Differential Encoding for Real-Time Status Updates

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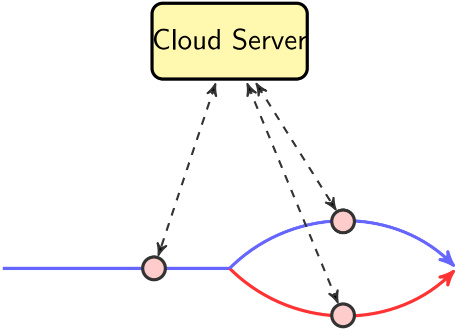
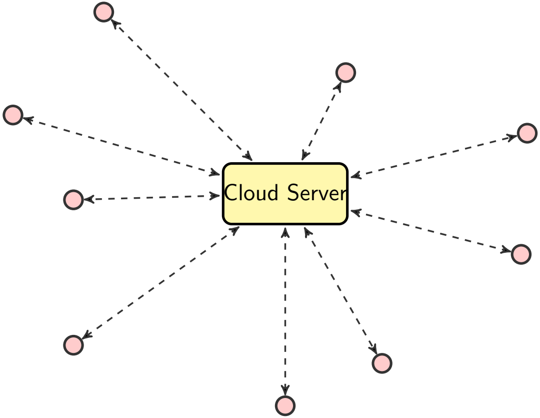
Why timely update?



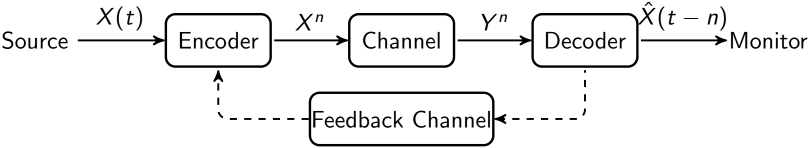
- Critical to know the status update before decision making

Potential Scenarios



- Cyber-physical systems: Environmental/health monitoring
- Internet of Things: Real-time actuation/control

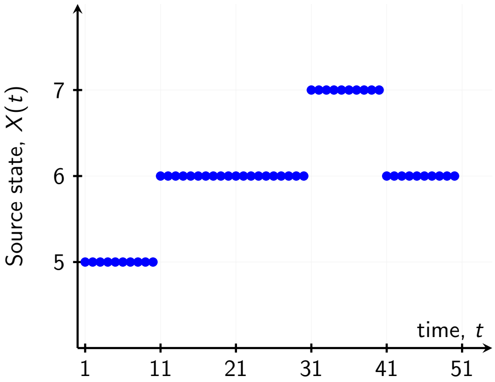
Link Model



Context

- Point-to-point communication with limited to no feedback
- Reliability through finite block-length coding

Source Model



- Source state $X(t)$ can be represented by m bits
- State difference between n realizations can be represented by $k < m$ bits

Problem Statement

Question

How to encode message at the temporally correlated source for timely update? Should one send the current state or the difference between the current and the past state?

Answer

It depends on the feedback

Coding Model

- Finite length code of n bits with permutation invariant code

Updates

- True Update: current state $X(t)$ of m bits is encoded to n bit codeword X^n
- Incremental Update: the state difference $X(t) - X(t-n)$ of k bits encoded to n bit codeword X^n

Channel Model

- Each transmitted bit of the codeword X^n erased *iid* with probability ϵ

Erasure Distribution

Number of erasures is Binomial with parameter (n, ϵ)

Decoding and Reception

Receiver Timing

Reception at time $t + n$ of n bits sent at time t after n channel uses

Probability of Decoding Failure

- True updates: $p_1 = \mathbb{E}P(n, n - m, E)$
- Incremental updates: $p_2 = \mathbb{E}P(n, n - k, E)$
- Monotonicity: $0 < p_2 < p_1 < 1$

Performance Metric

- Last successfully decoded source state at time t was generated at $U(t)$
- *Information age*[1] $A(t)$ at time t as

$$A(t) = t - U(t).$$

- Limiting value of average age

$$\lim_{t \to \infty} \frac{1}{t} \sum_{s=1}^{t} A(s).$$

[1] **Kaul, S., Yates, R., & Gruteser M., "Real-time status: How often should one update?". IEEE INFOCOM, 2012, pp. 2731–2735.**.

Update Transmission Schemes

True Updates

- Each opportunity send *true update*

Incremental Updates without Feedback

- Periodically send the *true update* after q updates
- In between true updates, send *incremental updates*.

Incremental Updates with Feedback

- Send the *true update* after each decoding failure
- In between true updates, send *incremental updates*.

Renewal Reward Theorem

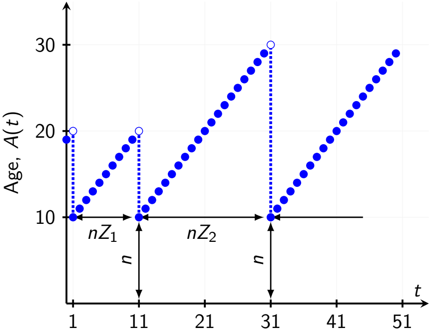
- Time instant S_i of the ith successful reception of the true update
- For all three schemes, the ith inter-renewal time $T_i = S_i - S_{i-1}$ is *iid*
- Accumulated age in ith renewal period

$$S(T_i) = \sum_{t=S_{i-1}}^{S_i - 1} A(t)$$

  is also *iid*
- By renewal reward theorem, the limiting average age is

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t} \sum_{s=1}^{t} A(s) = \mathbb{E}S(T_i)/\mathbb{E}T_i.$$

Age Sample Path: True Updates



- Inter-renewal time $T_i = nZ_i$
- Number of true update in ith renewal interval Z_i
- $\{Z_i : i \in \mathbb{N}\}$ is *iid* geometric with success parameter $(1 - p_1)$

Mean Age

Theorem

Limiting average age for the true update scheme is a.s.

$$\mathbb{E}A \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} A(s) = (n-1)/2 + n/(1-p_1).$$

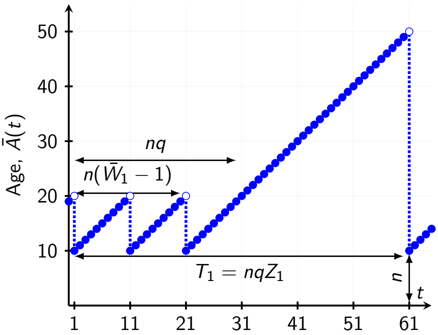
Proof.

Cumulative age for ith renewal interval is

$$S(nZ_i) = \sum_{j=0}^{nZ_i-1} (n+j) = n^2 Z_i + nZ_i(nZ_i - 1)/2.$$

□

Age Sample Path: Incremental Updates Without Feedback



- Inter-renewal time $T_i = nqZ_i$
- Number of successfully decoded contiguous incremental updates $\bar{W}_i - 1$ in the ith renewal interval
- $\bar{W}_i$ is the number of successfully decoded updates in ith renewal interval

Mean Age

Theorem

Limiting average age for the incremental updates without feedback is

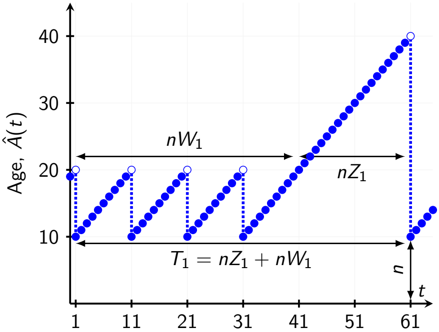
$$\mathbb{E}\bar{A} \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} \bar{A}(s) = \frac{\mathbb{E}\,T_i^2}{2\mathbb{E}\,T_i} + \frac{n^2\mathbb{E}\,\bar{W}_i(\bar{W}_i - 1)}{2\mathbb{E}\,T_i} - \left(n\mathbb{E}(\bar{W}_i - 2) + \frac{1}{2}\right).$$

Proof.

Cumulative age $S(T_i)$ in the ith renewal interval is

$$S(T_i) = \sum_{j=1}^{\bar{W}_i - 1}\sum_{k=0}^{n-1}(n+k) + \sum_{j=n(\bar{W}_i-1)}^{T_i - 1}(n + j - n(\bar{W}_i - 1)),$$
$$= \frac{n^2\bar{W}_i(\bar{W}_i - 1)}{2} + \frac{T_i^2}{2} - \left(n(\bar{W}_i - 2) + \frac{1}{2}\right)T_i.$$

Age Sample Path : Incremental Updates With Feedback



- Inter-renewal time $T_i = nZ_i + nW_i$
- Number of incremental updates W_i in ith renewal interval
- $\{W_i : i \in \mathbb{N}\}$ are *iid* geometric with success parameter p_2

Mean Age

Theorem

Limiting average age for the incremental updates with feedback is

$$\mathbb{E}\hat{A} \triangleq \lim_{t\to\infty} \frac{1}{t}\sum_{s=1}^{t} \hat{A}(s) = \frac{(3n-1)}{2} + \frac{n(\mathbb{E}Z_i^2 + \mathbb{E}Z_i)}{2(\mathbb{E}W_i + \mathbb{E}Z_i)}.$$

Proof.

Cumulative age $S(T_i)$ over the ith renewal period T_i is

$$S(T_i) = \sum_{j=1}^{W_i-1}\sum_{k=0}^{n-1}(n+k) + \sum_{k=0}^{T_i-n(W_i-1)-1}(n+k)$$
$$= (3n-1)T_i/2 + n^2(Z_i+1)Z_i/2.$$

□

Analytical Comparison

Theorem

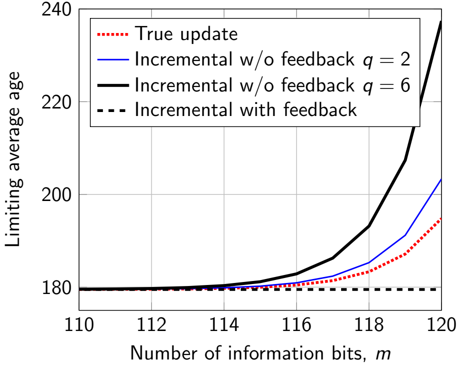
The mean age for the three schemes satisfy,

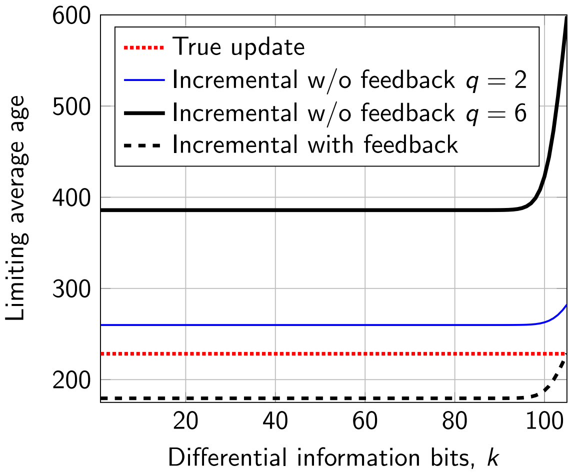
$$\mathbb{E}\hat{A} \leq \mathbb{E}A \leq \mathbb{E}\bar{A}.$$

Numerical Comparision

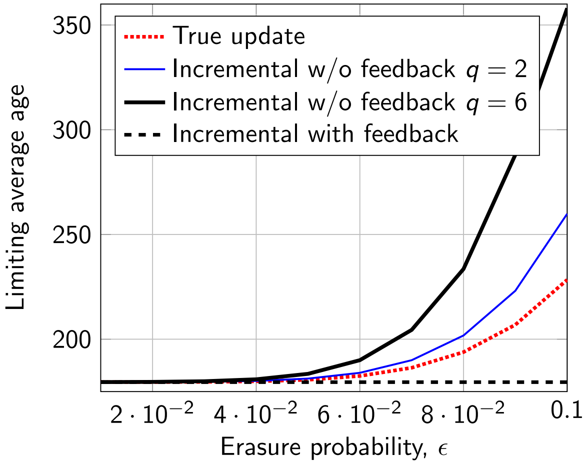
System Parameters

- Random coding scheme
- Code length $n = 120$
- Number of information bits $m = 105$
- $q \in \{2, 6\}$





600
500
400
300
200
Limiting average age
True update
Incremental w/o feedback $q = 2$
Incremental w/o feedback $q = 6$
Incremental with feedback
20
40
60
80
100
Differential information bits, k



True update
Incremental w/o feedback $q = 2$
Incremental w/o feedback $q = 6$
Incremental with feedback
Limiting average age
350
300
250
200
$2 \cdot 10^{-2}$
$4 \cdot 10^{-2}$
$6 \cdot 10^{-2}$
$8 \cdot 10^{-2}$
0.1
Erasure probability, ϵ

Discussion and Concluding Remarks

Main Contributions

- Integration of coding and renewal techniques to study timely communication for delay-sensitive traffic
- We model channel unreliability by the erasure channel
- Incremental updates only when there is feedback availability

Avenues of Future Research

- Extend results to structured sources
- Extend results to correlated finite-state erasure and error channels
- Impact of other coding schemes on timeliness