Due: 24 October 2013

Problem Set 4

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1. Consider a learning scheme where a forecaster uses the function $\Phi(R_t) = \Phi(R_{1,t}, \ldots, R_{N,t}) = \sum_{i=1}^{N}([R_{i,t}]_+)^2$ and the weights $w_{t+1} = \nabla\Phi(R_t)$. The loss function once again is bounded between 0 and 1 as in class. Argue that normalized worst-case regret

$$\max_{1 \le i \le N} \frac{R_{i,n}}{n}$$

continues to go to zero as $O(1/\sqrt{n})$. How does the normalized worst-case regret scale with N the number of experts?

2. Prove or disprove: The half planes $H_i = \{x \in R^n | x_i \le 0\}$ are approachable for each $i = 1, 2, \ldots, n$ if and only if the negative orthant is approachable.

3. Let Γ^e be a game in extensive form. Prove or disprove: If τ_i is a mixed representation of a behavioural strategy σ_i of player i of the game Γ^e, the σ_i is a behavioural representation of τ_i.

4. Consider a game Γ^e in extensive form *with perfect recall*. For a player i and an information state s in which player i moves, define $C_i^*(s)$ to be the set of all pure strategies in C_i that are compatible with the state s. Let x be a node where it is player i's turn to move and let the corresponding information state be s. Define $B_i(x)$ to be exactly those pure strategies c_i where player i makes all the moves necessary for the play to reach node x. (See Myerson, p.202. A $c_i \in B_i(x)$ if and only if for any information state r at which it is player i's turn and for any move d_r, if there is a branch on the path from the root node to x with move label d_r at a decision node $i.r$, then $c_i(r) = d_r$). Argue that $B_i(x) = C_i^*(s)$.