**E1–244** **Detection and Estimation Theory**

Discussion: After lectures
Due: No due date

# Problem Set 6

*Instructor: Rajesh Sundaresan* *TA: None*

**Problems:**

1. Let $(Y_1, \ldots, Y_n)$ be iid $N(\mu, \sigma^2)$. Assume that $\mu$ is known. What is the MVUE for $\sigma^2$? What is the minimum variance of this MVUE? (Compare this estimator with the one for the next problem).

2. For the same problem, assume that $\mu$ and $\sigma^2$ are both unknown. We saw the MVUE for $\sigma^2$ was $(n-1)^{-1}\sum_{k=1}^{n}(y_k - \hat{\mu}(y_1, \ldots, y_n))^2$ where $\hat{\mu}(y_1, \ldots, y_n)$ was the mean of the samples. Compute the minimum variance of this unbiased estimator. What is the penalty for not knowing $\mu$?

3. For the problem above ($\mu$ and $\sigma^2$ are both unknown), we identified the ML estimates $\hat{\mu}_{ML}(y)$ and $\hat{\sigma^2}_{ML}(y)$ in class. Are they efficient? If yes, why? If not, are they asymptotically efficient?

4. Give an example of a sequence $\{X_n, n \geq 1\}$ such that $X_n \to N(0,1)$ in distribution, and yet $\lim_n \mathsf{Var}(X_n) \neq 1$. Conclude that asymptotic normality does not imply asymptotic efficiency. (Advanced question: Can you think of some conditions under which this is true?).

5. Problem 13 in Section IV.F. Do your answers to part (b) generalise to the case when the alphabet set is not merely $A = \{\mathsf{heads}, \mathsf{tails}\}$, but now $A = \{1, 2, \ldots, a\}$? (First identify $\Lambda$, the parameter space). What about MVUE for a fixed component? Does it achieve the CRLB?

6. Let $g_0(Y)$ and $g_1(Y)$ be two unbiased estimators of $g(\theta)$. What all can you say about the new estimator $\alpha g_1(Y) + (1-\alpha)g_0(Y)$ of $g(\theta)$? Can you argue that the MVUE is unique?

7. Let $X_1, X_2, \ldots, X_n$ be iid $U[a,b]$. Let the ordered $X_i$'s be denoted $X_{(1)} \leq X_{(2)} \leq \cdots \leq X_{(n)}$. Argue that $(X_{(1)}, X_{(n)})$ is a sufficient statistic for the family $\{U[a,b],\ -\infty < a < b < +\infty\}$. Suppose now that the new family is $\{U[-b,b],\ b > 0\}$. Is there a further function of the above sufficient statistic that is also a sufficient statistic for the new family?