E1–254 **Game Theory**

Due: 12 September 2013

Problem Set 2

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1. Consider a two player zero sum game $(\{1,2\},(S_1,S_2),(u,-u))$. Show that if (x_1,y_1) and (x_2,y_2) are NE, then so is (x_1,y_2) [Interchangeability property]. As a consequence, there is no communication problem in reaching a NE, unlike in a nonzero-sum game (e.g., battle of the sexes).

2. (Aumann, 1976). Suppose agents 1 and 2 have uniform priors on the parameter (probability of 'head') of a coin. Let A be the event that the coin will come up 'head' in the next toss. Each person was permitted one private toss with the coin. Agent 1's private toss came up 'head'. Agent 2's private toss came up 'tail'. What are their posteriors for event A? If each informs the other of their posterior probabilities, what are their updated posteriors?

3. (Myerson, Exercise 6.2) Consider the two player game with payoffs as in the table below.

	L	C	R
U	$0,0$	$5,4$	$4,5$
M	$4,5$	$0,0$	$5,4$
D	$5,4$	$4,5$	$0,0$

   (a) Find all the Nash equilibria (pure and mixed).

   (b) Show that this game has a correlated equilibrium in which both players get expected payoffs strictly larger than in any NE.

   (c) Find the correlated equilibrium that maximizes the expected payoff for player 1.

4. Prove or disprove:

   - Every pure or mixed strategy NE is a correlated equilibrium.
   - Iterated best response need not always converge.
   - Suppose that the sequence of empirical distributions of a sequence of iterated best responses converges to some σ. Then σ must be a NE for the game.

5. Let $u^1(x,y)$ and $u^2(x,y)$ be the continuously differentiable utility functions of two players over a compact convex rectangle $S = S_1 \times S_2$. Write $J(x,y)$ for the Jacobian of $g(x,y) = [u^1_x, u^2_y]^T$, where $u^l_x(\cdot)$ and $u^l_y(\cdot)$ are the partial derivatives of u^l with respect to x and y, respectively. If $J(x,y) + J(x,y)^T$ is negative definite over S, then the family $u = (u_i, i = 1,2)$ is diagonally strictly concave. (Hint: If (x^0,y^0) and (x^1,y^1) are two points in S, write $(x^t,y^t) = t(x^1,y^1)+(1-t)(x^0,y^0)$. What is $dg(x^t,y^t)/dt$?).