Request completion times in coded parallel systems

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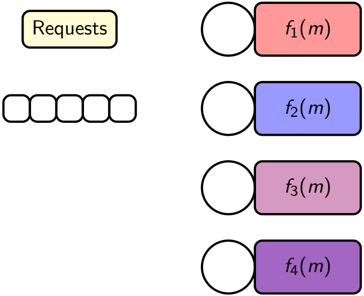
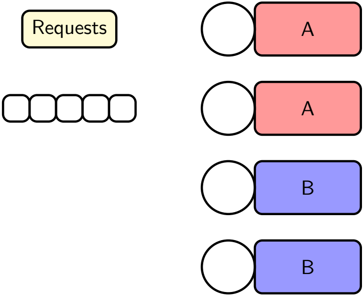
Problem Statement



Compute mean access time to download single message m

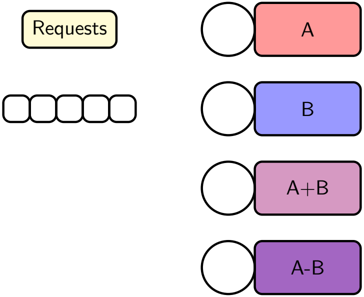
- with number of fragments k such that $m = (m_1, \ldots, m_k)$
- with encoding $(f_1(m), \ldots, f_n(m))$, and $f_i(m)$ stored at node i

Symmetric Codes



Replication (n, k)

Piece i stored at n/k servers



MDS (n, k)

Whole message can be decoded by any k out of n servers

Applications

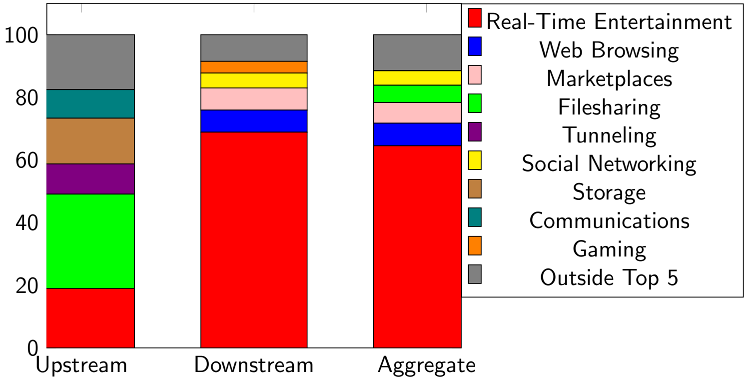
Distributed Storage

- **Content streaming:** NetFlix, HotStar, Eros Now, YouTube, Hulu, Amazon Prime Video
- **Cloud storage:** GitHub, DropBox, iCloud, OneDrive, UbuntuOne
- **Cloud service:** Facebook, Google Suite, Office365

Distributed Computation

- **Cloud computing:** Amazon Web Services, Microsoft Azure, Google Search
- **Cluster computing:** Hadoop, Spark
- **Distributed database:** Aerospike, Cassandra, Couchbase, Druid

Dominant traffic on Internet



Peak Period Traffic Composition (North America)

- Real-Time Entertainment: 64.54% for downstream and 36.56 % for mobile access[1]

[1] https://www.sandvine.com/downloads/general/global-internet-phenomena/2015/global-internet-phenomena-report-latin-america-and-north-america.pdf

System Model

File storage

- Each media file divided into k pieces
- Pieces encoded and stored on n servers

Arrival of requests

- Each request wants entire media file
- Poisson arrival of requests with rate λ

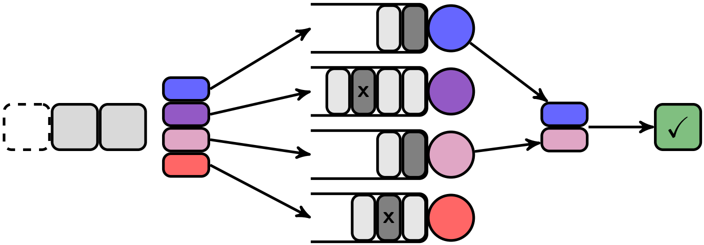
Time in the system

- Till the reception of whole file

Service at each server

- IID exponential service time with rate $\mu = k/n$

Storage Coding – (n, k) Fork-Join Model



exempli gratia: Joshi, Liu, Soljanin (2012, 2014), Joshi, Soljanin, Wornell (2015), Sun, Zheng, Koksal, Kim, Shroff (2015), Kadhe, Soljanin, Sprintson (2016), Li, Ramamoorthy, Srikant (2016)

Prior Work and Contributions

Kannan et al: join k queues for replication and MDS codes

- Numerical bounds using block Markov chains
- Trade-off between numerical accuracy and computational effort

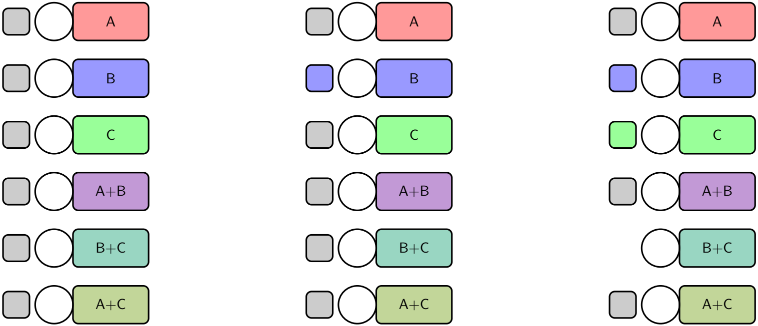
Soljanin, Wornell et al: fork-join (n, k) queues for MDS codes

- Closed-form upper and lower bounds
- Loose bounds for most of the rate region

This work: fork-join (n, k) queues for all symmetric codes

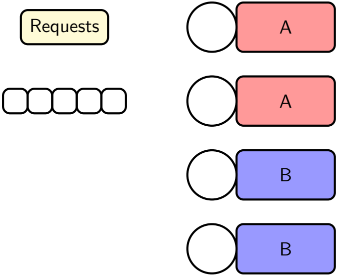
- Tight closed-form approximations for all rate regions
- Stability region for all symmetric codes
- Delay minimising symmetric code

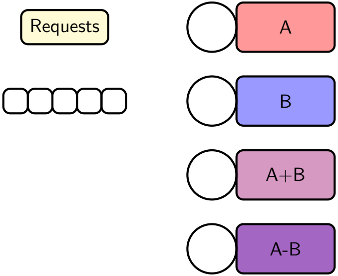
Coding Model



- Information sets $\mathcal{I} = \{S \subset [n] : |S| = k, f_S \text{ reconstructs } m\}$
- Observed servers $T \subset S$ for some info set $S \in \mathcal{I}$
- Useful servers $M(T) = \bigcup_{S \in \mathcal{I}} S \setminus T$
- **Symmetric codes:** number useful servers $N_{|T|} = |M(T)|$

Symmetric Codes





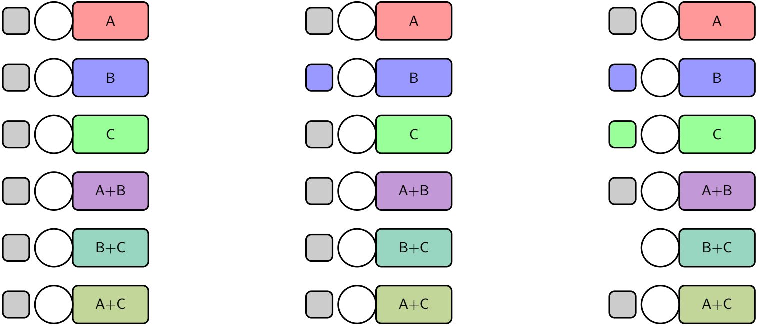
Replication (n, k)

Number of useful servers
$N_i = (k - i)n/k$

MDS (n, k)

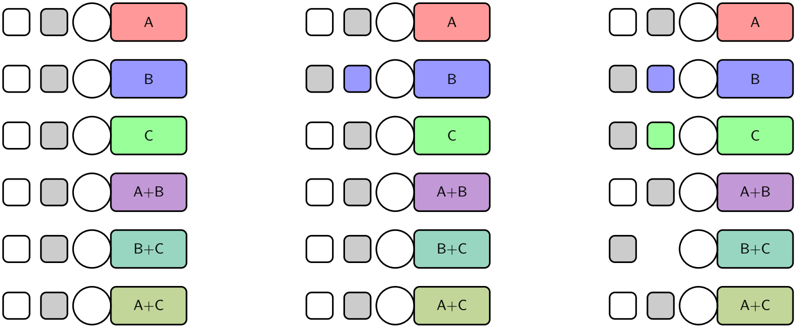
Number of useful servers
$N_i = (n - i)$

Single Request



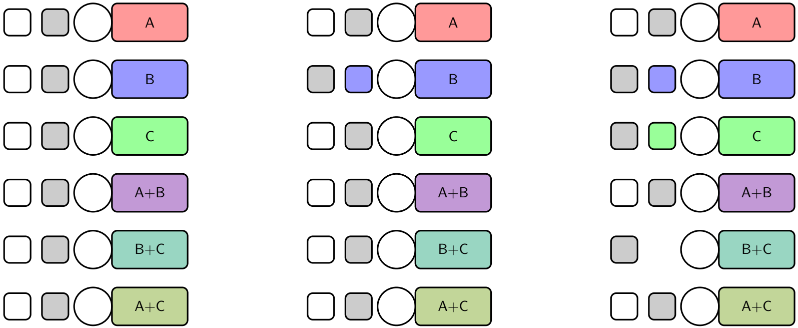
- $\mathbf{T}(t) = \{T \subset S : S \in \mathcal{I}\}$ is a Markov process

Two Requests



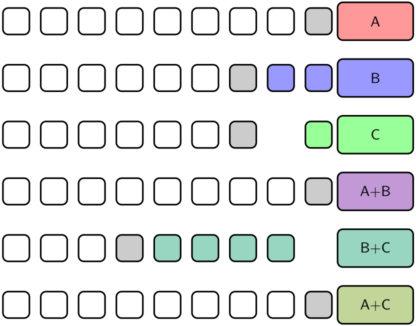
- $\mathbf{T}(t) = \{(T_1, T_2) \subset S \times S : S \in \mathcal{I}\}$ is a Markov process
- $|T_1| \geq |T_2|$ and $M_{T_1} \subset M_{T_2}$
- FIFO service: number of available servers $M_{T_2} \setminus M_{T_1}$

State Transitions



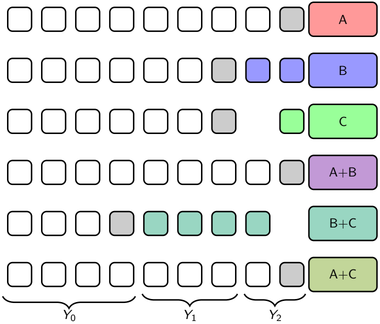
- Arrival rate: $(T_1, T_2) \to (T_1, T_2, \emptyset)$ at rate λ
- Departure rate: $(T_1, T_2) \to (T_2)$ at rate $N_{|T_1|}\mu$
- Service rate: $(T_1, T_2) \to (T_1, T_2 \cup B)$ at rate μ

State Space Collapse



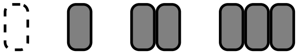
- $\mathbf{L}(t) = \{(\ell_1, \ldots, \ell_r) : \ell_i = |T_i|, \ell_1 \geq \ell_2\}$ is a Markov process
- **Arrival:** $(\ell_1, \ldots, \ell_r) \to (\ell_1, \ldots, \ell_r, 0)$ at rate λ
- **Departure:** $(\ell_1, \ldots, \ell_r) \to (\ell_2, \ldots, \ell_r)$ at rate $N_{\ell_1}\mu$
- **Service:** $(\ldots, \ell_i, \ldots) \to (\ldots, \ell_i + 1, \ldots)$ at rate $(N_{\ell_i} - N_{\ell_{i+1}})\mu$

State Space Transformation



- $\mathbf{Y}(t) = \{Y_0, Y_1, \ldots, Y_{k-1}\}$ is a Markov process
- **Arrival:** $Y_0 \to Y_0 + 1$ at rate λ
- **Departure:** $Y_{k-1} \to Y_{k-1} - 1$ at rate $N_{k-1}\mu$
- **Service:** $(Y_{i-1}, Y_i) \to (Y_{i-1} - 1, Y_i + 1)$ at rate $(N_{i-1} - N_{l_{i-1}})\mu$

State Transitions of Collapsed System



Arrival of requests at rate λ

- Unit increase in $Y_0(t) = Y_0(t-) + 1$ with rate λ

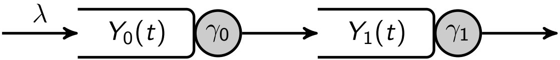
Getting additional symbol at rate $\gamma_i = (N_{i-1} - N_i)\mu$

- Unit increase in $Y_i(t) = Y_i(t-) + 1$
- Unit decrease in $Y_{i-1}(t) = Y_{i-1}(t-) - 1$

Getting last missing symbol at rate $\gamma_{k-1} = N_{k-1}\mu$

- Unit decrease in $Y_{k-1}(t) = Y_{k-1}(t-) - 1$

Tandem Queue Interpretation (No Empty States)



Duplication

- n/k available servers at level i
- Normalized service rate at level i

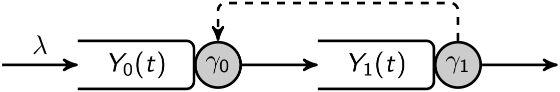
$$\gamma_i = 1$$

MDS Coding

- Single server at level $i \neq k-1$
- Normalized service rate at level i

$$\gamma_i = \begin{cases} \frac{k}{n} & i < k-1 \\ \frac{k}{n}(n-k+1) & i = k-1 \end{cases}$$

Tandem Queue Interpretation (General Case)



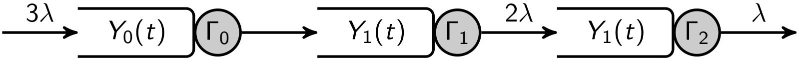
Tandem Queue with Pooled Resources

- Servers with empty buffers help upstream
- Aggregate service at level i becomes

$$\sum_{j=i}^{I_i(t)-1} \gamma_j \qquad \text{where} \qquad I_i(t) = k \wedge \{I > i : Y_I(t) > 0\}$$

- No explicit description of stationary distribution for multi-dimensional Markov process

Stability Region For Pooled Tandem Queues

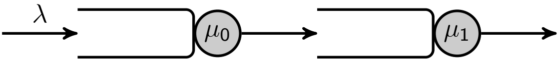


For a distributed storage system with symmetric codes and fork-join queues with FCFS service, the stability region is equal to

$$\lambda < \min\left\{ \frac{\Gamma_i}{k-i} : i \in \{0, \dots, k-1\} \right\},$$

where $\Gamma_i \triangleq \sum_{j=i}^{k-1} \gamma_j$ is the useful service rate for level i.

Bounding and Separating



Theorem[†]

When $\lambda < \min \mu_i$, tandem queue has product form distribution

$$\pi(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\mu_i} \left(1 - \frac{\lambda}{\mu_i}\right)^{y_i}$$

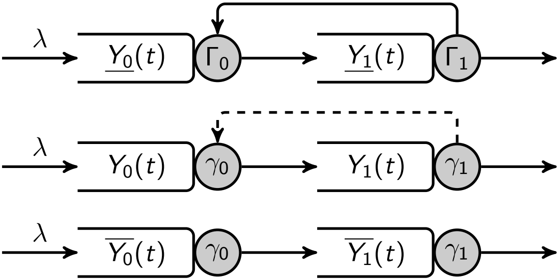
Uniform Bounds on Service Rate

Transition rates are uniformly bounded by

$$\gamma_i \leq \sum_{j=i}^{l_i(y)-1} \gamma_j \leq \sum_{j=i}^{k-1} \gamma_j \triangleq \Gamma_i$$

[†] F. P. Kelly, Reversibility and Stochastic Networks. New York, NY, USA: Cambridge University Press, 2011.

Bounds on Tandem Queue



Lower Bound

Higher values for service rates yield lower bound on queue distribution

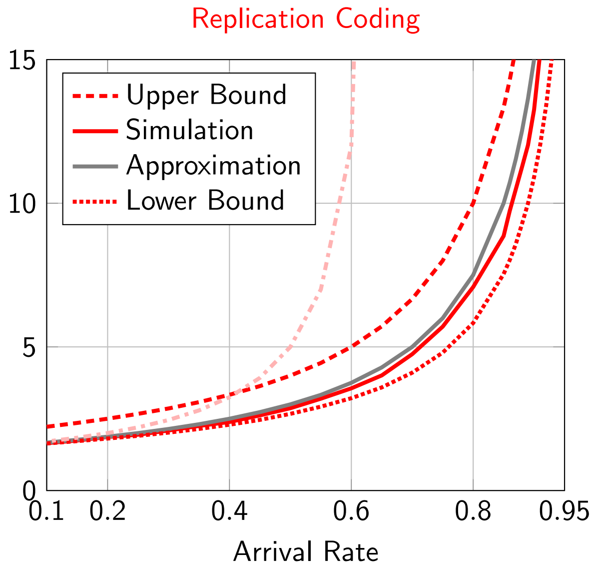
$$\underline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\Gamma_i} \left(1 - \frac{\lambda}{\Gamma_i}\right)^{y_i}$$

Upper Bound

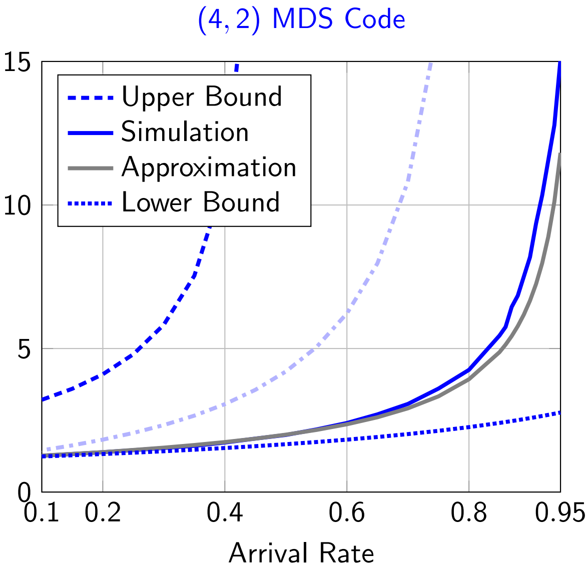
Lower values for service rate yield upper bound on queue distribution

$$\overline{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\gamma_i} \left(1 - \frac{\lambda}{\gamma_i}\right)^{y_i}$$

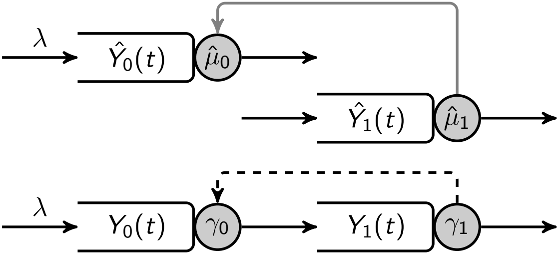
Mean Sojourn Time



Mean Sojourn Time



Approximating Pooled Tandem Queue



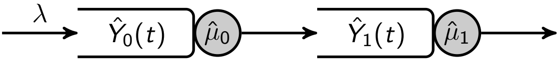
Independence Approximation with Statistical Averaging

Service rate is equal to base service rate γ_i plus cascade effect, averaged over time

$$\hat{\mu}_{k-1} = \gamma_{k-1}$$
$$\hat{\mu}_i = \gamma_i + \hat{\mu}_{i+1}\hat{\pi}_{i+1}(0)$$

$$\hat{\pi}(y) = \prod_{i=0}^{k-1} \frac{\lambda}{\hat{\mu}_i}\left(1 - \frac{\lambda}{\hat{\mu}_i}\right)^{y_i}$$

Delay Minimizing Storage Code

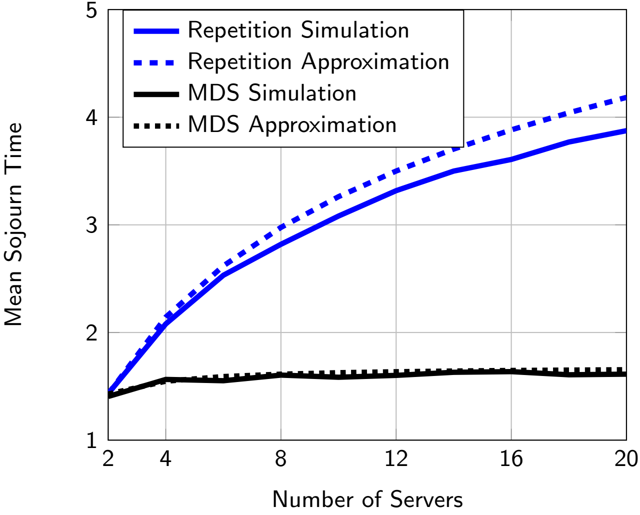


Optimizer to the objective function

$$\gamma^* = \arg\min \left\{ \sum_{i=1}^{k-1} \frac{1}{\Gamma_i - (k-i)\lambda} : \gamma \in \mathcal{A} \right\}.$$

The MDS coding scheme minimizes the approximate mean sojourn time for a fork-join queueing system with identical exponential servers among all symmetric codes.

Comparing Replication versus MDS Coding



Arrival rate 0.3 units and coding rate $n/k = 2$

Summary and Discussion

Main Contributions

- Analytical framework for study of distributed computation and storage systems
- Upper and lower bounds to analyze replication and MDS codes
- A tight closed-form approximation to study distributed storage codes
- MDS codes are better suited for large distributed systems
- Mean access time is better for MDS codes for all code-rates