E2 205 Error-Control Coding
Lecture 16

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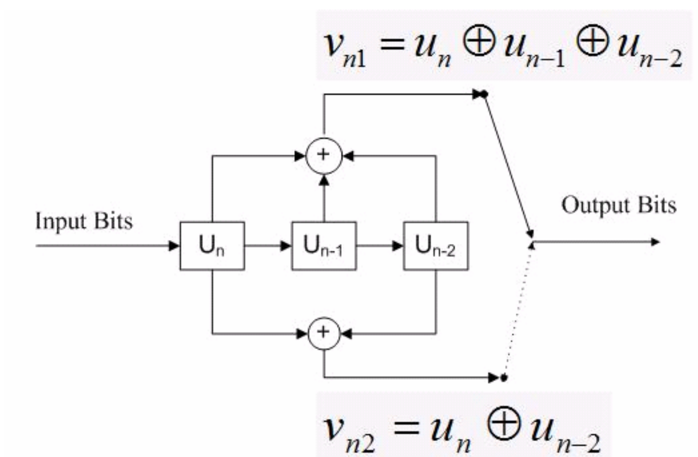
1 Viterbi Decoding



$$V^2(D) = \sum_{t=0}^{\infty} V_t^2 D^t$$

$$\sum_{t=0}^{\infty} V_t^1 D^t = \sum_{t=0}^{\infty} (U_t + U_{t-1} + U_{t-2}) D^t$$

$$= \sum_{t=0}^{\infty} U_t D^t + \sum_{t=0}^{\infty} U_{t-1} D^t + \sum_{t=0}^{\infty} U_{t-1} D^t$$
$$= U(D)[1 + D + D^2]$$

1.1 Polynomial Generator Matrix (PGM)

$$U(t) = (1, 1, 1, ..., 1)$$

Then

$$[V^1(D), V^2(D)] = \frac{1}{1+D}[1 + D^2, 1 + D]$$
$$= [1 + D, 1]$$

$$W_H\{U(t)\}_0^{\infty} = \infty$$
$$W_H\{V^1(t), V^2(t)\}_0^{\infty} = 2 + 1$$
$$= 3$$

1.2 Catastrophic Error Propagation (CEP)

CEP is said to take place in a convolutional code, when the input has infinite hamming weight but output has finite hamming weight.

$LetG(D) = [g_1(D), g_2(D), ..., g_n(D)]gcd[g_1(D), g_2(D), ..., g_n(D)] = P(D)P(D) = D^l[1 + q(D)]the$

	$1+D^2$	$1+D$	Quotient
$1+D^2$	1	0	0
$1+D$	0	1	$1+D$

ASIDE:Euclidean Distance Algorithm

Thus the gcd of two polynomial can be expressed as a linear combination of the two polynomials.

$$1 + D = 0 * (1 + D^2) + 1 * (1 + D)$$

To find gcd of

$$[g_1(D), g_2(D), g_3(D)]$$

and

$$\sum_{i=1}^{n} a_i(D)g_i(D) = D^l$$

Then

$$U(D)[g_1(D), g_2(D), ..., g_n(D)] \begin{bmatrix} a_1(D) \\ a_2(D) \\ . \\ . \\ a_n(D) \end{bmatrix} = U(D) \sum_{i=1}^{n} a_i(D)g_i(D)$$

$$= D^l U(D).$$

Hence it is not possible for input to have ∞ hamming and the output to have finite hamming weight

Theorem

To ensure that a rate $\frac{1}{n}$ convolutional code having PGM

$$G(D) = [g_1(D), g_2(D), ..., g_n(D)]$$

To avoid CEP, it is necessary and sufficient that

$$gcd[g_1(D), g_2(D), ..., g_n(D)] = D^l \qquad l \geq 0$$

Example: rate $\frac{1}{n}$ convolutional code

$$\begin{bmatrix} 1+D & 1 & 1+D \\ D & 1+D & 0 \end{bmatrix}$$

Theorem

A necessary and sufficient condition on the PGM of a general , rate $\frac{k}{n}$ convolutional encoder to avoid CEP is that

$$gcd[\triangle_0(D), \triangle_1(D), ..., \triangle_{\binom{n}{k}}(D) \qquad l \geq 0$$

where

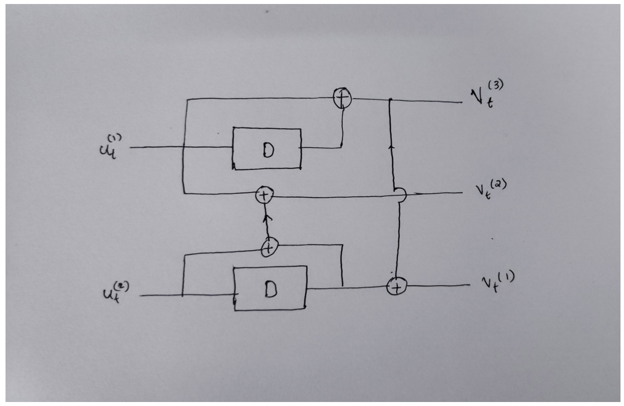
$$\{\triangle_i(D)_{i=1}^{\binom{n}{k}}\}$$

is the collection of determinants of the $\binom{n}{k}$ $[k * k]$ submatrix.

In our example

$$\triangle_1(D) = \begin{bmatrix} 1+D & 1 \\ D & 1+D \end{bmatrix} = 1 + D + D^2$$

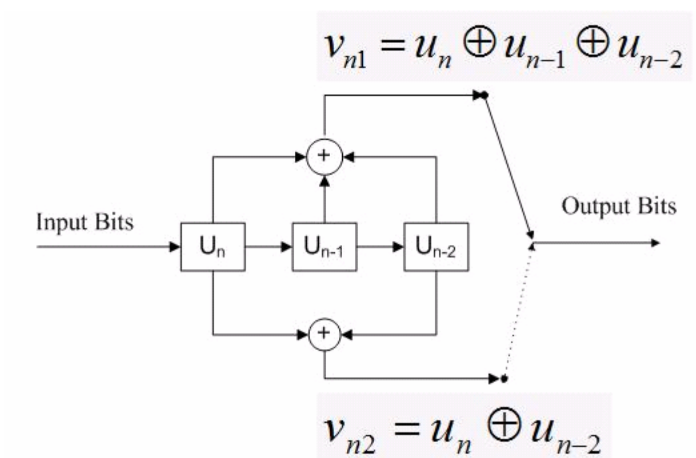
$$\triangle_2(D) = \begin{bmatrix} 1+D & 1+D \\ D & 0 \end{bmatrix} = D + D^2$$



$$\triangle_2(D) = \begin{bmatrix} 1 & 1+D \\ 1+D & D \end{bmatrix} = 1 + D^2$$

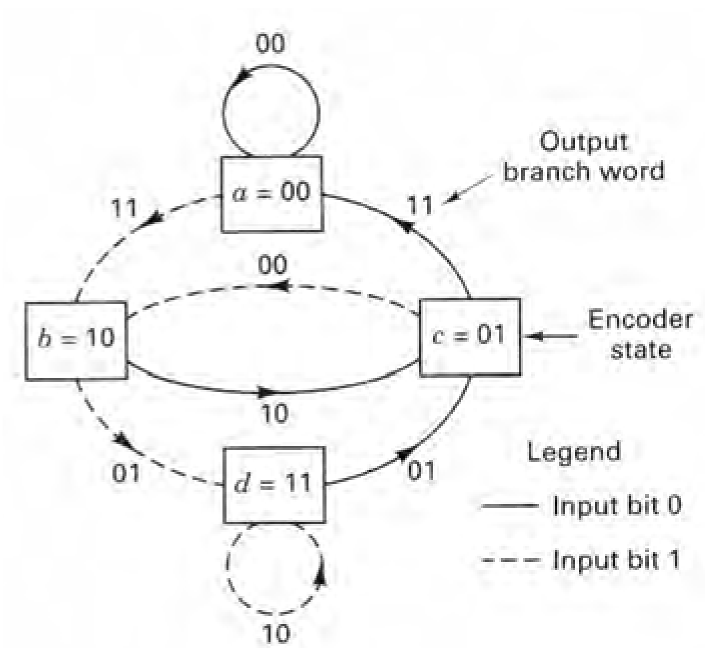
$$gcd[1 + D + D^2, D + D^2, 1 + D^2] = 1$$

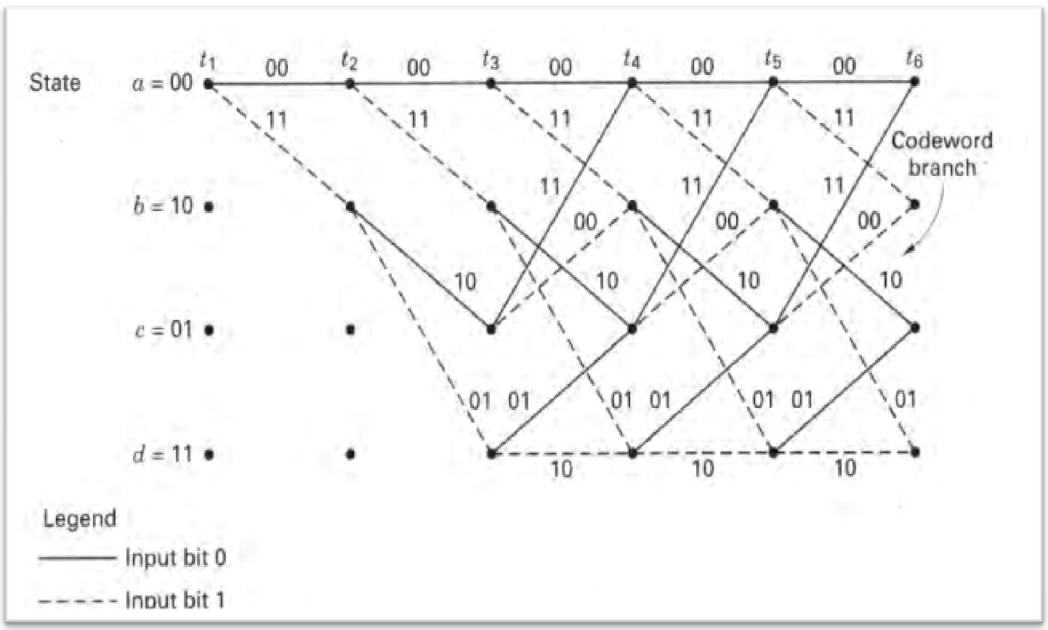
1.3 Finite State Machine



$$G(D) = [1 + D + D^2, 1 + D^2]$$

U_t	U_{t-1} U_{t-2}	V_t^1 V_Tt^2
0	00	00
1	00	11
0	01	11
1	01	00
0	10	10
1	10	01
0	11	01
1	11	10




State
a = 00
b = 10
c = 01
d = 11
t1
t2
t3
t4
t5
t6
00
11
10
01
Codeword branch
Legend
Input bit 0
Input bit 1