E1–254	**Game Theory**	
		Due: 12 December 2013

Problem Set 6

Instructor: Rajesh Sundaresan

1. Recall the revenue optimal mechanism with iid valuations. The function $m_i(x_i)$ is the *expected* payment from buyer i, conditioned on buyer i valuation of x_i, for the revenue optimal mechanism. Suppose now that the payment rule is modified to $P_i(x_i)\mathbf{1}\{i \text{ wins}\}$, where the function P_i is chosen so that $\mathbb{E}[P_i(X)\mathbf{1}\{i \text{ wins}\}|X_i = x_i] = m_i(x_i)$. Thus buyer i knows exactly what he will pay if he wins, at the time of bidding. Is truthful reporting a weakly dominant strategy? Is truthful reporting by all buyers a Bayes equilibrium?

2. For the revenue optimal mechanism, prove that the expected value of the virtual valuation is 0.

3. Suppose that a VCG mechanism is used to sell two objects $\mathcal{O} = \{a, b\}$ to three buyers. Each buyer can buy none, one, or both of the objects. Each buyer is asked to report his valuation function, i.e., $u_i = (u_i(\emptyset), u_i(\{a\}), u_i(\{b\}), u_i(\{a, b\}))$. Clarke's pivotal mechanism is used to allocate the objects and determine the payments. Suppose that

$$\begin{aligned} u_1 &= (0, 10, 3, 13) \\ u_2 &= (0, 2, 8, 10) \\ u_3 &= (0, 3, 2, 14). \end{aligned}$$

   Determine the assignment of objects and payments. Why might buyer 3 object to the outcome?

4. Consider two buyers for a good and the English auction for its sale. The signal X_i to buyer i is binary valued. The joint distribution is given by

$$p(0, 0) = 1/10, \quad p(0, 1) = p(1, 0) = 1/5, \quad p(1, 1) = 1/2.$$

   Verify that the joint distribution is affiliated.

   The valuation function for each buyer is $v_i(x) = x_{-i}$, i.e., each buyer's valuation is the other buyer's signal. Compute the equilibrium strategy. (a) When buyer 1 sees a signal 0, at what price will he drop out of the auction? When he sees a signal 1, at what price will he now drop out?