Optimal Pricing in Finite Server Systems

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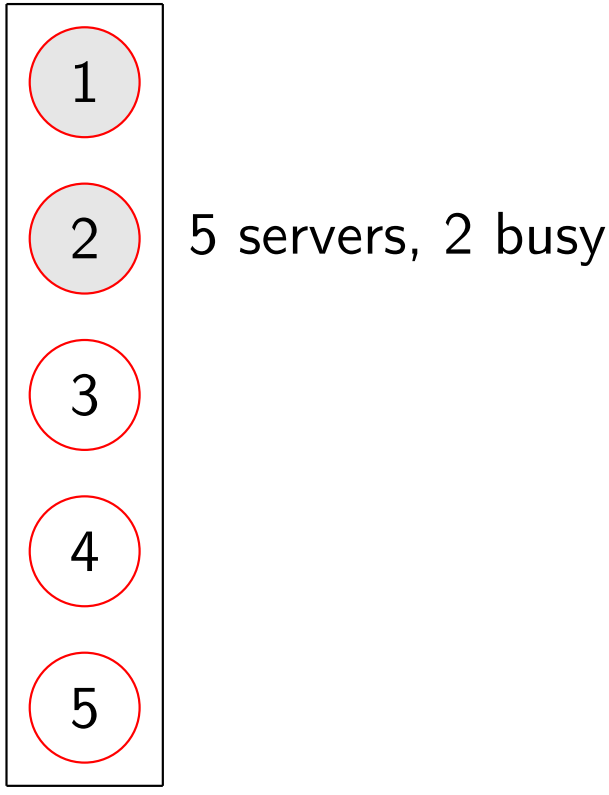
The Problem



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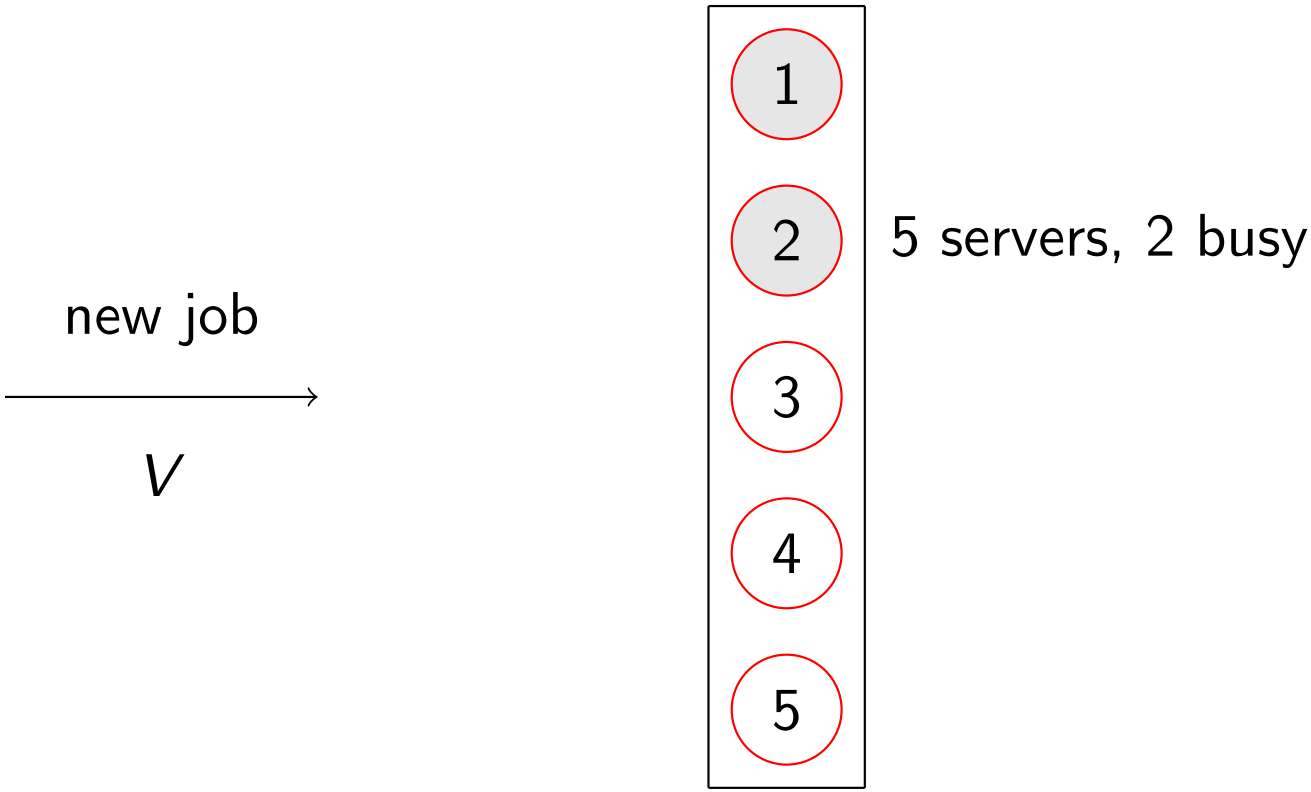

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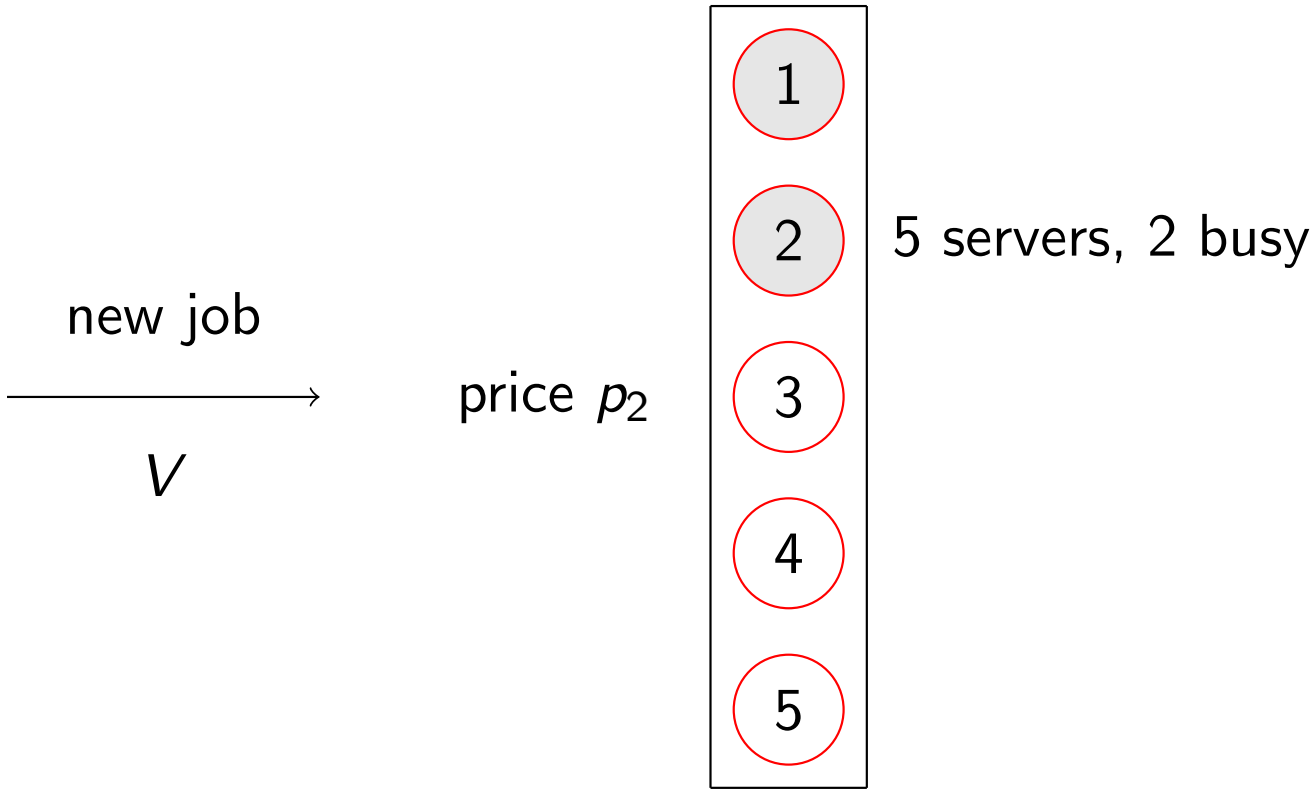

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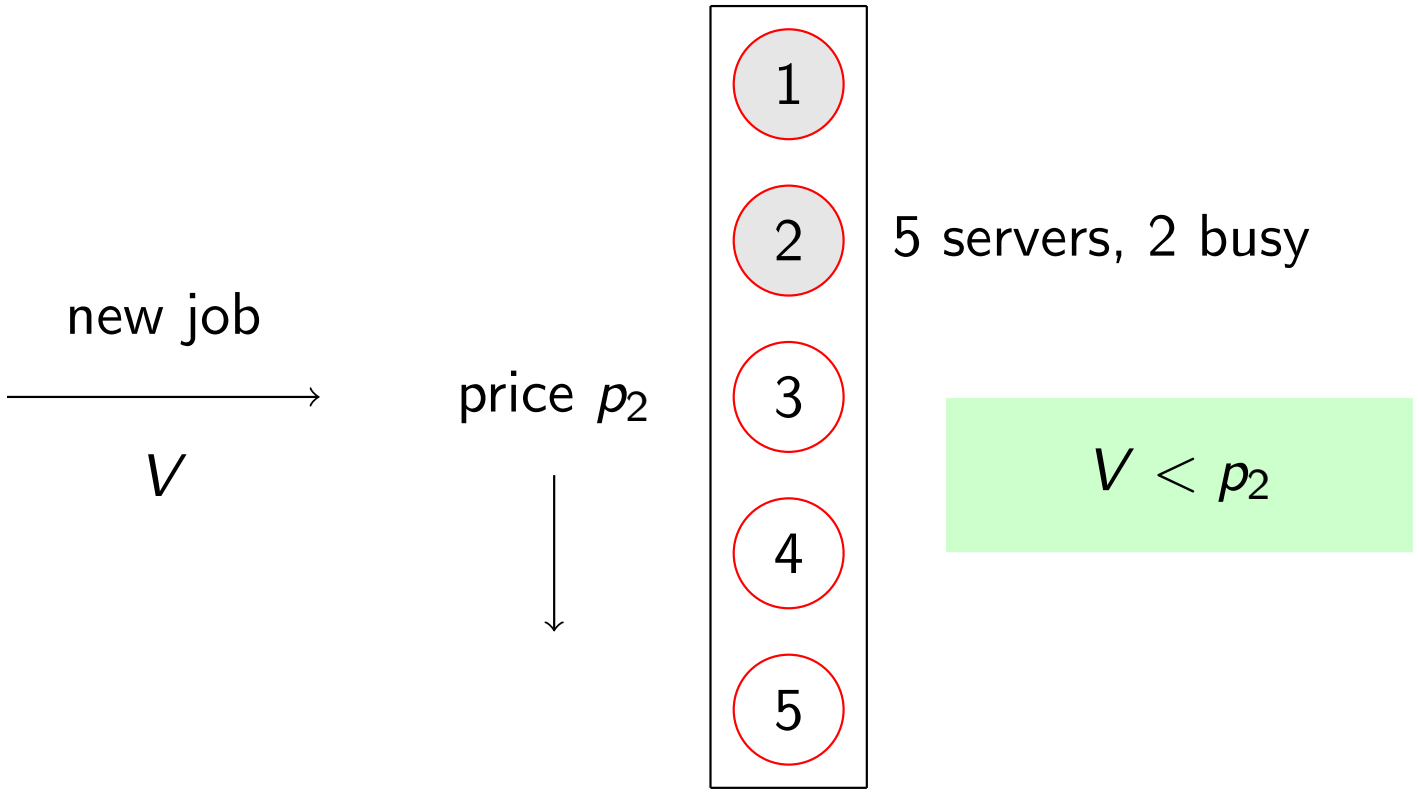

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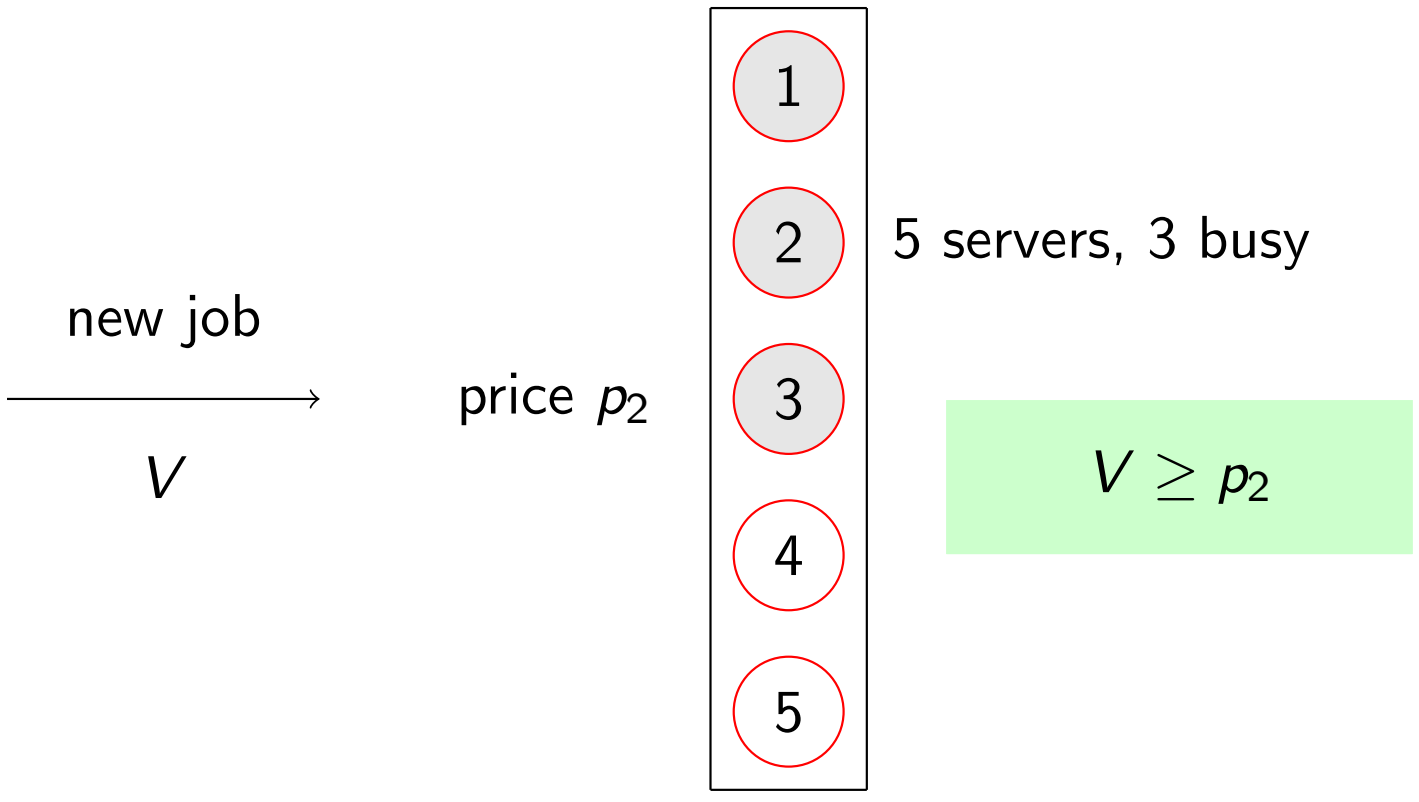

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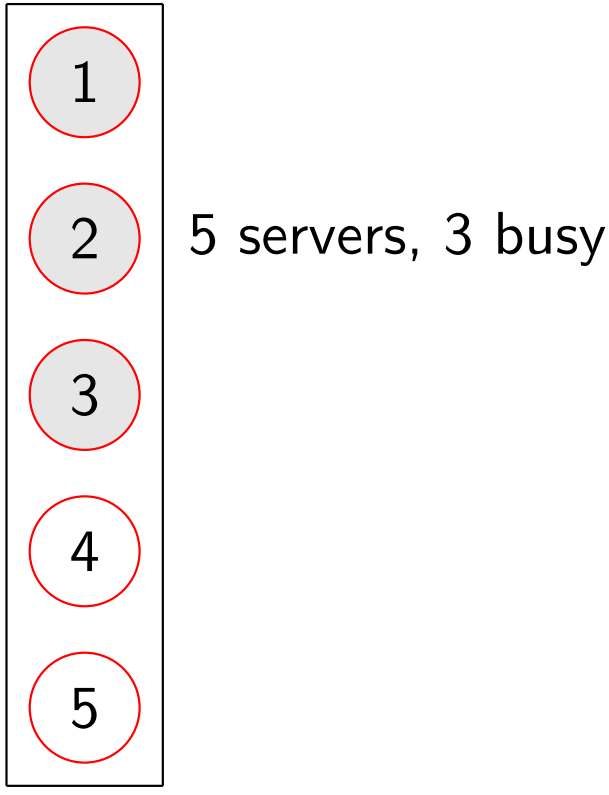

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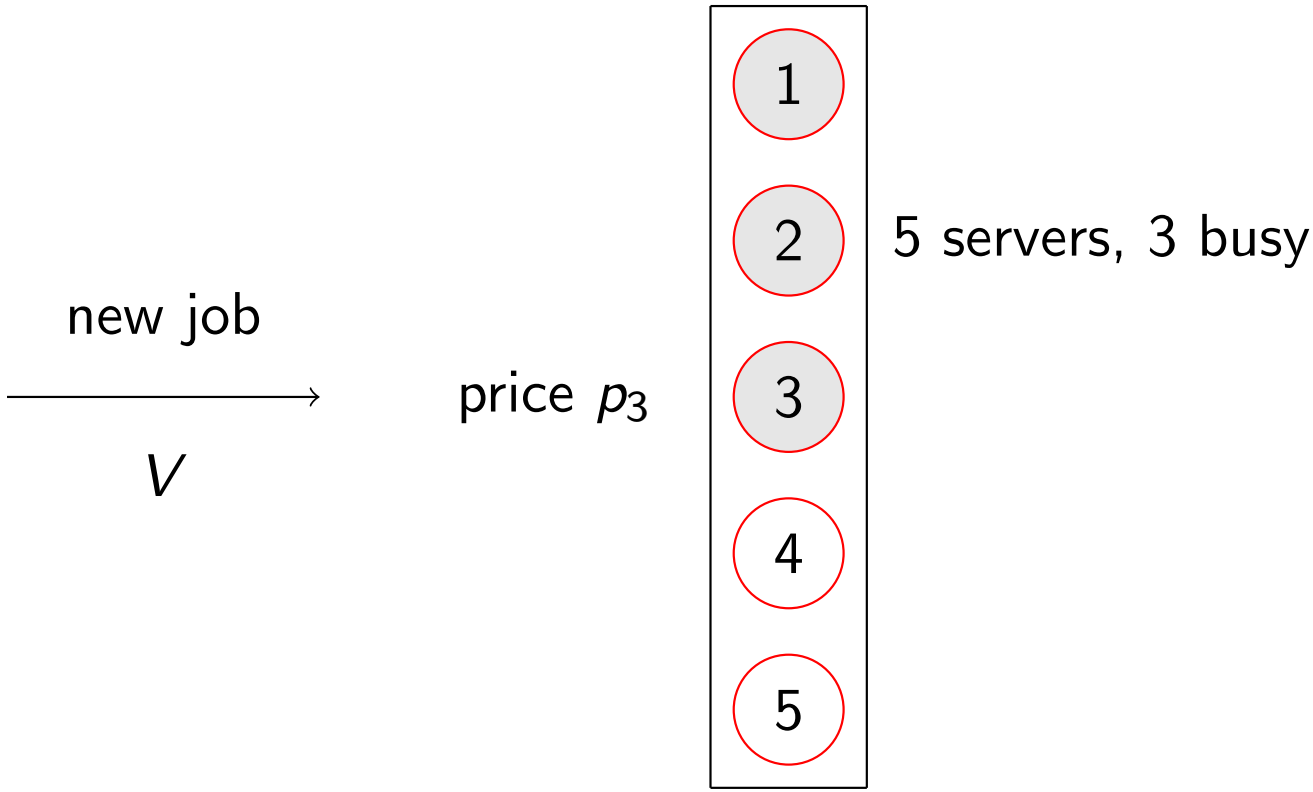

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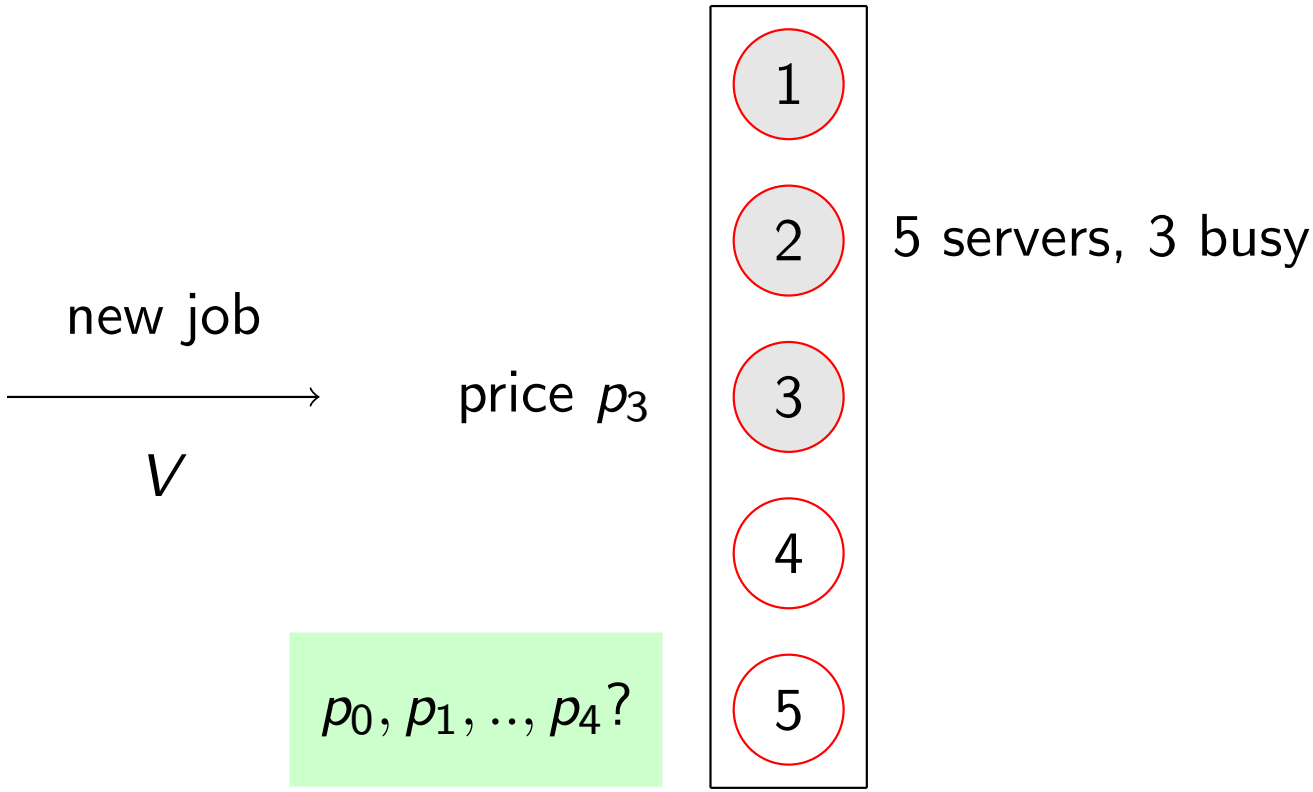
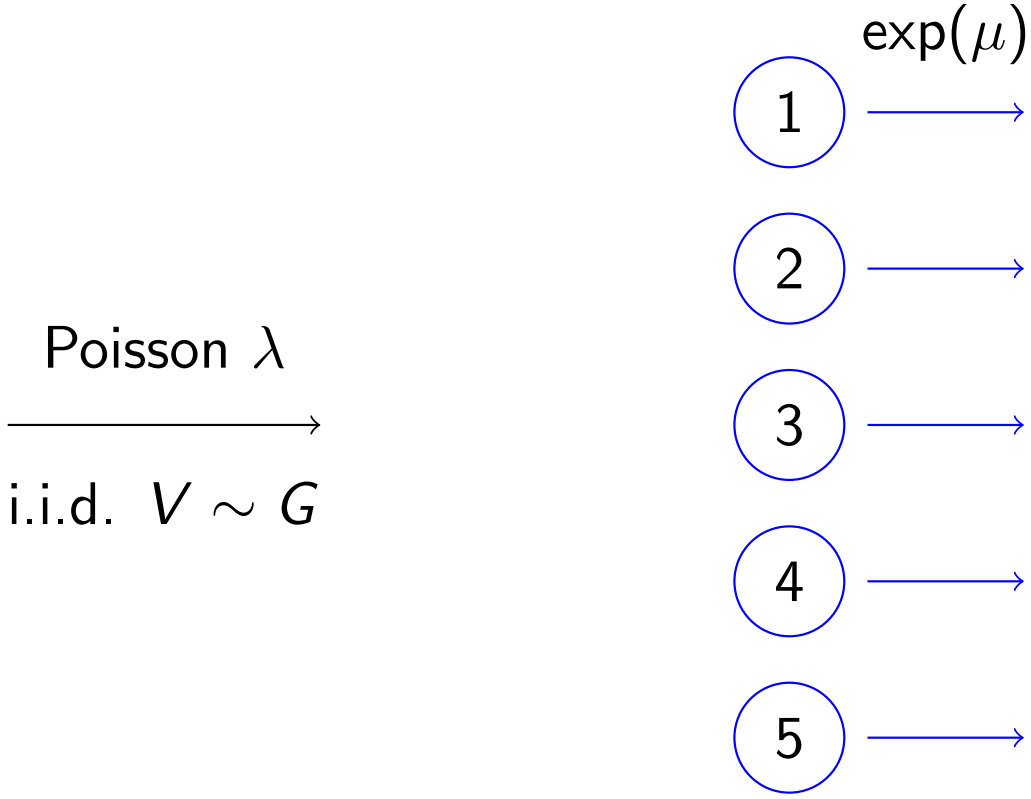


Previous Work and Our Setting

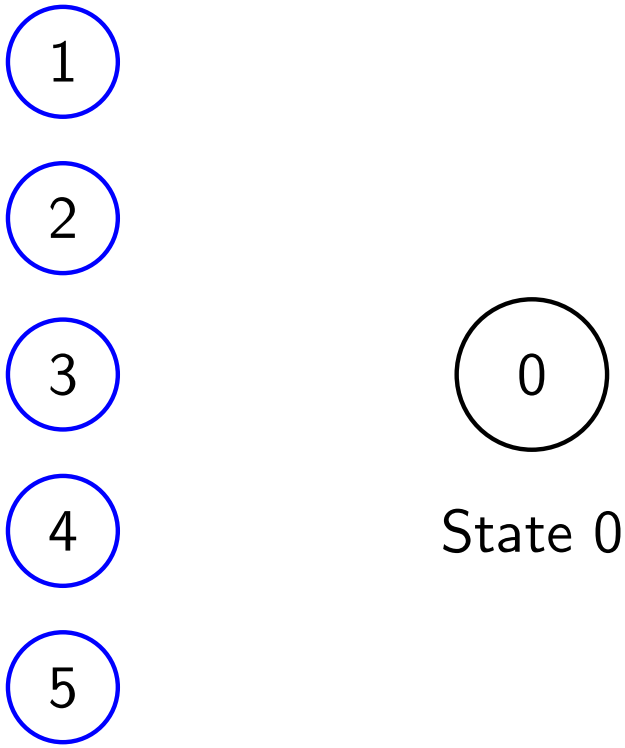
- Social optimum vs Revenue Maximization
  - Naor '69, Chen '01, Borgs '14
- Homogeneous vs heterogeneous customers
  - Whang '90, Shimkin '00, Mandelbaum '02
- Single vs Multi Servers
  - Haviv '94, Bradford '96, Dumas '11
- Our setting:
  - Revenue maximization
  - Heterogeneous customers
  - Multi server system

System Model



price vector $(p_0, p_1, p_2, p_3, p_4)$

State Evolution



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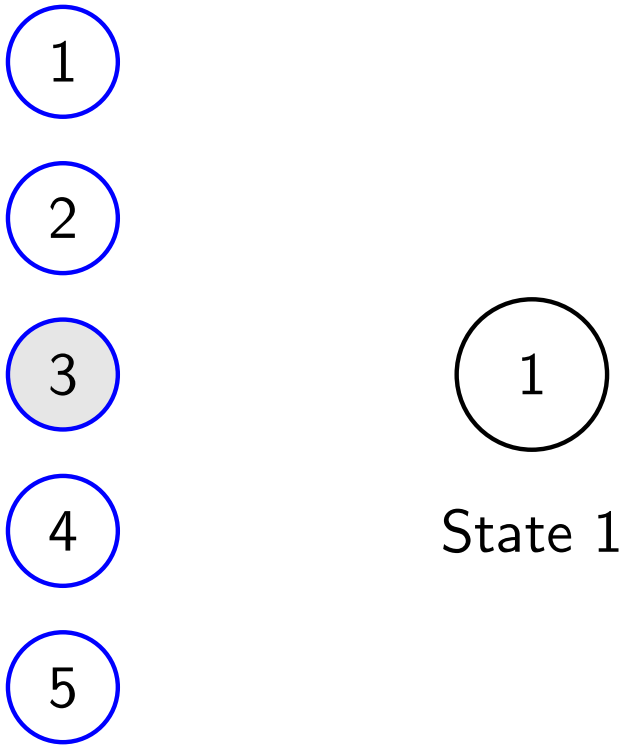

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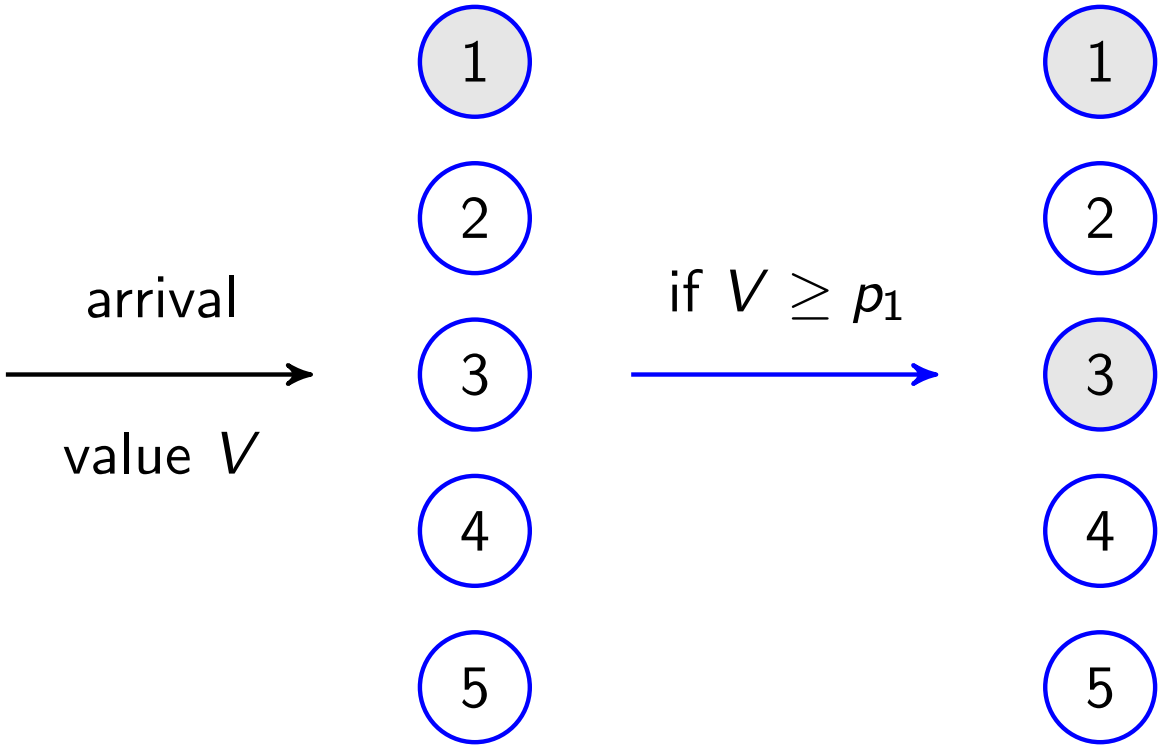

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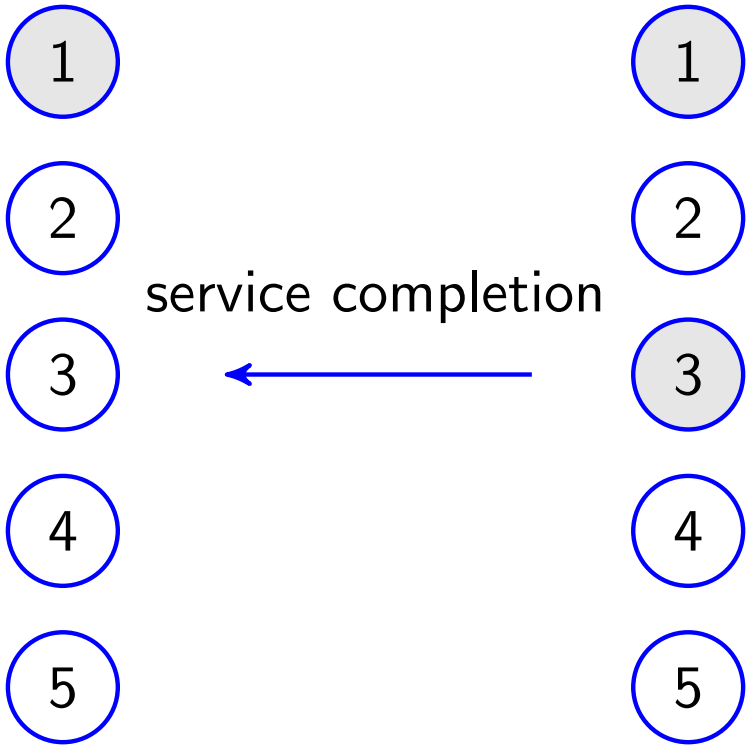

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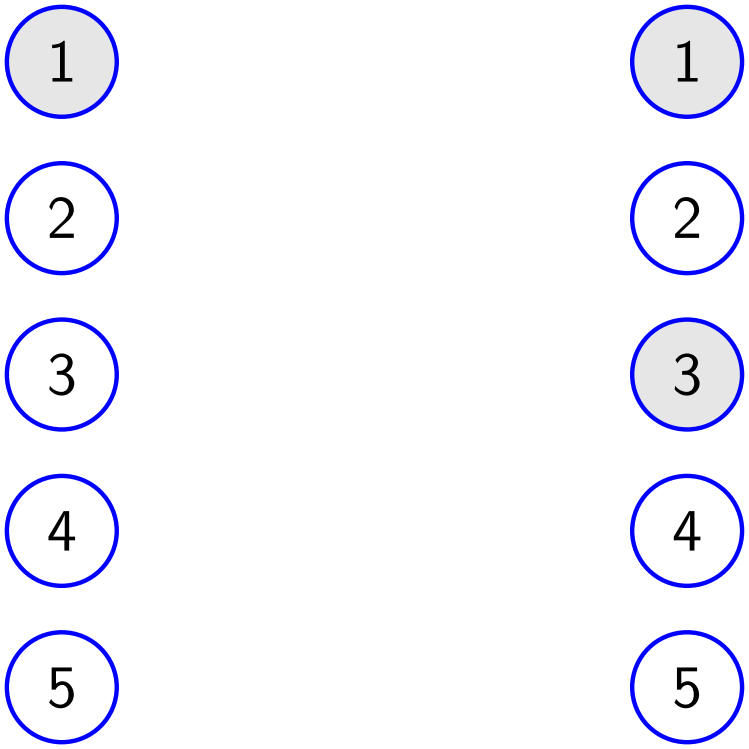
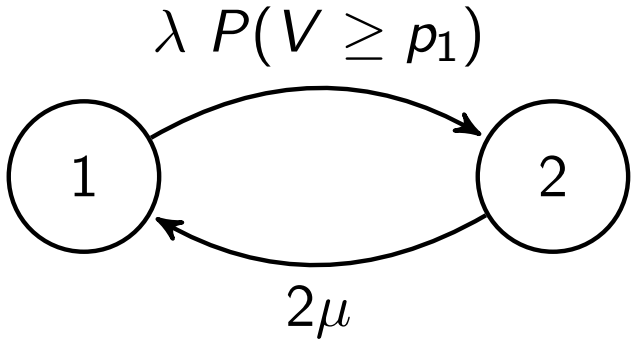
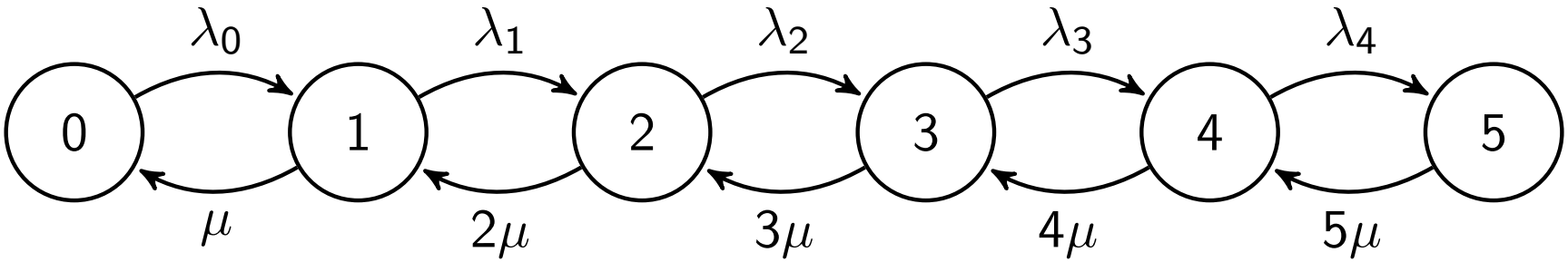




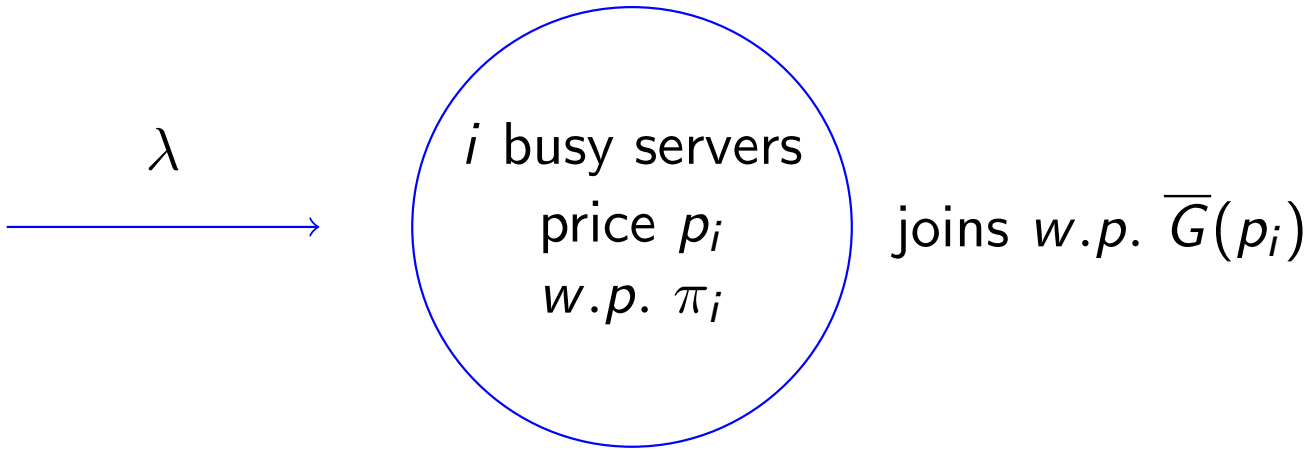
State evolution



$$\lambda_i = \lambda P(V \geq p_i) = \lambda(1 - G(p_i)) = \lambda \overline{G}(p_i)$$

Gives stationary distribution π

Revenue



$$\text{Revenue} = \lambda \sum_{i=0}^{K-1} \pi_i \overline{G}(p_i) p_i$$

The Infinite Server Case

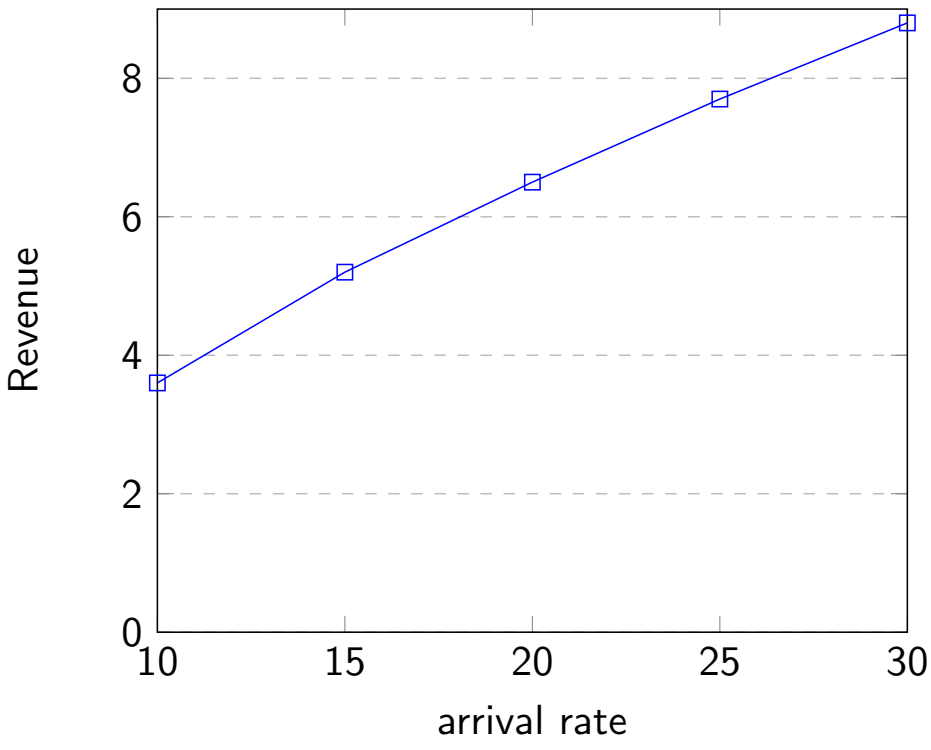
1. Infinite Servers
2. Revenue $= \lambda \sum_{i=0}^{\infty} \pi_i \overline{G}(p_i) p_i$
3. $\leq \lambda \overline{G}(p^*) p^* \sum_i \pi_i$
4. $p^* = \arg\max p \overline{G}(p)$
5.

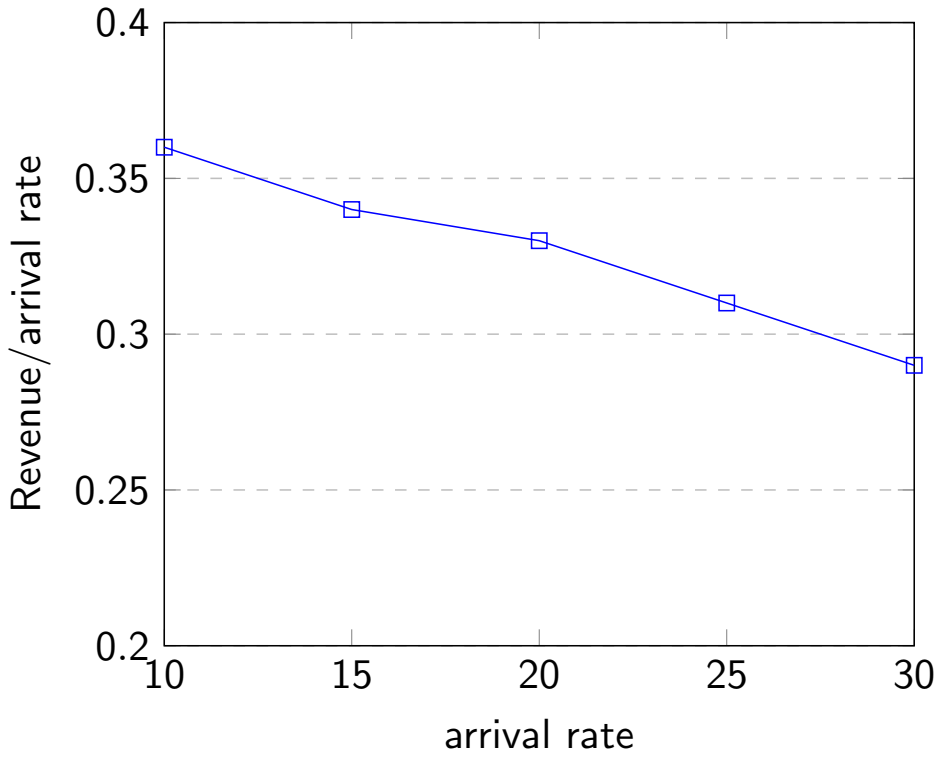
A Sub-Optimal Scheme

1. Uniform Pricing
2. All states have same price
3. $p_5^* = \arg\max p\overline{G}(p)(1 - \pi_5(p))$
4.
5.

Optimal Price

Properties of the Optimal Solution





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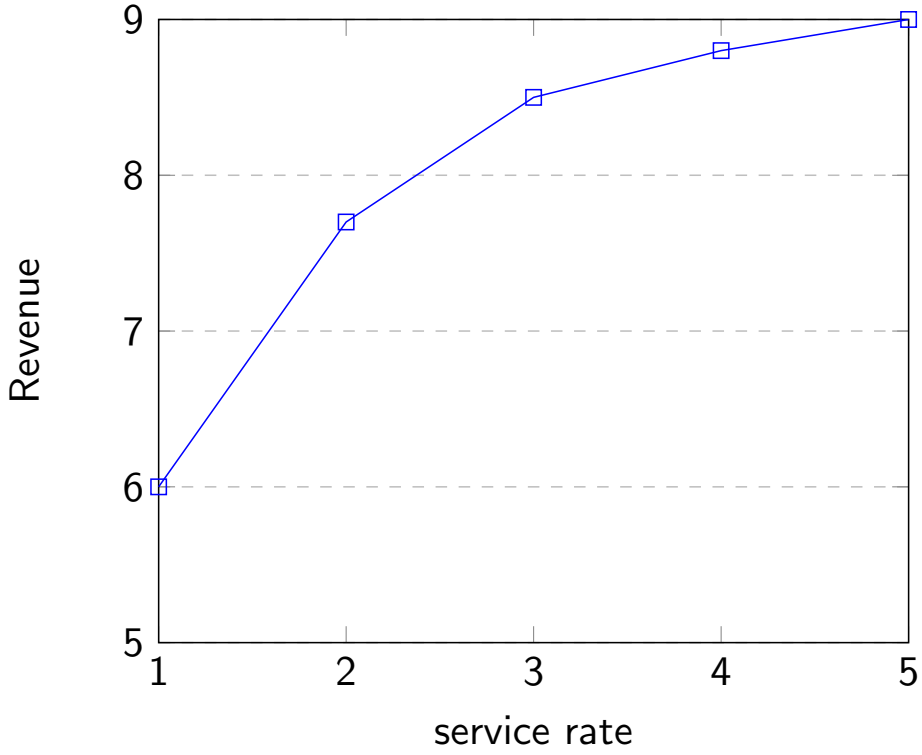

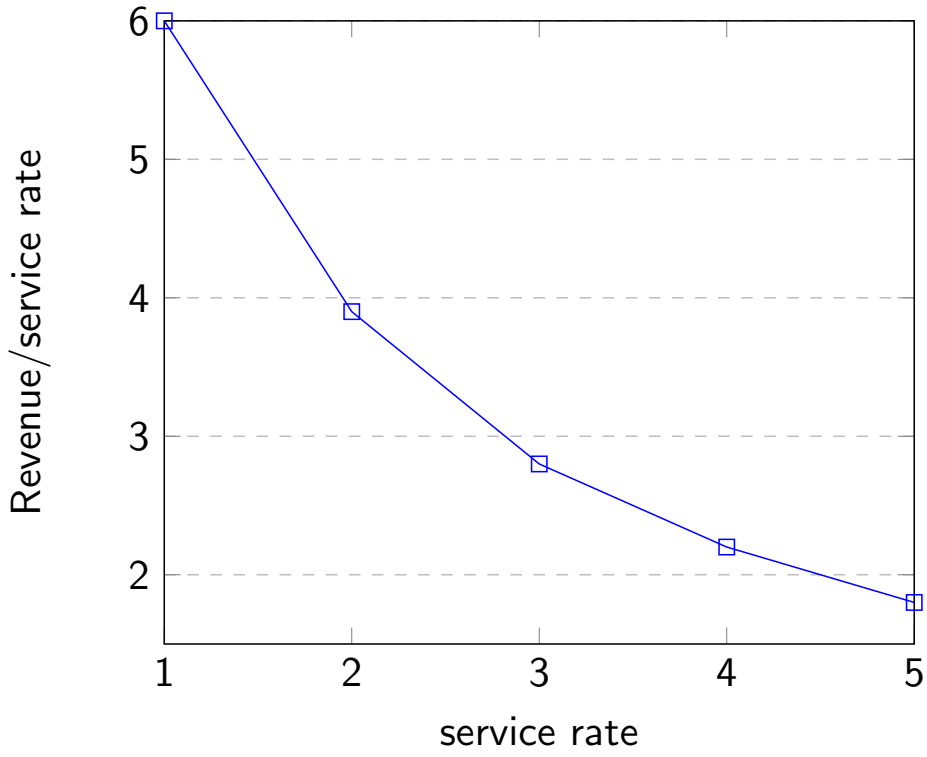

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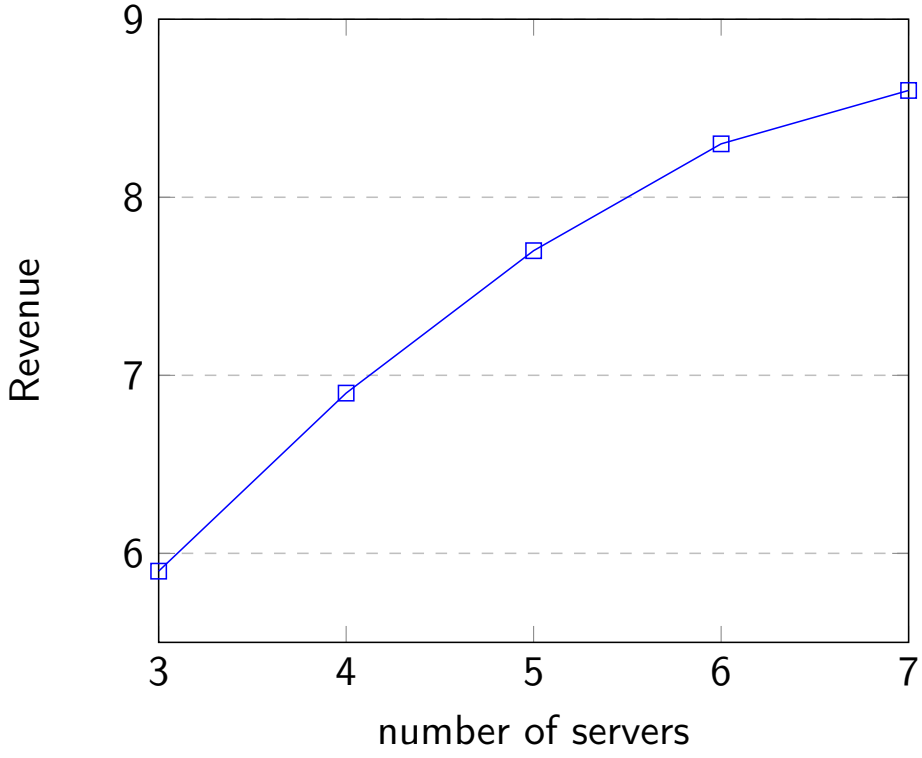

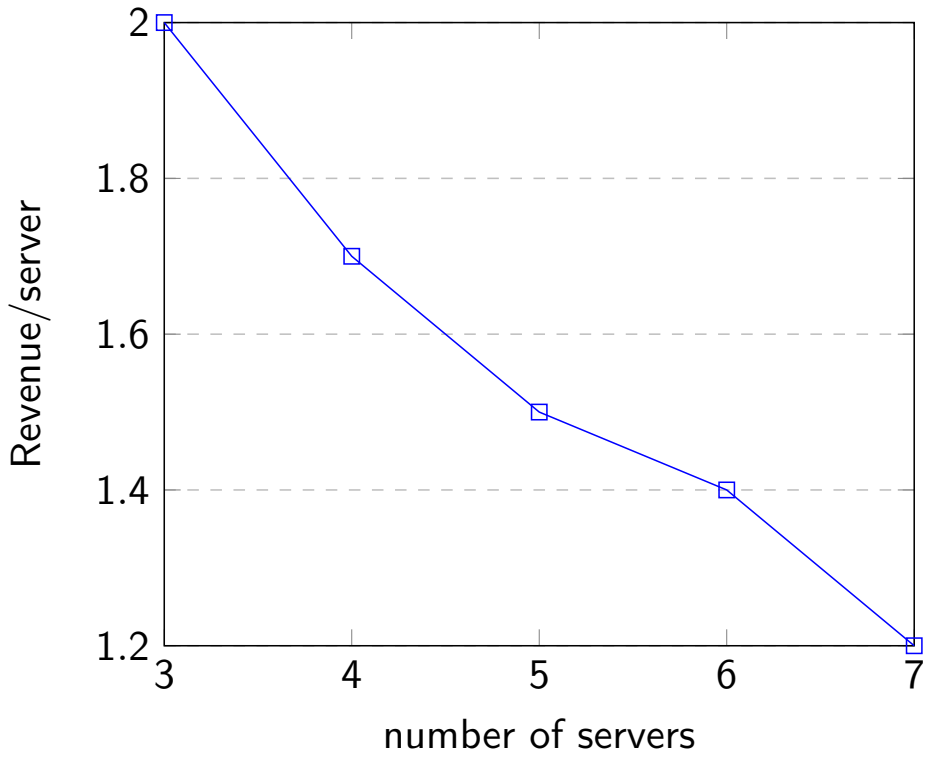

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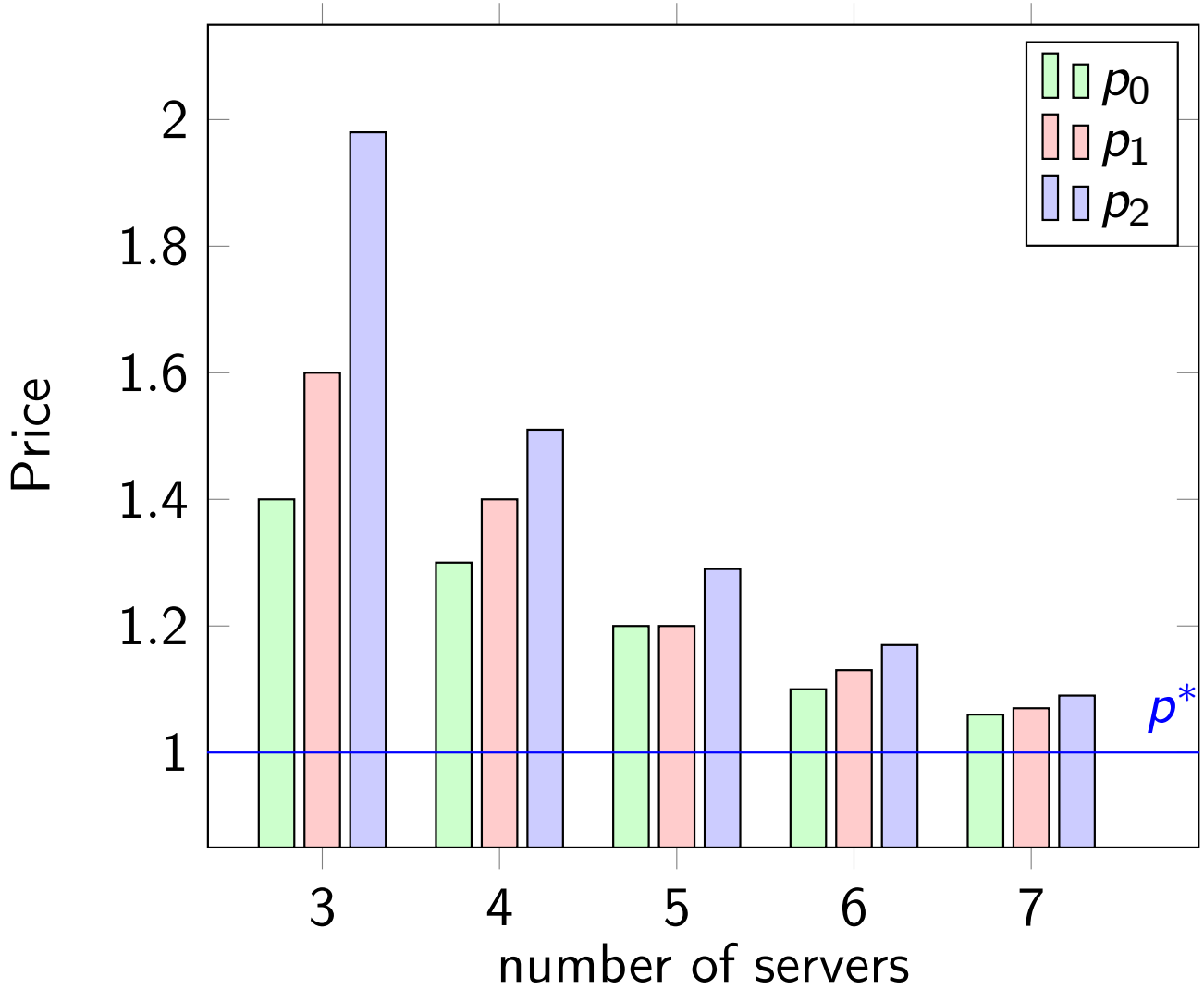

Revenue Gain

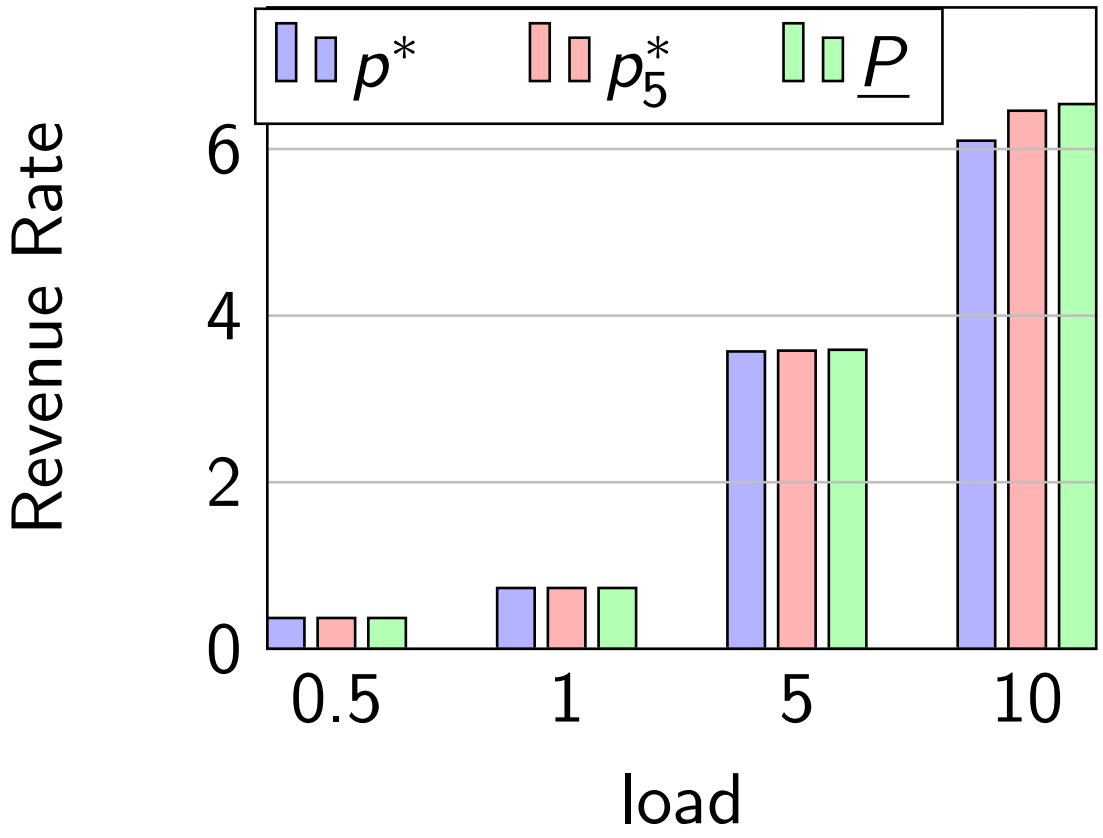

Figure: Revenue rate as a function of load

Summary

- Analysis of system with heterogeneous customers
- Solution to the server pricing problem for revenue maximization
- Uniform pricing is optimal for infinite server system
- Analytical MDP solution to obtain the optimal pricing for a finite server system
- Two simple heuristic algorithms for pricing a finite server system
- Properties of optimal pricing for finite server systems
- Performance comparison between the optimal and heuristic algorithms for finite server systems