E2 205 Error-Control Coding
Homework 4

1. Consider the rate $1/2$ convolutional code with

$$G(D) = [1+D+D^2 \quad 1+D^2].$$

   (a) Draw a complete trellis diagram up to node level 6 (beginning at node level 0). Label all branches with code symbols.

   (b) Use the trellis to determine the free distance d_{free} of the code.

   (c) If the received sequence (across a BSC) is

$$\underline{r} = (01 \ 00 \ 01 \ 00 \ 00 \ 00\ldots...)$$

   find (the information sequences associated to) all survivors at node level 6.

   (d) If the received sequence (across an AWGN channel) is

$$\underline{r} = (4 \ -1 \quad -3 \ 2 \quad 6 \ -5 \quad 2 \ 4 \quad 5 \ 3 \quad 5 \ 5\ldots...)$$

   find (the information sequences associated to) all survivors at node level 6.

2. The polynomial generator matrices (PGM)

$$G_1(D) = [1+D^2, \quad 1+D]$$
$$G_2(D) = [1+D, \quad 1]$$

   generate the same convolutional code $\mathscr{C}$. If the (infinite length) all-1 input sequence $\{u(k)\}_{k=0}^{\infty} = (111\ldots)$ produces the codeword ($V^{(1)}(D), V^{(2)}(D)$) when PGM $G_1(D)$ is employed, identify the first 10 symbols of the input sequence $\{u_k^{'}\}_{k=0}^{\infty}$ which will yield the same codeword when PGM $G_2(D)$ is employed. Make clear your reasoning.

3. Consider the convolutional code having polynomial generator matrix (PGM)

$$G(D) = [1, \quad 1+D].$$

   (a) Draw the state diagram of the (finite state machine associated to the code) using dotted lines to denote a 1 at the input.

   (b) Determine the generating function $A_{\text{END}}(L, D, I)$ associated to this code.

   (c) What is the minimum free distance d_{free} of this code ?

4. In the field of formal power series $\mathbb{F}_2((X))$, what is the inverse of $(x+x^2)$?

5. Given a binary convolutional code $\mathscr{C}$ having polynomial generator matrix (PGM)

$$G(D) = [1+D+D^3, \quad D+D^4]$$

   how many states are there in the corresponding trellis diagram of the code ?

6. Will the choice of generator matrix,

$$G(D) \;=\; \left[\; 1+D+D^2+D^3, \;\; 1+D^2+D^3+D^5, \;\; 1+D^4 \;\right],$$

cause the associated convolutional code $\mathscr{C}$ to exhibit catastrophic error propagation ? Explain fully your answer.

7. A PGM $G(D)$ represents one possible encoder for a convolutional code. For the code that has the PGM

$$G(D) \;=\; [1+D+D^3, \;\; D+D^4]$$

as one possible encoder, provide a second, not necessarily distinct encoder that has catastrophic error propagation. Explain your reasoning.

8. In the field of of formal power series $F_2[[D]]$, find the first 7 terms in the power-series expansion of

   (a) $\frac{1}{1+D^2}$

   (b) $\frac{D}{1+D+D^2}$

   (c) $\frac{D^2}{1+D^2+D^5}$

9. Determine whether the convolutional codes encoded using the $G(D)$ below given below are catastrophic. If so, find an infinite weight input sequence that generates a codeword of finite weight.

   (a) $G(D) \;=\; [1+D+D^3,\; 1+D+D^2,\; 1+D^2+D^3]$.

   (b) $G(D) \;=\; [1+D^3,\; 1+D+D^2+D^4,\; 1+D^2+D^3+D^4]$.

   Hint: The irreducible polynomials of degree ≤ 3 over $GF(2)$ are listed below:

   degree 1: $D, 1+D$
   degree 2: $1+D+D^2$
   degree 3: $1+D+D^3, 1+D^2+D^3$.

10. Use Euclid's algorithm to find the greatest common divisor $g(D)$ of

   (a) $g_1(D) \;=\; 1+D^{20}$, $g_2(D) \;=\; 1+D^{15}$.

   (b) $g_1(D) \;=\; D+D^2+D^4$, $g_2(D) \;=\; 1+D^8$.

   In each case also find a pair $\{a_1(D), a_2(D)\}$ of polynomials such that

$$a_1(D)g_1(D) \;+\; a_2(D)g_2(D) \;=\; g(D).$$

11. Determine an upper bound to the bit error probability P_{be} of the rate $1/3$ convolutional code having

$$G(D) \;=\; \left[\; 1+D \quad 1+D^2 \quad 1+D+D^2 \;\right].$$