j \in \Delta_m$ Markov with transition matrix P
- Probability of the state difference $M_{j+1} - M_j \in \Delta_k$ independent of the initial state

Update Protocol

True Update

Encode current state $M_j \in \Delta_m$ to n bit codeword X^n

Incremental Update

- State difference $M_j - M_{j-1} \in \Delta_k$ almost surely
- If last update successfully received, then encode state difference to n bit codeword X^n

Generalized Incremental Update

If state difference $M_j - M_{j-1} \in \Delta_k$ and last update successfully received, then send incremental update, else send true update

Problem Statement

Question

Find the differential encoding threshold k for timely update of a Markov source that minimizes limiting average age

Answer

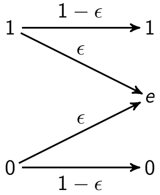
- **Higher threshold:** more differential encoding opportunities
- **Lower threshold:** more error protection

Coding and Channel Model

Finite-length Code

- Finite length code of n bits with permutation invariant code

Bit-wise Erasure Channel



- Each transmitted bit of the codeword X^n erased *iid* with probability ϵ
- Number of erasures per codeword E Binomial (n, ϵ)

Decoding and Reception

Receiver Timing

Reception at time $t + n$ of n bits sent at time t after n channel uses

Probability of Decoding Failure

- True updates: $p_t = \mathbb{E}P(n, n - m, E)$
- Incremental updates: $p_d = \mathbb{E}P(n, n - k, E)$
- Monotonicity: $0 < p_d < p_t < 1$

Differential Encoding Probability

Probability of source state difference being represented by k bits, $p_e = P_i(\Delta_k)$ for all states i

Renewal Reward Theorem

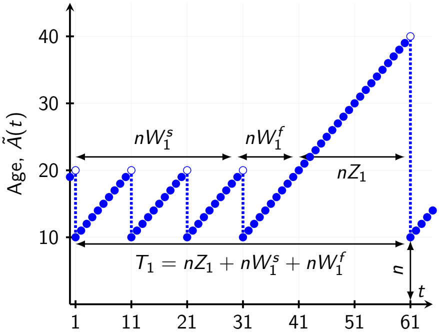
- Time instant S_i of the ith successful reception of the true update
- For all three schemes, the ith inter-renewal time $T_i = S_i - S_{i-1}$ is *iid*
- Accumulated age in ith renewal period also *iid*

$$S(T_i) = \sum_{t=S_{i-1}}^{S_i - 1} A(t)$$

- By renewal reward theorem, the limiting average age is

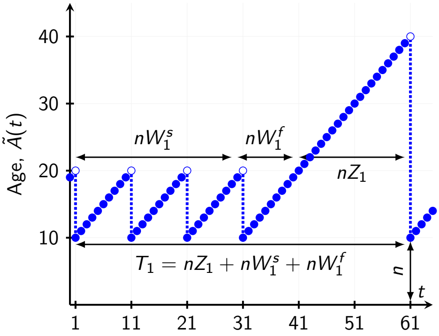
$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s) = \frac{\mathbb{E}S(T_i)}{\mathbb{E}T_i}.$$

Age Sample Path: True Updates



- True updates Z_i, *iid* geometric with success prob $(1 - p_t)$
- Successful incremental updates W_i^s, *iid* geometric with success probability $p_e(1 - p_d)$
- Failed incremental updates W_i^f, *iid* Bernoulli with success probability $\frac{p_e p_d}{1 - p_e(1 - p_d)}$

Mean Age

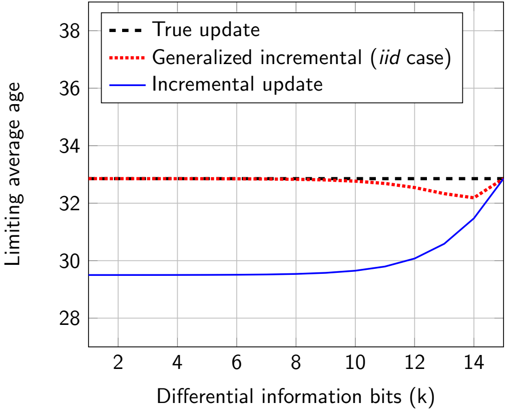


Theorem

Limiting average age for the true update scheme is a.s.

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s) = n - \frac{1}{2} + \frac{n\mathbb{E}(W_i^s)^2 + n\mathbb{E}(W_i^f + Z_i)^2}{2(\mathbb{E}W_i^s + \mathbb{E}W_i^f + \mathbb{E}Z_i)}.$$

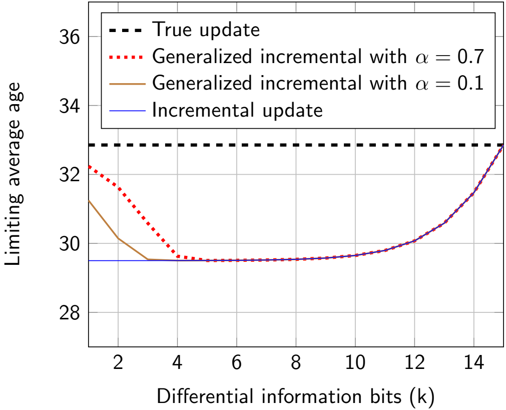
Uniform IID Source



System Parameters

- Differential encoding prob $p_e = \frac{2^k}{2^m}$
- Random coding, erasure probability $\epsilon = 0.1$
- Code length $n = 20$, information bits $m = 15$

State Homogeneous Markov Source



System Parameters

- Transition probability $P_{i,i\pm1} = \frac{\alpha}{2}, P_{i,i} = 1 - \alpha$
- Random coding, erasure probability $\epsilon = 0.1$
- Code length $n = 20$, information bits $m = 15$

Discussion and Concluding Remarks

Main Contributions

- Integration of coding and renewal techniques to study timely communication for delay-sensitive traffic
- We model channel unreliability by the erasure channel
- Model source correlation by Markov process
- True and incremental updates are special cases

Avenues of Future Research

- Extend results to correlated finite-state erasure and error channels
- Impact of other coding schemes on timeliness