On Real-Time Status Updates over Symbol Erasure Channels

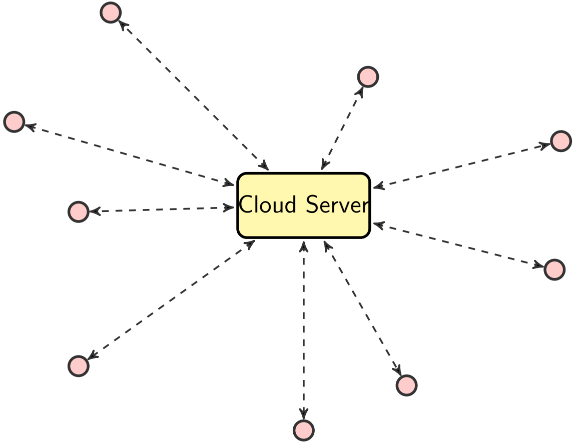
Parimal Parag
Austin Taghavi
Jean-Francois Chamberland

Electrical Communication Engineering
Indian Institute of Science

Electrical and Computer Engineering
Texas A&M University

Wireless Communications and Networking Conference
Mar 22, 2017

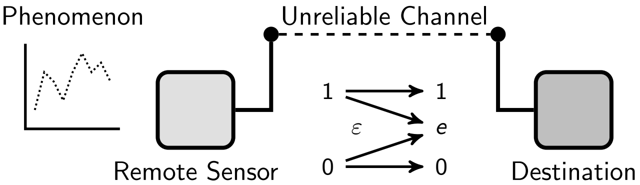
Timely Status Updates



Potential Scenarios

- Cyber-physical systems: Environmental/health monitoring
- Internet of Things: Real-time actuation/control

Updates over Erasure Channel



System Model

- Source has K bit message to send at all times
- One bit per channel use can be sent
- Bit-wise erasures *iid* Bernoulli with probability ϵ
- Number of i erasures in N channel usage is Binomial (N, ϵ)

Problem Statement

Compare the optimal coding for finite block length and hybrid ARQ schemes for timely update.

Timeliness Metric

$U(t)$ generation time of last successfully received message, then information age

$$A(t) = t - U(t)$$

Empirical information age

Limiting empirical information age is

$$\bar{A} \triangleq \lim_{T\to\infty} \frac{1}{T} \int_0^T A(t)dt$$

Renewal Reward Theorem

For a renewal interval T and accumulated reward $\int_0^T A(t)dt$

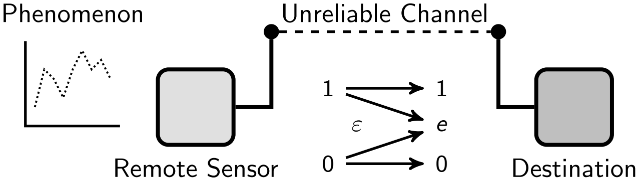
$$\mathbb{E}A = \bar{A} = \frac{\mathbb{E}\int_0^T A(t)dt}{\mathbb{E}T}$$

Finite Block Length Coding

- Maps message $m \in \mathbb{F}_2^K$ to codeword $x \in \mathbb{F}_2^N$
- Parity check matrix H such that $Hx = 0$ for any codeword x
- Number of erasures $|E|$ in a block N are Binomial (N, ϵ)
- Decoding failure event is *iid* Bernoulli with probability

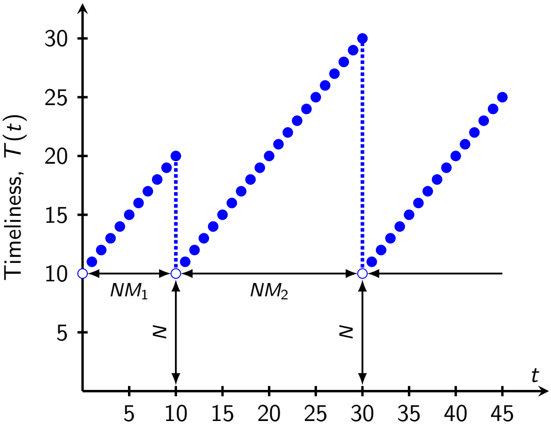
$$\rho = \mathbb{E}1\{\hat{x}(y) \neq x\} \triangleq \mathbb{E}P_f(N - K, |E|)$$

Single Transmission Scheme



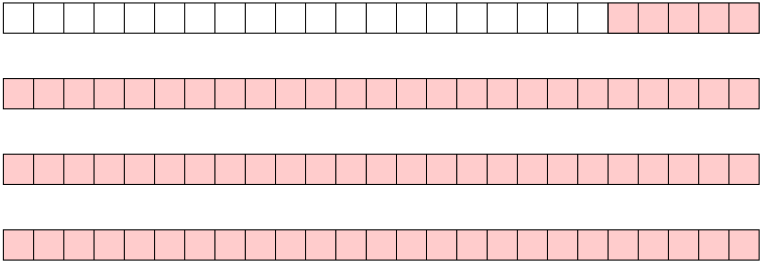
- Encode K length message to N length codeword
- Transmit N length codeword over N channel usage
- Transmit new codeword at next transmission opportunity
- Number of transmission attempts M before a success is Geometric with success probability $1 - \rho$

Single Transmission Scheme



- Mean age is $N + \frac{\mathbb{E}[M_i(NM_i+1)]}{2\mathbb{E}M_i}$

Hybrid Automatic Repeat Requests

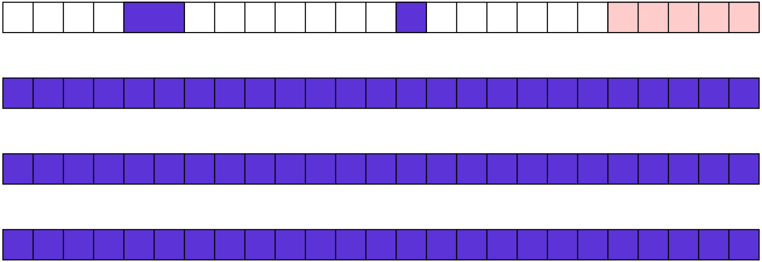


- Maps message $m \in \mathbb{F}_2^K$ to codeword $x \in \mathbb{F}_2^{aN}$ of depth a
- Decoding failure event is *iid* Bernoulli with probability

$$\bar{f}_a \triangleq \mathbb{E} P_f(aN - K, |E|)$$

- Number of transmission attempts M before a success is Geometric with success probability $1 - \bar{f}_a$

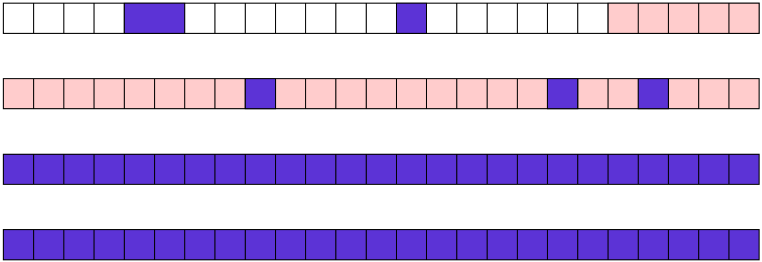
Hybrid Automatic Repeat Requests



- Successive decoding failures with effective number of erasures $|E|_i + (a-i)N$
- Decoding failure event after transmission of depth i of aN-length codeword is Bernoulli with probability

$$\bar{f}_i = \mathbb{E}P_f(aN - K, |E|_i + (a-i)N)$$

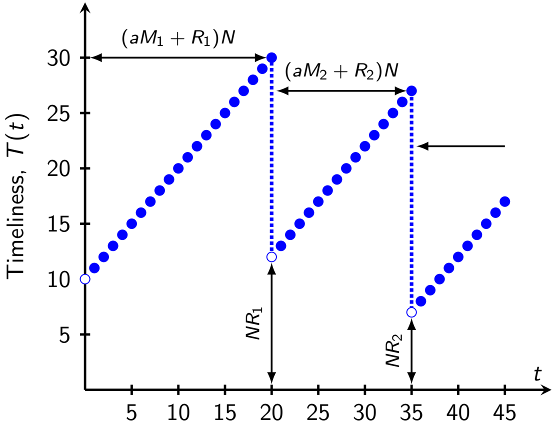
Hybrid Automatic Repeat Requests



- Number of erasures in first iN bits is $|E|_i \leq |E|_{i-1} + N$
- Number of effective erasure decreasing with each transmission

$$|E|_1 + (a-1)N \geq |E|_2 + (a-2)N \geq \cdots \geq |E|_a$$

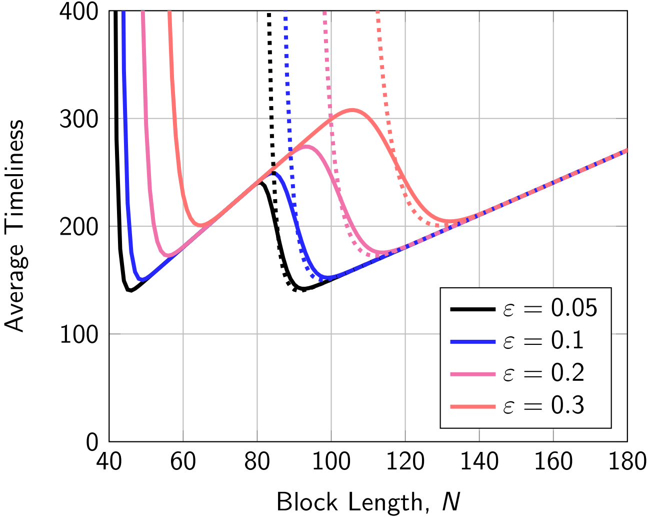
Hybrid Automatic Repeat Requests



- Mean age is $N\mathbb{E}R_{i-1} + \frac{\mathbb{E}[(aM_i+R_i)((aM_i+R_i)N+1)]}{2\mathbb{E}(aM_i+R_i)}$

Mean Timeliness Result

Performance for number of information bits $K = 80$ per message



Conclusions

- An analytical framework for parameter selection for timely updates over unreliable channels
- Demonstration of natural tradeoff between erasure resilience and timely delivery
- Optimal code rate very sensitive to channel characteristics
- Limited feedback does not improve timeliness update performance