1. Consider n strings with two ends each. We assume that each end can connect with any open end uniformly at random. Let the number of loops (cycles) formed be L_n. Calculate $\mathbb{E}[L_n]$.

2. At a party, N men throw their hats into the centre of a room. The hats are mixed up and each man randomly selects one. Find the expected number of men who select their own hats. Find the conditional expected number of matches given that the first person did not have a match.

3. Let $(\Omega, \mathcal{F}, P)$ be a probability space. Show that $\mathcal{G} = \{A \in \mathcal{F} : P(A) = 0 \text{ or } 1\}$ is a σ-algebra

4. If $X_1, X_2, \ldots$ are random variables then show that the following are also random variables.
   (a) $\inf_n X_n$,
   (b) $\sup_n X_n$,
   (c) $\limsup_n X_n$, and
   (d) $\liminf_n X_n$.

5. Assume that the area of a circle A on a plane is distributed as

$$f(A) \triangleq \alpha A e^{-A^2} \mathbb{1}_{\{A \geqslant 0\}}.$$

   (a) Calculate α.
   (b) Find the distribution of the circumference C and the radius r?

6. You know that a certain letter is equally likely to be in any one of three different folders. Let α_i be the probability that you will find your letter upon making a quick examination of folder i if the letter is, in fact, in folder $i \in [3]$. It is possible that $\alpha_i < 1$. Suppose you look in folder 1 and do not find the letter. What is the probability that the letter is in folder 1?

7. Consider an independent sequence of trials, each of which is a success with probability p. The trials are performed until there are k consecutive successes. Compute the mean number of necessary trials using conditional expectation.

8. Consider zero-mean random variables X, Y with correlation ρ. Prove that $\mathbb{E}[\mathrm{Var}[Y|X]] \leqslant (1-\rho^2)\,\mathrm{Var}[Y]$.