E1–244	**Detection and Estimation Theory**	
		Discussion: Mondays 13,20 April 2015, 11:30-12:30 Final exam: Friday 24 April 2015

Problem Set 7

Instructor: Rajesh Sundaresan — *TA: None*

Problems:

1. For the Kalman-Bucy filter, verify that the innovations sequence $\{I_n, n \geq 0\}$, where $I_n = Y_n - H_n \hat{X}_{n|n-1}$, is an independent sequence (under Gaussian assumptions).

2. Exercise 2 in Section V.E.

3. Exercise 5 in Section V.E.

4. Exercise 11 in Section V.E.

5. Consider the model $Y = sX + Z$ where $s \in \mathbb{R}^N$ is some known signature vector, X is ± 1 with equal probability, and Z has mean 0 and covariance Σ. The goal is to estimate X having observed Y with estimates of the form $\hat{X} = h^T Y$. Consider now the following iterative procedure with training where you are given $\{(X_i, Y_i), i = 1, 2, \ldots, r\}$ for r training instants. Define $\phi(h) := \mathbb{E}[(X - h^T Y)^2]$ and fix $\mu > 0$. Start with a candidate $h(0)$, and update as follows:

$$h(k) = h(k-1) - \mu \nabla \phi(h(k-1)), \quad k \geq 1.$$

   Towards which point h^* are these iterates taking you? Give a heuristic justification for your answer.

   To get $\nabla \phi(h)$, we need to know the statistics of (X, Z). Replace this by a noisy estimate of the gradient coming from the sample (X_k, Y_k), and write down a noisy iterative scheme for proceeding towards h^*.

6. Let $Y_n = X_n + Z_n$ where $-\infty < n < \infty$. Assume that $\{X_n\}$ and $\{Z_n\}$ are uncorrelated, zero mean, and have power spectral densities $\phi_X(\omega)$ and $\phi_Z(\omega)$, respectively. Find the transfer function for the noncausal Wiener-Kolmogorov filter in terms of $\phi_X(\omega)$ and $\phi_Z(\omega)$. Give an expression for the (linear) minimum mean squared error in terms of these power spectral densities.

7. Let $\tilde{X}_t$ be the best linear noncausal estimate of X_t given the observations $\{Y_n, -\infty < n < \infty\}$. Let $\hat{X}_t$ denote the best linear causal estimate of X_t given the observations $\{Y_n, -\infty < n \leq t\}$. Argue that $\hat{X}_t$ is the linear MMSE of $\tilde{X}_t$ given $\{Y_n, -\infty < n \leq t\}$, and interpret.