Load-balancing on heterogeneous parallel servers

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Acknowledgements








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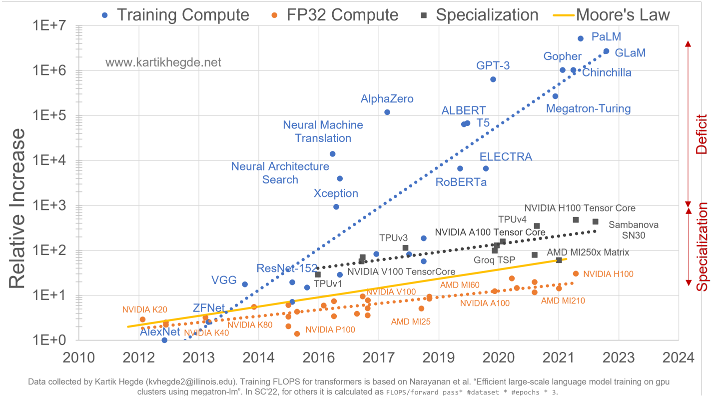
NRF National Research Foundation

The growing performance deficit

Deep learning compute demand[1]



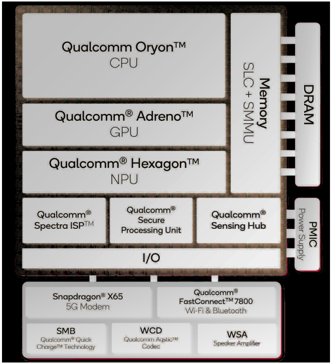
- End of Moore's law and Dennard scaling
- Can no longer keep adding more transistors and increasing core frequencies
- We're accustomed to more and more compute

[1] Image credit: https://kartikhegde.substack.com/p/accelerating-deep-learning-in-the

Heterogeneous Computing[2]

Sources of heterogeneity

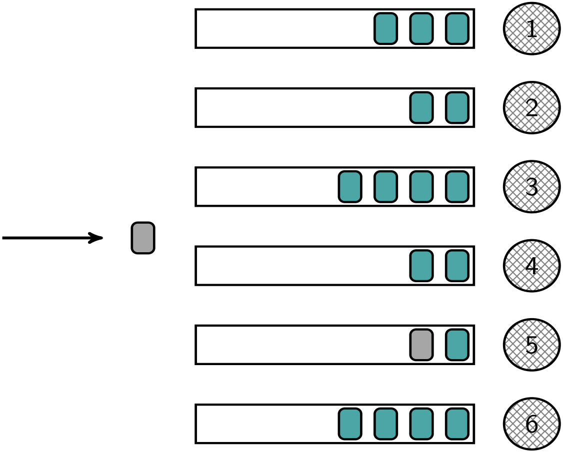
- Different generations of servers and accelerators
- Pooling of all available compute resources (CPUs, GPUs, NPUs)
- Compute resources may be run at different speeds for energy conservation
- Compute cores optimized for different operating regions, to deal with dynamic workloads
- External factors—network bottlenecks, data affinity, etc.



[2] Image: https://www.anandtech.com/show/21445/qualcomm-snapdragon-x-architecture-deep-dive

Load balancing policies—homogeneous servers

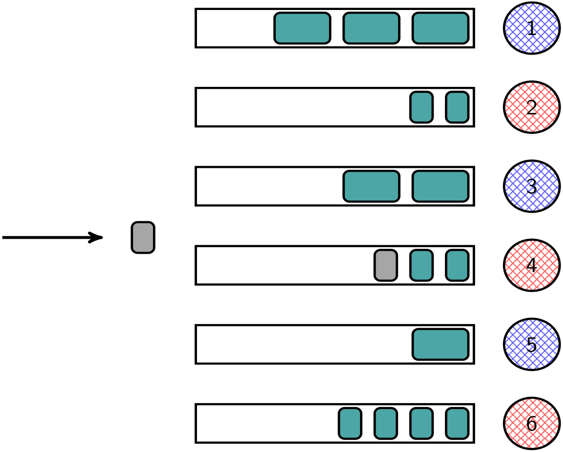
Join shortest queue (1) [3]



[3] W. Winston, "Optimality of the shortest line discipline," J. App. Prob., 14(1), 181–189, Mar 1977.

Load balancing policies—heterogeneous servers

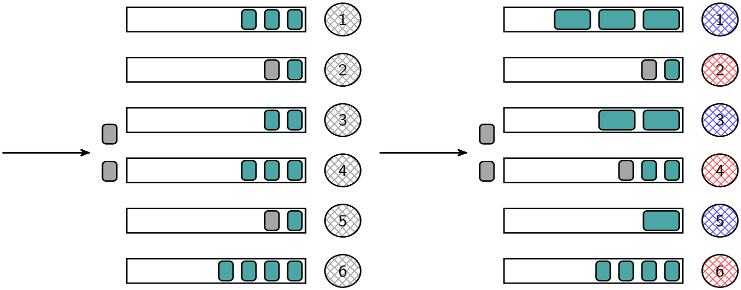
Join smallest workload queue (1) [4]



[4] R. R. Weber, "On the optimal assignment of customers to parallel servers," J. App. Prob., 15(2), 406–413, Jun 1978.

Load balancing policies—parallel processing of subtasks

Join the shortest queue (k)



- Equivalent to (n, k) fork-join system [5]

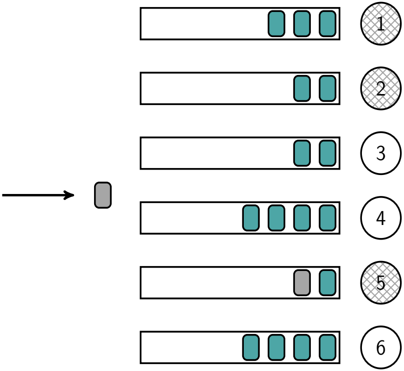
Pro Minimizes the mean task completion time

Con Feedback overhead linearly scaling in the number of servers

[5] A. Badita, P. Parag, and J.-F. Chamberland. Latency analysis for distributed coded storage systems. IEEE Transactions on Information Theory. 65(8):4683–4698, Aug 2019.

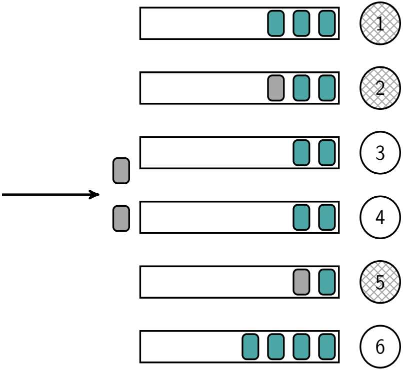
Load balancing policies—low overhead alternative [6]

Power-of-d (1)



- Equivalent to $(d, 1)$ fork-join queue
- When $d = n$, it is JSW

Power-of-d (k)

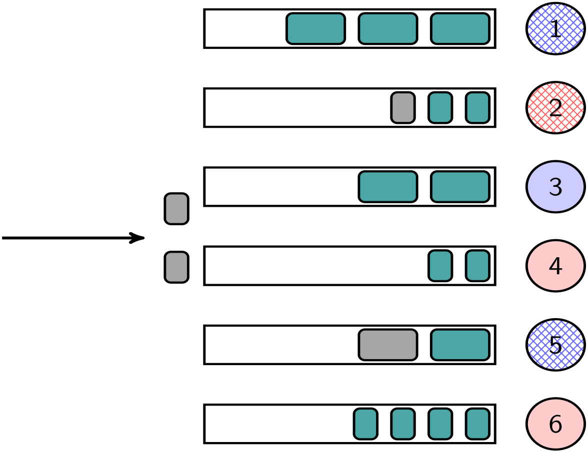


- Equivalent to $(d, 1)$ fork-join queue
- When $d = n$, it is (n, k) fork-join

[6] M. Mitzenmacher, "The power of two choices in randomized load balancing," IEEE Trans. Parallel Distrib. Syst., vol. 12, no. 10, pp. 1094–1104, Oct. 2001.

Heterogeneous servers— low overhead alternative

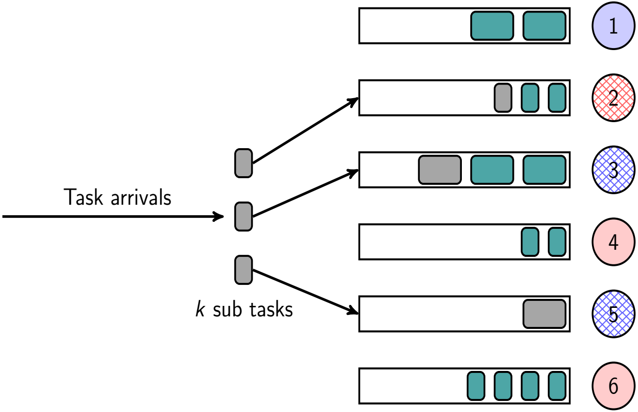
(d, k) fork-join system



- Sample d servers, join k smallest
- Task departs on completion of all k sub-tasks

Heterogeneous servers— zero overhead alternative[7]

(k, k) fork-join system with probabilistic scheduling



Objective Find the optimal slow server selection probability p_s^* that minimizes the mean task completion time

[7] R. Jinan, A. Badita, T. P. Bodas, and P. Parag. Load balancing policies without feedback using timed replicas. *Performance Evaluation*. 162, 102381, Nov 2023.

Related Works

Load balancing strategies in homogeneous system

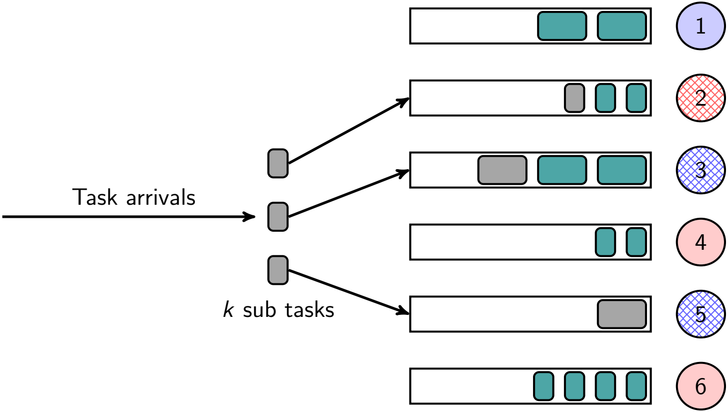
- **Without sub-division of tasks** [M. Mitzenmacher et al., 2001], [U.Ayesta et al., 2019], power-of-d variants
- **With sub-division of tasks** [A.Badita et al., 2019] [R.Jinan et al., 2022],

Load balancing strategies in heterogenous system

- **Without sub-division of tasks**[Der Boor et al., 2021], [Jaleel et al., 2022]: power-of-d variants
- **With sub-division of tasks:** Our work

SystemParameters

Random selection of slow and fast servers

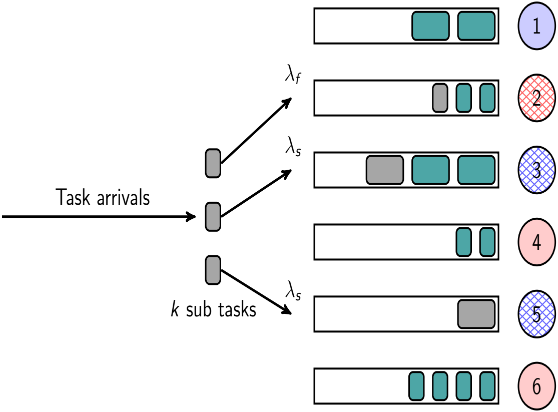


- Probability of selecting k_s slow and $k - k_s$ fast servers for any task is

$$q(k_s) = \binom{k}{k_s} p_s^{k_s} (1 - p_s)^{k-k_s}$$

System Parameters

Arrival rate at individual servers

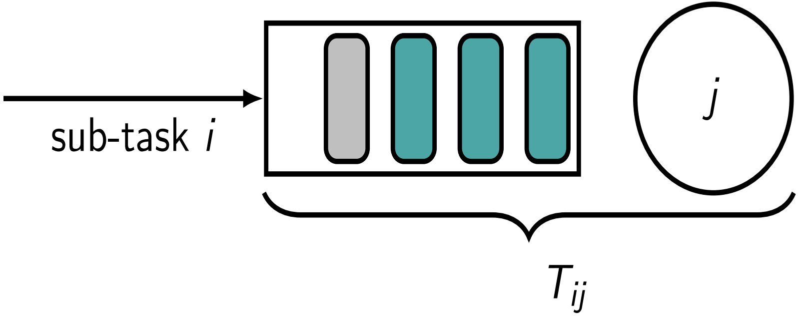


- Arrival at each server is a thinned Poisson process with the arrival rate at slow and fast servers are

$$\lambda_s \triangleq \frac{\lambda n}{k}\left(\frac{kp_s}{nf_s}\right) = \frac{\lambda p_s}{f_s}, \qquad \lambda_f \triangleq \frac{\lambda \bar{p}_s}{\bar{f}_s}.$$

Performance metrics

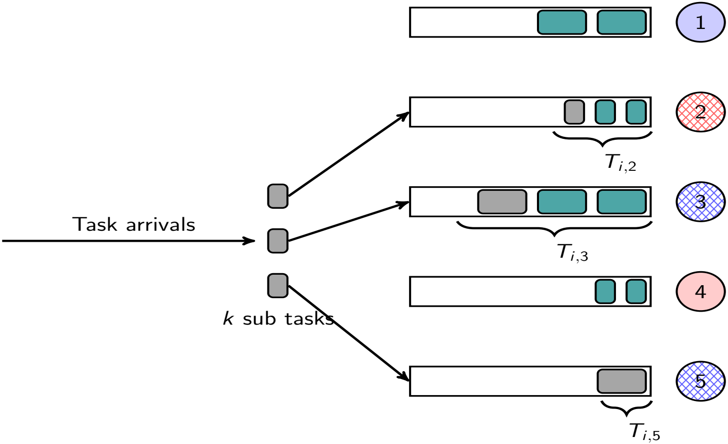
Sub-task completion time



- Sub-task completion time at server j is $T_{i,j} \triangleq W_{i,j} + X_{i,j}$
- For slow server j, $L_s(x) \triangleq \lim_{i\to\infty} P\{T_{i,j} \leqslant x\}$
- For fast server j, $L_f(x) \triangleq \lim_{i\to\infty} P\{T_{i,j} \leqslant x\}$

Performance metrics

Task completion time



$$T_i \triangleq \max_{j \in I^i} T_{i,j}$$

Key contributions

- Establishing asymptotic independence of the stationary workload distribution in a heterogeneous server system with two classes of heterogeneity
- Analytical computation of the limiting mean task completion time for systems with an arbitrarily large number of servers
- A tight closed form approximation for the optimal selection probability

Asymptotic Independence

Theorem

If $\pi^k, \hat{\pi}^k$ are the equilibrium distributions for workloads in the first $k = o(n^{\frac{1}{4}})$ servers of systems $\mathcal{S}$ and $\hat{\mathcal{S}}$ respectively, the total variation distance

$$\lim_{n\to\infty} d_{\mathrm{TV}}(\pi^k, \hat{\pi}^k) = 0$$

Theorem

If asymptotic independence of equilibrium workload at any k queues for a large number of servers holds, then

$$P\left\{T_\infty \leqslant x\right\} = P \cap_{j\in I^\infty} \left\{T_{\infty,j} \leqslant x\right\} = \mathbb{E} \prod_{j\in I^\infty} P\left\{T_{\infty,j} \leqslant x\right\}$$

Therefore, the mean completion time is

$$\mathbb{E}[T] = \sum_{k_s=0}^{k} \binom{k}{k_s} p_s^{k_s}(1-p_s)^{k_s} L_s(x)^{k_s} L_f(x)^{k-k_s} = \int_{x\in\mathbb{R}_+} [1-(p_s L_s(x) + \bar{p}_s L_f(x))^k]dx$$

Asymptotic Independence

When $k(n) = \left\lceil n^{\frac{2}{3}} \right\rceil$

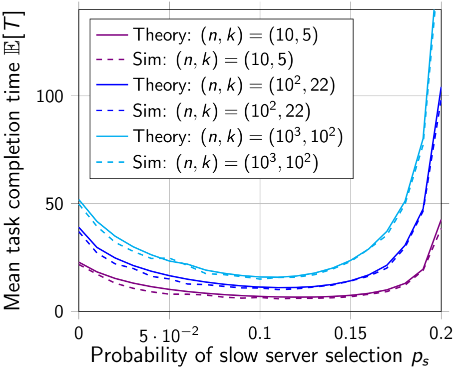


Figure: System with $f_s = 0.5$, $\lambda = 1.2$, $(\mu_s, \mu_f) = (0.5, 2.5)$.

Bound on Mean Task Completion Time

Upper and Lower Bound

- Average smaller than maximum smaller than sum, i.e.

$$\frac{1}{|I^i|}\sum_{j\in I^i} T_{i,j} \leqslant \max_{j\in I^i} T_{i,j} \leqslant \sum_{j\in I^i} T_{i,j}$$

- Mean of sum of sub-task completion times

$$\lim_{i\to\infty} \mathbb{E}\sum_{j\in I^i} T_{i,j} = kp_s \int_{x\in\mathbb{R}_+} \bar{L}_s(x)dx + k\bar{p}_s \int_{x\in\mathbb{R}_+} \bar{L}_f(x)dx$$

- For general service times

$$\lim_{i\to\infty} \mathbb{E}\sum_{j\in I^i} T_{i,j} = \frac{kp_s}{\lambda_s}\left(\rho_s + \frac{\lambda_s^2\mathbb{E}X_s^2}{2(1-\rho_s)}\right) + \frac{k\bar{p}_s}{\lambda_f}\left(\rho_f + \frac{\lambda_f^2\mathbb{E}X_f^2}{2(1-\rho_f)}\right),$$

  where the load $\rho_s = \lambda_s\mathbb{E}X_s < 1$ and $\rho_f = \lambda_f\mathbb{E}X_f < 1$.

Optimal slow server selection probability

Minimizing mean task completion time

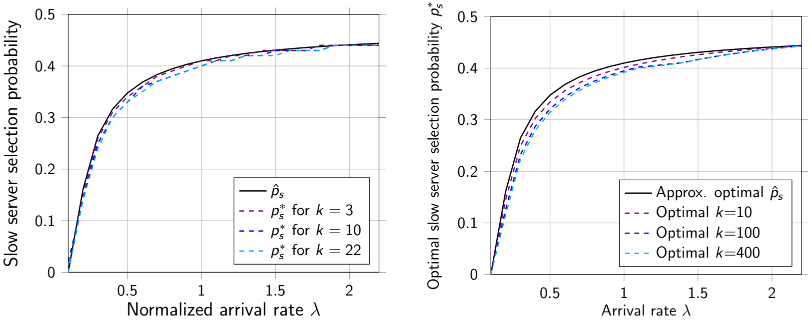
- Find optimal probability p_s^* that minimizes $\mathbb{E}T$ difficult to compute analytically
- Find probability $\hat{p}_s$ that minimizes both the upper and the lower bound
- Approximate p_s^* by $\hat{p}_s$

Exponentially distributed sub-task completion times

Defining $\tau_1 \triangleq \bar{f}_s(\mu_f - \sqrt{\mu_s \mu_f})$,

$$\hat{p}_s = \begin{cases} 0, & \lambda \leqslant \tau_1, \\ \frac{1 - \frac{\tau_1}{\lambda}}{1 + \frac{\bar{f}_s}{f_s}\sqrt{\frac{\mu_f}{\mu_s}}}, & \tau_1 \leqslant \lambda < \mu_s f_s + \mu_f \bar{f}_s. \end{cases}$$

Slow server selection probabilities

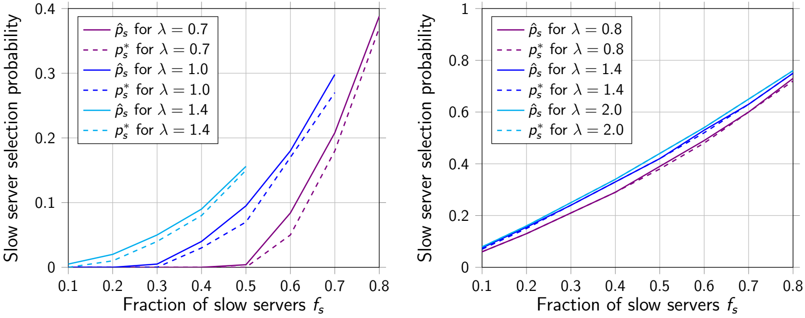


(a) $n = 10^2, k \in \{3, 10, 22\}$.

(b) $n = 10^3, k \in \{10, 100, 400\}$.

Figure: System with $(\mu_s, \mu_f) = (2, 2.5)$.

Variation with the fraction of slow servers f_s



(a) $(\mu_s, \mu_f) = (0.5, 2.5)$. (b) $(\mu_s, \mu_f) = (2.0, 2.5)$.

Figure: System with $n = 10^3$, $k = 10$

Deterministic Scheduling

Deterministic selection

Select k_s slow and $k - k_s$ fast servers

Theorem

The optimal selection probability of slow servers, converges to

$$\lim_{k\to\infty} p_s^* = \frac{k_s^*}{k},$$

where k_s^* is the optimal deterministic selection of slow servers. The optimal probability of choosing ℓ servers converges to

$$\lim_{k\to\infty} q(\ell) = \mathbb{1}_{\{\ell = kp_s^*\}}.$$

Mean number of slow servers

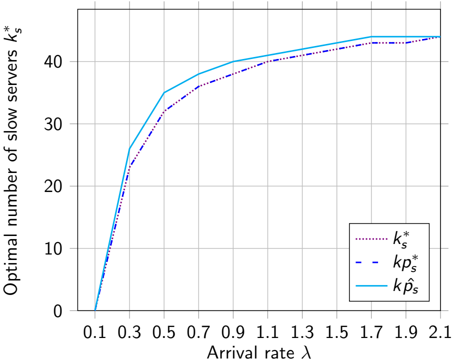
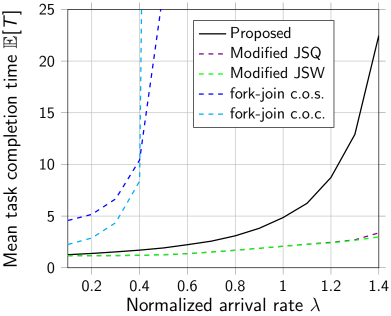
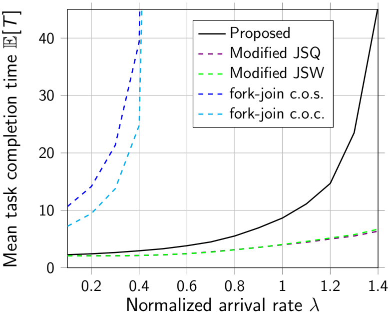


Figure: System with $n = 10^3$, $k = 100$, $f_s = 0.5$,$(\mu_s, \mu_f) = (2, 2.5)$.

Comparison with other Load Balancing Policies



(a) $n = 10^2,\ k = 10$



(b) $n = 10^3,\ k = 10^2$

Figure: System with $f_s = 0.5$, $(\mu_s, \mu_f) = (0.5, 2.5)$

Asymptotic Independence

Theorem

If$\pi^k, \hat{\pi}^k$ are the equilibrium distributions for workloads in the first $k = o(n^{\frac{1}{4}})$ servers of systems $\mathcal{S}$ and $\hat{\mathcal{S}}$ respectively, the total variation distance

$$\lim_{n \to \infty} d_{\mathrm{TV}}(\pi^k, \hat{\pi}^k) = 0.$$

Asymptotic Independence

Proof sketch

We look at three systems with n servers:

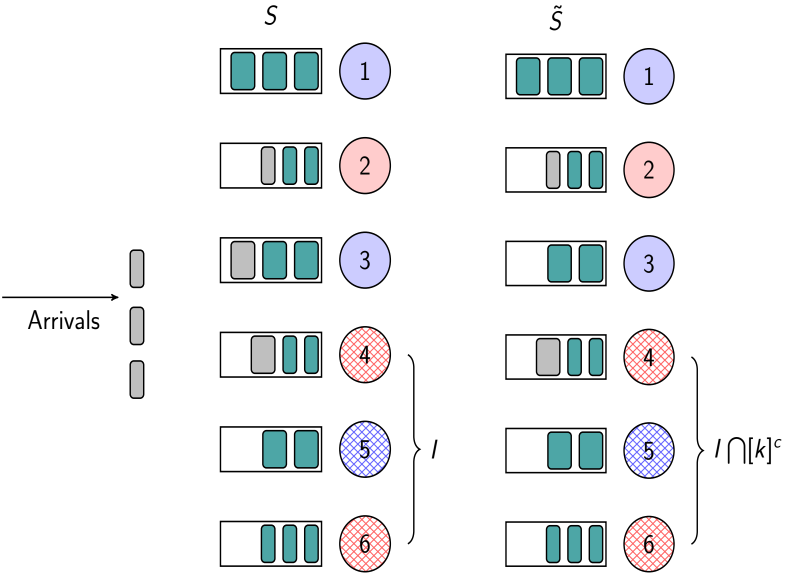
- Original system S under consideration
- Independent system $\hat{S}$, whereall the queues are independent
- Coupled system$\tilde{S}$, where no more than one arrival is allowed in the first k queues

Focus

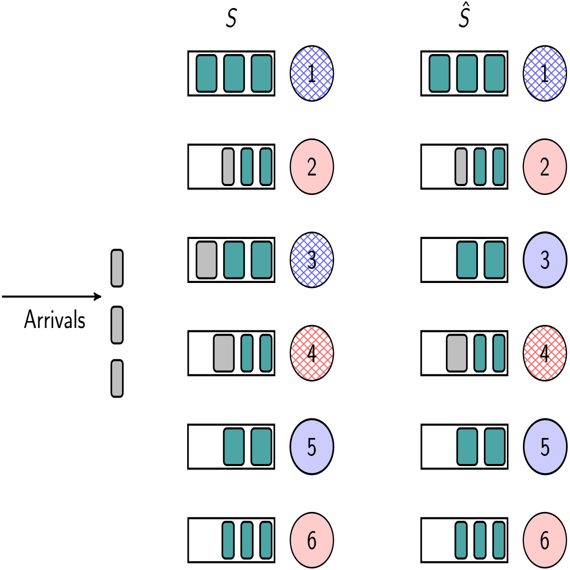
Joint distribution of queues at the first k servers of the original system S, an independent system $\hat{S}$ and a coupled system $\tilde{S}$

$$\begin{aligned} d_{\mathrm{TV}}(\pi^k, \hat{\pi}^k) \leqslant & d_{\mathrm{TV}}(\pi^k, \pi^k_\tau) + d_{\mathrm{TV}}(\pi^k_\tau, \tilde{\pi}^k_\tau) + d_{\mathrm{TV}}(\tilde{\pi}^k_\tau, \tilde{\pi}^k) \\ & + d_{\mathrm{TV}}(\tilde{\pi}^k, \hat{\pi}^k). \end{aligned}$$

Asymptotic Independence—coupled system evolution



Asymptotic Independence—coupled system evolution



Proof steps

- Original and coupled system differ when there is more than one arrival at first k servers
- Coupled system has thinned arrivals since some arrivals to first k servers are dropped
- For a finite time, bound the probability that the workload at first k servers differs between two systems
- At any finite time, workload distribution at first k servers for the original and the coupled systems are close
- For sufficiently large time, the workload distribution is close to stationary distribution
- Workload distribution for coupled and independent systems is close since the load differences are bounded

Conclusion

- We show that the joint distribution of the stationary workload across k queues becomes asymptotically independent as the number of servers, n, grows and $k = o(n^{\frac{1}{4}})$
- We derive an upper and lower bound on the limiting mean response time and identify the selection probability, p_s, that minimizes this bound
- Numerical experiments confirm that the selected probability provides near-optimal performance

How to reduce CPU power?

Energy proportionality

- Push CPUs to sleep.
- Dynamic voltage and frequency scaling (DVFS).

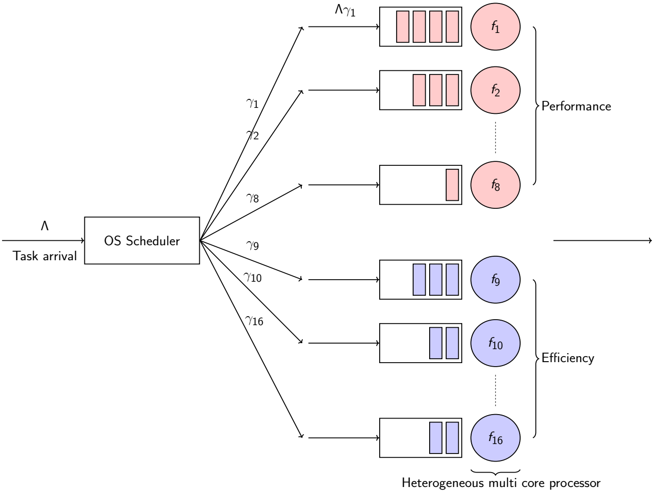
Challenges in Homogeneous multi-cores

- High performance at the cost of high power consumption.
- Power efficient with degraded performance.

Heterogeneous multi-core processor (HMP)

A new architecture consisting of CPUs with heterogeneous cores having different power-performance trade-offs.

Key questions

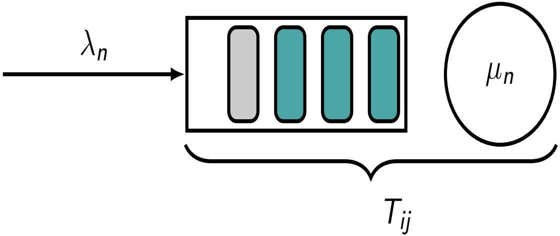


- What is the optimal workload split ?
- What is the operating frequency for all cores?

Key contributions

- Power and performance model for CPUs with heterogeneous cores.
- Problem formulation for the workload splitting and operating frequencies.
- HEMP—*Heterogeneity enabled Energy-Minimizer with Performance constraints*.
- Comparison with Linux frequency governors **(upto 80% reduction in energy-delay product)**.

Problem Formulation



$$\mu_n \triangleq \alpha_{c_n} f_n,$$

$$\rho_n = \frac{\lambda_n}{\mu_n},$$

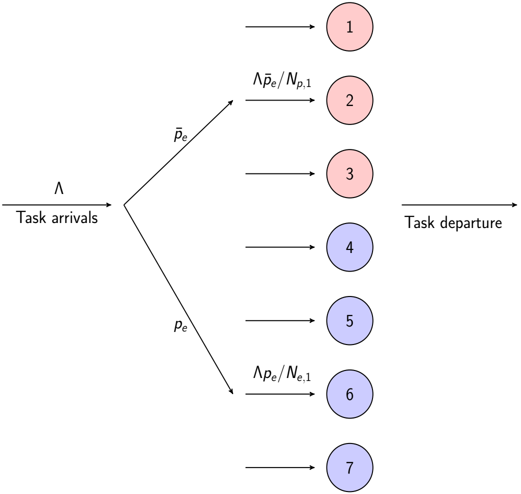
$$\bar{W}_n(c_n, f_n, \gamma_n) = \frac{1}{\mu_n - \lambda_n},$$

$$\bar{P}_n = P_{\text{sta}} + \rho_n P_{\text{dyn}}.$$

$$(\gamma^*, f^*) \triangleq \arg \min_{(\gamma, f) \in A} \sum_{n \in [N]} \bar{P}_n.$$

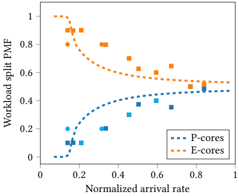
$$\bar{W}_n(c_n, f_n, \gamma_n) \leq w.$$

Optimal selection probability and frequency allocation

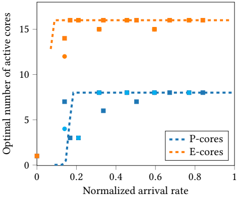


$$f^* \triangleq \frac{1}{\alpha}\Big(\Lambda\gamma + \frac{1}{w}\Big), \qquad \gamma_j^* = \frac{p_e^*}{N_{e,1}^*}\mathbb{1}_{\{j\in E\}} + \frac{1-p_e^*}{N_{p,1}^*}\mathbb{1}_{\{j\in f\}}.$$

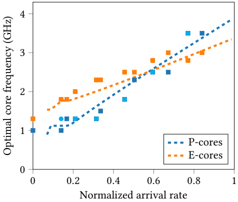
Optimal splitting between classes



Optimal workload split



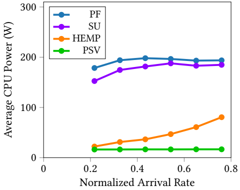
Optimal number of active cores



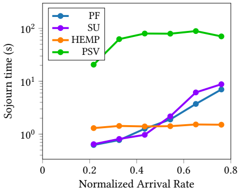
Optimal frequency

- Probabilistic split between two core types
- Constant frequency for all active cores of one type

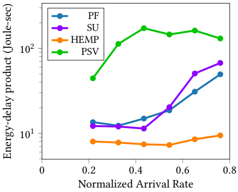
Comparison with Linux frequency governors



Total power



Sojourn time

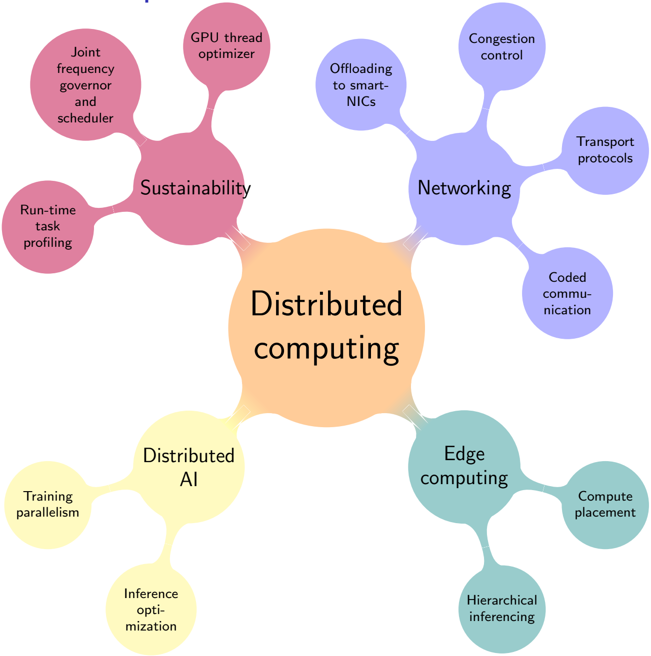


Energy-delay product

TensorFlow Lite workload

PF: Performance **SU**: Schedutil **PSV**: Powersave

Research Landscape



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