Mean Delay for File Access in Distributed Coded Storage

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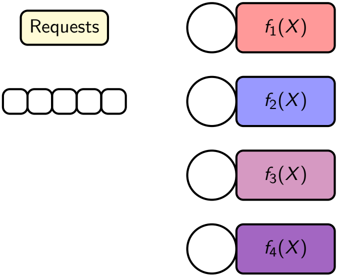
Problem Statement



Quantify mean access time

- with number of fragments for a single message X,
- with encoding and storage $f_i(X)$ for fragmented message $X = (X_1, \dots, X_k)$ at n distinct nodes.

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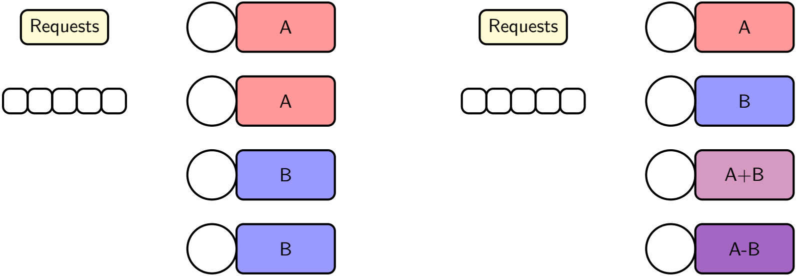


Problem

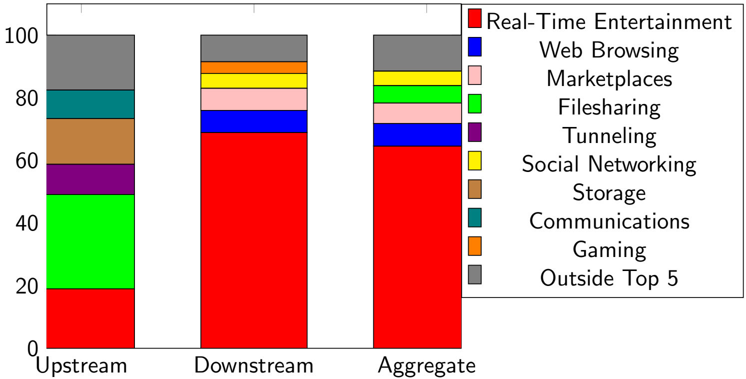
Quantify the latency gains offered by distributed coding

Solution

Coded storage offers scaling gains over replication

Dominant traffic on Internet

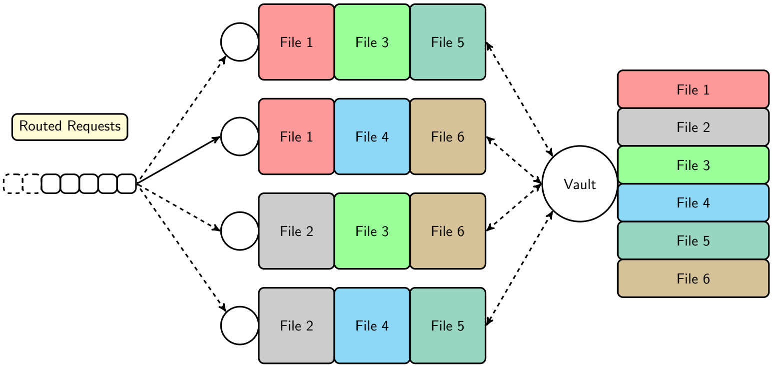
Peak Period Traffic Composition (North America)



- Real-Time Entertainment: 64.54% for downstream and 36.56 % for mobile access[1]

[1] https://www.sandvine.com/downloads/general/global-internet-phenomena/2015/global-internet-phenomena-report-latin-america-and-north-america.pdf

Established Solutions – Content Delivery Network



Congestion Prevention and Outage Protection

- Mirroring content with local servers
- Media file on multiple servers

System Model

File storage

- Each media file divided into k pieces
- Pieces encoded and stored on n servers

Arrival of requests

- Each request wants entire media file
- Poisson arrival of requests with rate λ

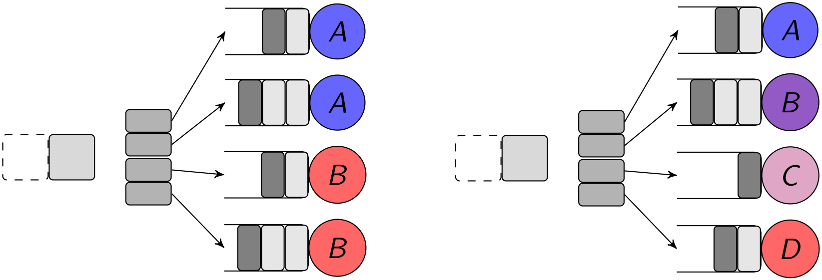
Time in the system

- Till the reception of whole file

Service at each server

- IID exponential service time with rate k/n

Question: Duplication versus MDS Coding



Reduction of access time

- How to select number of fragments for a single message?
- How to encode and store at the distributed storage nodes?

Pertinent References (very incomplete)

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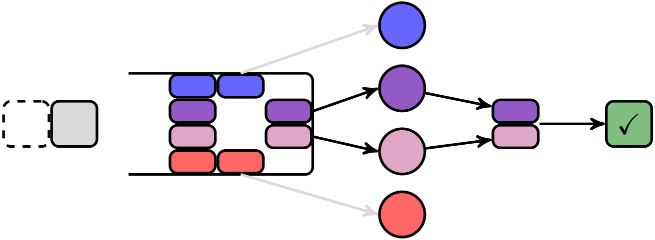
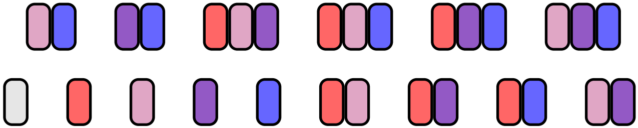
- B. Li, A. Ramamoorthy, R. Srikant, "Mean-field-analysis of coding versus replication in cloud storage systems", INFOCOM, 2016.

Storage Coding – The Centralized MDS Queue



exempli gratia: Shah, Lee, Ramchandran (2013), Lee, Shah, Huang, Ramchandran (2017), Vulimiri, Michel, Godfrey, Shenker (2012), Ananthanarayanan, Ghodsi, Shenker, Stoica (2012) Baccelli, Makowski, Shwartz (1989)

State Space Structure



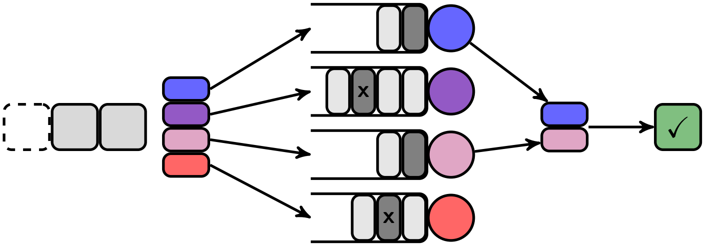
Keeping Track of Partially Fulfilled Requests

- Element of state vector $Y_S(t)$ is number of users with given subset S of pieces

Continuous-Time Markov Chain

- $\mathbf{Y}(t) = \{Y_S(t) : S \subset [n], |S| < k\}$ is a Markov process

Storage Coding – (n, k) Fork-Join Model



exempli gratia: Joshi, Liu, Soljanin (2012, 2014), Joshi, Soljanin, Wornell (2015), Sun, Zheng, Koksal, Kim, Shroff (2015), Kadhe, Soljanin, Sprintson (2016), Li, Ramamoorthy, Srikant (2016)

State Space Collapse

Theorem

For duplication and coding schemes under priority scheduling and parallel processing model, collection

$$\mathcal{S}(t) = \{S \subset [n] : Y_S(t) > 0, |S| < k\}$$

of information subsets is totally ordered in terms of set inclusion

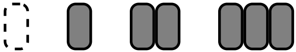
Corollary

Let $Y_i(t)$ be number of requests with i information symbols at time t, then

$$\mathbf{Y}(t) = (Y_0(t), Y_1(t), \ldots, Y_{k-1}(t))$$

is Markov process

State Transitions of Collapsed System



Arrival of Requests

- Unit increase in $Y_0(t) = Y_0(t-) + 1$ with rate λ

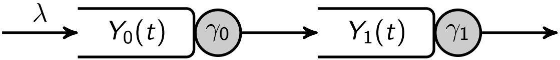
Getting Additional Symbol

- Unit increase in $Y_i(t) = Y_i(t-) + 1$
- Unit decrease in $Y_{i-1}(t) = Y_{i-1}(t-) - 1$

Getting Last Missing Symbol

- Unit decrease in $Y_{k-1}(t) = Y_{k-1}(t-) - 1$

Tandem Queue Interpretation (No Empty States)



Duplication

- When all states non-empty
- No. servers available at level i is n/k
- Normalized service rate at level i

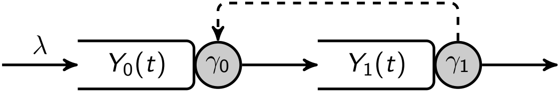
$$\gamma_i = 1 \quad i = 0, \dots, k-1$$

MDS Coding

- When all states non-empty
- One server available at level $i \neq k-1$
- Normalized service rate at level i

$$\gamma_i = \begin{cases} \frac{k}{n} & i < k-1 \\ \frac{k}{n}(n-k+1) & i = k-1 \end{cases}$$

Tandem Queue Interpretation (General Case)



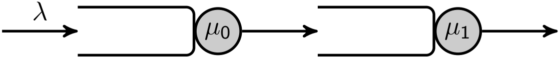
Tandem Queue with Pooled Resources

- Servers with empty buffers help upstream
- Aggregate service at level i becomes

$$\sum_{j=i}^{l_i(t)-1} \gamma_j \qquad \text{where} \qquad l_i(t) = k \wedge \{l > i : Y_l(t) > 0\}$$

- No explicit description of stationary distribution for multi-dimensional Markov process

Bounding and Separating



Theorem†

When $\lambda < \min \mu_i$, tandem queue has product form distribution

$$\pi(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\mu_i} \left(1 - \frac{\lambda}{\mu_i}\right)^{y_i}$$

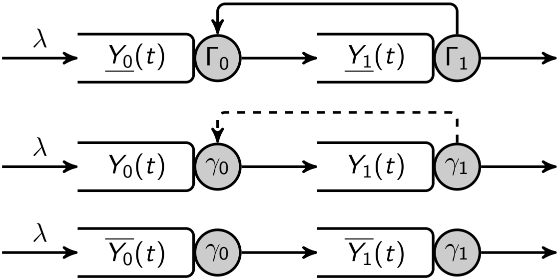
Uniform Bounds on Service Rate

Transition rates are uniformly bounded by

$$\gamma_i \leq \sum_{j=i}^{l_i(y)-1} \gamma_j \leq \sum_{j=i}^{k-1} \gamma_j \triangleq \Gamma_i$$

† F. P. Kelly, Reversibility and Stochastic Networks. New York, NY, USA: Cambridge University Press, 2011.

Bounds on Tandem Queue



Lower Bound

Higher values for service rates yield lower bound on queue distribution

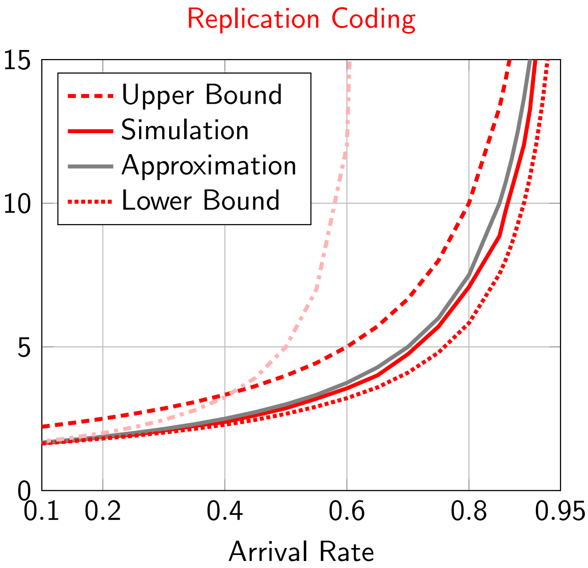
$$\underline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\Gamma_i}\left(1 - \frac{\lambda}{\Gamma_i}\right)^{y_i}$$

Upper Bound

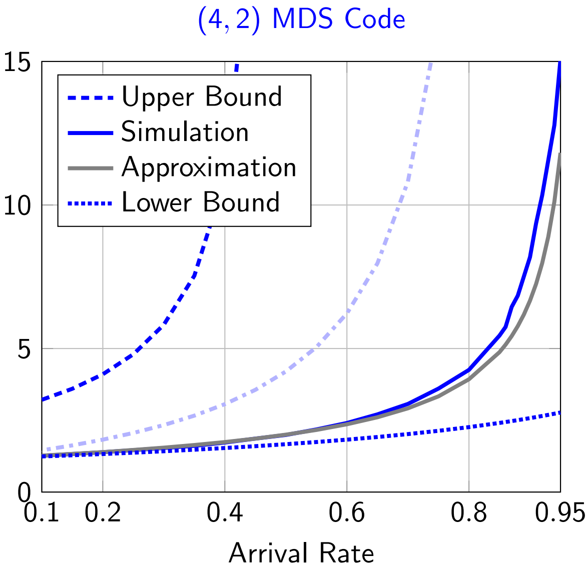
Lower values for service rate yield upper bound on queue distribution

$$\overline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\gamma_i}\left(1 - \frac{\lambda}{\gamma_i}\right)^{y_i}$$

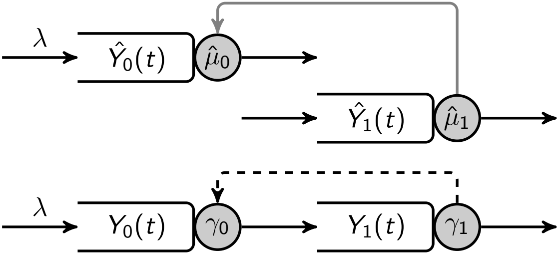
Mean Sojourn Time



Mean Sojourn Time



Approximating Pooled Tandem Queue



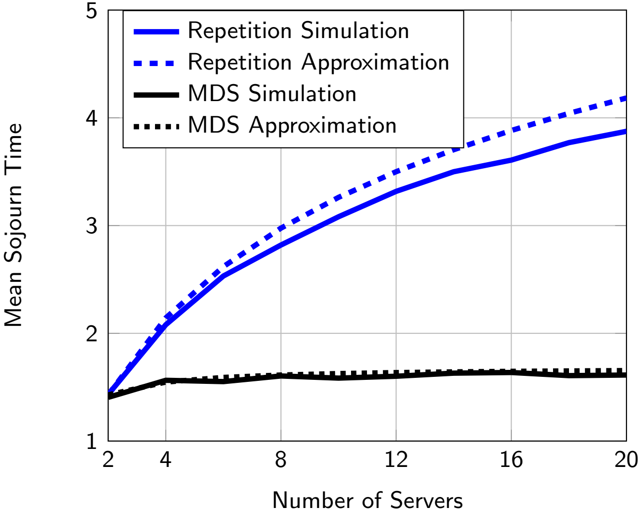
Independence Approximation with Statistical Averaging

Service rate is equal to base service rate γ_i plus cascade effect, averaged over time

$$\hat{\mu}_{k-1} = \gamma_{k-1}$$
$$\hat{\mu}_i = \gamma_i + \hat{\mu}_{i+1}\hat{\pi}_{i+1}(0)$$

$$\hat{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\hat{\mu}_i}\left(1 - \frac{\lambda}{\hat{\mu}_i}\right)^{y_i}$$

Comparing Replication versus MDS Coding



Arrival rate 0.3 units and coding rate $n/k = 2$

Summary and Discussion

Main Contributions

- Analytical framework for study of distributed computation and storage systems
- Upper and lower bounds to analyze replication and MDS codes
- A tight closed-form approximation to study distributed storage codes
- MDS codes are better suited for large distributed systems
- Mean access time is better for MDS codes for all code-rates