Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.

- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

- You can solve the programming problems in teams of three. One submission per team suffices. Indicate team members.

Problems:

1. Show that for any index set Δ, the function $V : [0,1] \to \mathbb{R}$ given by

$$x \mapsto V(x) = \inf_{\delta \in \Delta} [a_\delta x + b_\delta]$$

   is a concave function.

2. Let $V : [0,1] \to \mathbb{R}$ be a concave function. Show the following.

   - f is continuous in the open set $(0,1)$.
   - Left and right derivatives exist at every point in $(0,1)$.

3. Show that the minimum Bayes risk $V(\pi_0)$ is continuous at the end-points 0 and 1, that it has a right derivative at 0 and a left derivative at 1. Find the slopes. (*Hint*: Write out $V(\pi_0)$. For the right derivative at 0, use $p_1(y) \leq \tau(\pi_0) p_0(y)$ for the decision region for H_0, where $\tau(\pi_0)$ is $O(\pi_0)$ as π_0 approaches 0).

4. Problem 2 in Section II.F.

5. Problem 11 in Section II.F.

6. Problem 16 in Section II.F.

7. Write a program in matlab to generate a Gaussian random variable with mean 0 and variance 1 using only the function rand that generates a uniform random variable between $[0,1]$. (*Hints*: It is easier to generate two such Gaussians. What is the distribution of $R^2 = X_1^2 + X_2^2$? Of the angle $\theta = \arctan(X_2/X_1)$? How will you generate samples for R and Θ using rand? Transform the generated samples into samples for X_1 and X_2. Vectorise your code.)

8. Let $d = (\mu_1 - \mu_0)/\sigma$ in the location testing problem with Gaussian errors. Plot the receiver operating characteristics for $d = 0$, $d = \infty$ and three other intermediate values of d. (You can use erf or erfc or build your Q function.)