Real-Time Status Updates for Markov Sources over Unreliable Channels

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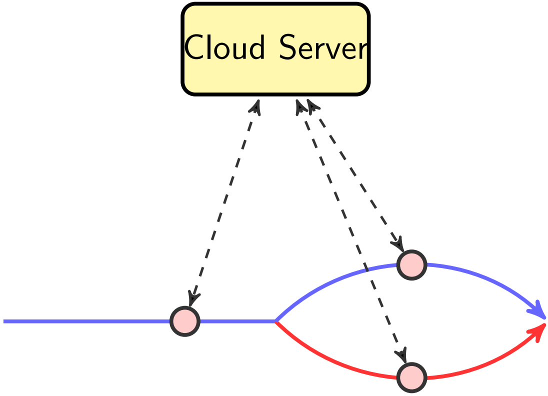
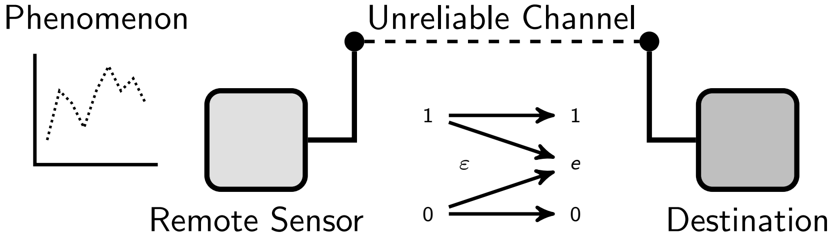
Real-time decision systems



- Cyber-physical systems: Environmental/health monitoring
- Internet of Things: Real-time actuation/control
- Critical to know the status update before decision making

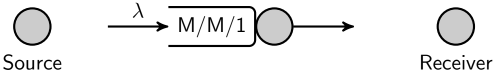
Status updates over unreliable channels



Question

How to encode message at the temporally correlated source for timely update over an iid binary symmetric erasure channel?

Quantifying timeliness: delay versus age



Information packet delay at transmitter

- If the arrival rate is higher than the service rate, then unstable queue and infinite delay

Information age at receiver

- Generation time of last received information packet $U(t)$
- Age of information $A(t) = t - U(t)$
- Finite average age for always on source

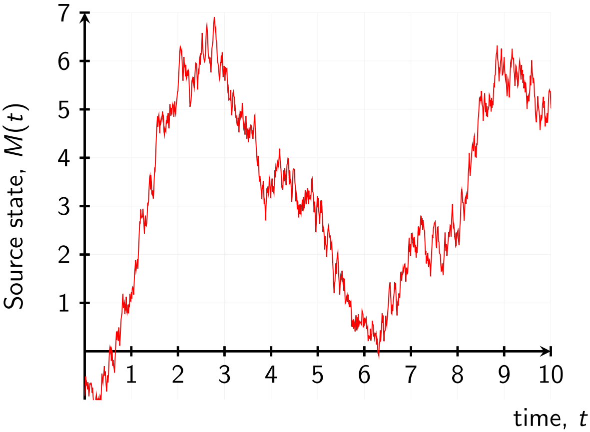
Prior work

- 'Age of information' introduced by Kaul and R. Yates in 2012
- Different queueing models and queueing disciplines are used to analyze update system [A. Ephermides, N. B. Shroff, Y. Sun, E. Modiano, M. Costa]
- M/M/1 queue with FCFS/LCFS [Kaul, Gruster, S. Kompella, R. Yates]

Summary

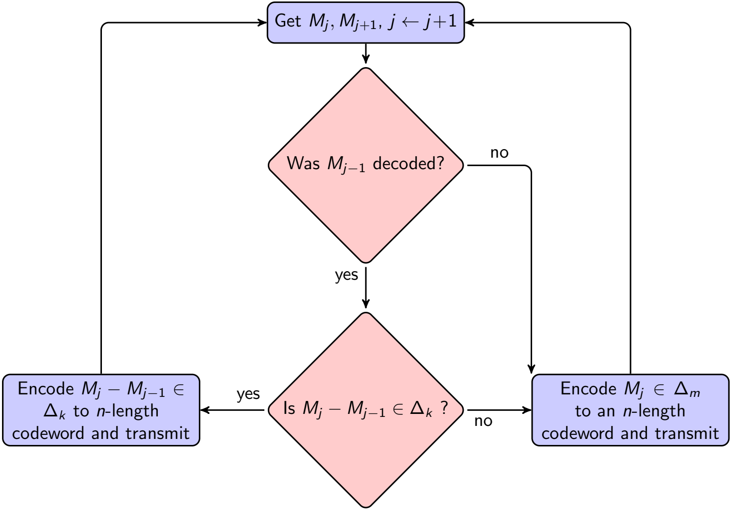
- Limiting average age minimizing update rate
- Channel uncertainty modeled by random service time
- A scheduled update is delivered successfully

Source Model



- Discrete sampled source $M_j = M(jn) \in \Delta_m$ Markov with transition matrix P and invariant distribution ν

Update Protocol



- Special case when $\Delta_k = 0$, always send actual update

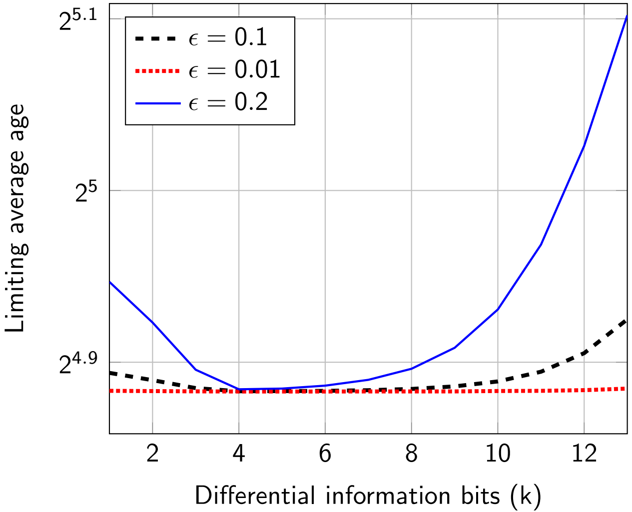
Problem Statement

Question

For a fixed code-length n, find the code-rate $\frac{k}{n}$ for the given Markov source $(M_j \in \Delta_m : j \in \mathbb{N})$ over iid binary symmetric erasure channel $\sim$ Bernoulli(ϵ) that minimizes the limiting average age

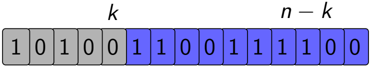
$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} A(t).$$

Optimal code-rate

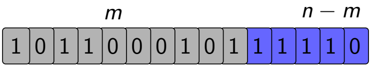


- Code length $n = 20$, number of information bits $m = 15$

Permutation invariant code



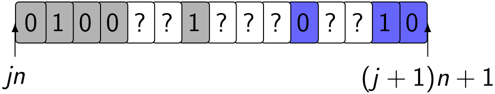
Incremental Update



True Update

Bit-wise erasure channel

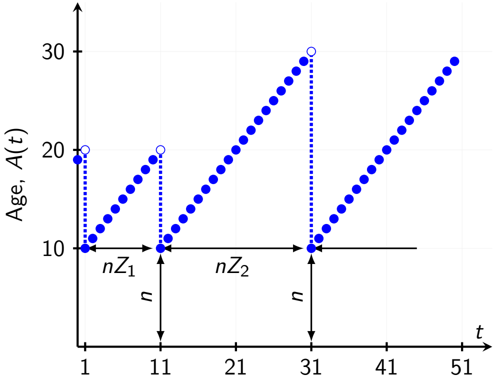
- Number of erasures per codeword $E \sim \text{Binomial}\ (n, \epsilon)$



Probability of decoding failure

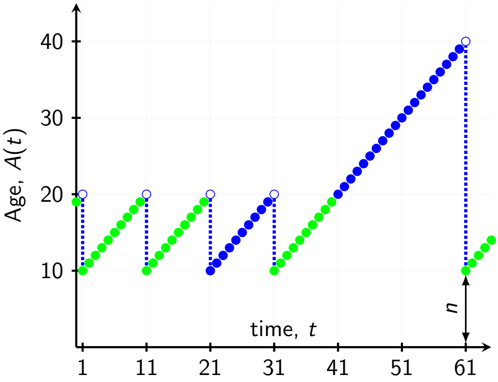
- Actual state updates: $p_a = \mathbb{E}P(n, n - m, E)$
- Incremental updates: $p_d = \mathbb{E}P(n, n - k, E)$

Information age process



- Age determined by scaled samples $(A_j = \frac{1}{n}A(jn+1) : j \in \mathbb{N})$
- Mean average age is $\frac{\mathbb{E}\sum_{t=1}^{T_1} A(t)}{\mathbb{E}\, T_1} = n\mathbb{E}A_j + \frac{(n-1)}{2}$

Age process for incremental updates



- Decoding success iff $A_j = 1$, Bernoulli indicators with p_a, p_d
- Incremental update if $\{A_{j-1} = 1\} \cap \{M_j - M_{j-1} \in \Delta_k\}$
- Process $((M_j, A_j) : j \in \mathbb{N})$ is jointly Markov

Highly Correlated Source

- $M_j - M_{j-1} \in \Delta_k$ almost surely, then $P_i(\Delta_k) = \sum_{l-i \in +\Delta_k} P_{il} = 1$ or $\tilde{D} = (p_a - p_d)P$

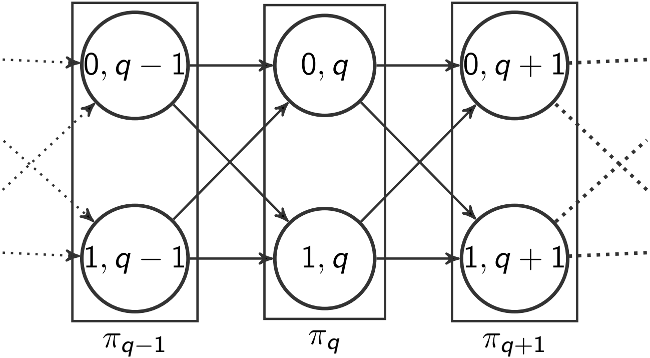
$$\begin{bmatrix} P_{00} & P_{01} & 0 & 0 & \dots & \dots \\ P_{10} & P_{11} & P_{12} & 0 & \dots & \dots \\ 0 & P_{21} & P_{22} & P_{23} & \dots & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \end{bmatrix}$$

Uniform Markov Source

- $P_i(\Delta_k)$ is identical to all states i

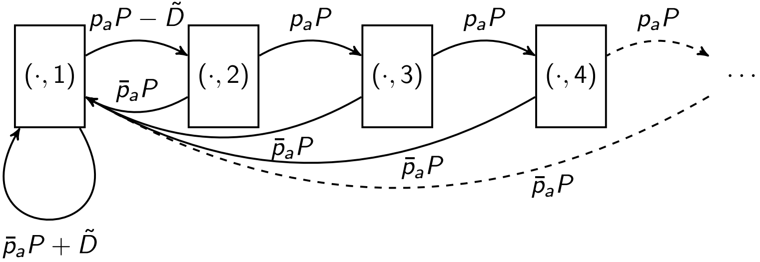
$$\begin{bmatrix} P_{00} & P_{01} & P_{02} & P_{03} & \dots & \dots \\ P_{10} & P_{11} & P_{12} & P_{13} & \dots & \dots \\ P_{20} & P_{21} & P_{22} & P_{23} & \dots & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \end{bmatrix}$$

Quasi birth-death process



- Transition probability $(i, q) \to (j, q+1)$: $p_a P_{ij}$
- Transition probability $(i, 1) \to (i, 2)$: $-(p_a - p_d) P_{ij} 1_{\{j-i \in \Delta_k\}} + p_a P_{ij}$

Block transitions



- $\tilde{D}_{ij} = (p_a - p_d)P_{ij}1_{\{j-i\in\Delta_k\}}$
- Transition matrix for the joint Markov process

$$\begin{bmatrix} p_aP + \tilde{D} & p_aP - \tilde{D} & 0 & 0 & \dots & \dots \\ \bar{p}_aP & 0 & p_aP & 0 & \dots & \dots \\ \bar{p}_aP & 0 & 0 & p_aP & \dots & \dots \\ \bar{p}_aP & 0 & 0 & 0 & p_aP & \dots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \end{bmatrix}$$

Equilibrium distribution

Geometric form in sampled-scaled age

$$\pi_q = \bar{p}_a \nu (I - \tilde{D})^{-1} (p_a P - \tilde{D})(p_a P)^{q-2}, \qquad q \geq 2.$$

Mean sampled and scaled age

$$\mathbb{E}A = \langle -\nu (I - \tilde{D})^{-1}, \mathbf{1} \rangle + 1 + \frac{1}{\bar{p}_a}.$$

Tail decay-rate

$$\theta = - \lim_{q \to \infty} \frac{1}{q} \log \sum_{u=q}^{\infty} \langle \pi_u, \mathbf{1} \rangle = \log \frac{1}{p_a}.$$

Discussion and Concluding Remarks

Main Contributions

- Integration of coding and block Markov chain techniques to study timely communication for delay-sensitive traffic
- We model channel unreliability by the erasure channel
- Model source correlation by Markov process
- True and incremental updates are special cases