E1–254 **Game Theory**

Due: 29 August 2013

Problem Set 1

Instructor: Rajesh Sundaresan

1. Consider the game of matching pennies. The winner gets 1 and the loser gets -1. Identify all the pure and mixed strategy Nash equilibria.

2. Consider player i. For a probability distribution σ_i on the finite set of actions S_i, write $\delta(\sigma_i) = \{s_i \in S_i : \sigma_i(s_i) > 0\}$ for the support set of σ_i. Prove the following statement.
   The mixed strategy profile $(\sigma_i^*, i \in I)$ is a mixed strategy Nash equilibrium if and only if the following hold for each i:

   - $u_i(s_i, \sigma_{-i}^*)$ is the same for all $s_i \in \delta(\sigma_i^*)$,
   - $u_i(s_i', \sigma_{-i}^*) \leq u_i(s_i, \sigma_{-i}^*)$ for each $s_i' \notin \delta(\sigma_i^*)$ and $s_i \in \delta(\sigma_i^*)$.

3. Prove or disprove:

   - For a finite two-player game, if iterated elimination of weakly dominated strategies results in elimination of all but one strategy profile, then that survivor is a Nash equilibrium.
   - For a finite two-player game, iterated elimination of weakly dominated strategies cannot eliminate a Nash equilibrium.

4. Consider the finite game $(I, (S_i, i \in I), (u_i, i \in I))$, where $|I| = n$. For a mixed strategy profile $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_n)$ define $f_i(\sigma) \in \Sigma_i$ as follows

$$f_i(\sigma)(s_i) = \frac{\sigma_i(s_i) + [u_i(s_i, \sigma_{-i}) - u_i(\sigma_i, \sigma_{-i})]_+}{\sum_{s_i' \in S_i} [\sigma_i(s_i') + [u_i(s_i', \sigma_{-i}) - u_i(\sigma_i, \sigma_{-i})]_+]}, \quad s_i \in S_i.$$

   and further define $f = (f_1, f_2, \ldots, f_n)$. This time, apply Brouwer's fixed point theorem (every continuous function from a compact convex subset of a Euclidean space to itself has a fixed point) to the function f and establish Nash's theorem.