

## Problem Set 3

*Instructor: Rajesh Sundaresan*

1. Prove that in Shapley's example of a two-player nonzero-sum game, a modification of the rock-paper-scissors game, fictitious play does not converge.
2. For any finite game in strategic form, prove that there is at least one (trembling-hand) perfect equilibrium.
3. Simulate the replicator dynamics for the dove-hawk game. The pay-off matrix is as follows.

	<i>D</i>	<i>H</i>
<i>D</i>	3, 3	5, 1
<i>H</i>	1, 5	0, 0

At time  $t = 0$ , assume  $n^0(D)$  and  $n^0(H)$  to be the initial condition. At time  $t$ , sample two individuals from the population (without replacement) and make them play. Their strategy is their type (dove or hawk). Replace these two birds, and additionally, add as many birds of the same feather as that bird's reward. If you sampled a dove and a hawk, you'd restore the two birds to the population, and add a dove and five hawks to the population. Provide a print out of your code along with two plots. The first plot should have two curves for the population evolution of dove and hawks for  $1 \leq t \leq 100$ . The second plot should be a curve of the evolution of the fraction of doves.

4. Suppose that the strategy set of any player is an interval of real numbers. Let the payoff function for each player be continuously differentiable. Let  $P$  be a continuously differentiable function from  $S = S_1 \times \dots \times S_n$  to  $R$ . Find a necessary and sufficient condition for  $P$  to be a potential function for the game.