E1–254 **Game Theory**

Due: 12 November 2013

Problem Set 5

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1. *Ultimatum game*: There is a pile of 100 gold coins. Player 1 can offer i gold coins to player 2, $0 \leq i \leq 100$. Player 2 either accepts the offer, in which case player 1 takes the remaining coins, or player 2 rejects the offer, in which case both get 0. Identify all the subgame perfect equilibria of the extensive form game and the trembling hand perfect equilibria of the normal form game.

2. *Half Kuhn poker*: There are two players A and B, and a deck of three cards numbered 1,2,3. Before the game begins, each player adds one rupee to the pot. The deck is shuffled, each player is dealt a card. The third is put away. Player A plays first, and can either check (with no addition of money to the pot) or raise (by adding 1 rupee to the pot). If A checks, the player with the higher card wins the pot. If A raises, B can either check (by adding 1 rupee to the pot) or fold. If B checks, the player with the higher card wins the pot. If B folds, player 1 takes the pot. Draw the game tree. Find an equilibrium for the normal form representation of the game.

3. *Location game*: Consider a continuum of customers of unit density on the line segment $L = [-1/2, 1/2]$. There are two grocery stores that wish to set up shop. Store A goes first and decides on a location on L. Store B comes in later and decides on its location on L knowing store A's location. Given the locations of the stores A and B, customers choose the nearest store for their purchases. Each store's payoff is the volume of customers that comes to the store for purchases. (Half of those customers equidistant from A and B go to A and the remaining go to B). Identify the equilibria for this game.

4. *Cournot game with incomplete information*: Suppose that firm 1 is of type $\theta_1 = 1$ with probability p and of type $\theta_2 = 0$ with probability $1-p$. Firm 1 knows its type, but not firm 2. Firm 2 is always of type 1. If firm i's type is θ_i, it incurs a product cost per unit good of $\theta_i c$. Let q_1 and q_2 be the productions (actions) of firms 1 and 2, respectively. Then the demand is $d(q_1, q_2) = [a - q_1 - q_2]_+$. Revenue for firm i is $q_i[d(q_1, q_2) - c\theta_i]$. Find all the Bayesian equilibria for this game. Assume $0 < p < 1$ and $a \geq 2c$.