E2–201 Information Theory Discussion: Saturday 03 January 2025

Problem Set 1

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, practice the solution to each problem in your own words without referring to a friend, text, class notes, or AI engines.

Problems:

1. Exercises on sequences.
   - Recall the notation that $\leq_n$ stands for the relation "is less than or equals for all sufficiently large n". Recall that the definition of lim sup of a sequence: $\limsup_{n\to\infty} x_n = \inf_{n\geq 0} \sup_{m\geq n} x_m$.
     Suppose that for each $\varepsilon > 0$, we have $a_n \leq_n a + \varepsilon$. Show that $\limsup_{n\to\infty} a_n \leq a$.
   - Let $a_n \leq_n b_n$. Show that $\limsup_{n\to\infty} a_n \leq \limsup_{n\to\infty} b_n$.
   - Let $a = \limsup_{n\to\infty} a_n \in \mathbb{R}$. Show that for every $\varepsilon > 0$, the inequality $a_n > a - \varepsilon$ occurs infinitely often.
   - What are the analogous statements for $\liminf$?
   - Show that $\liminf_{n\to\infty} a_n \leq \limsup_{n\to\infty} a_n$.
   - Show that $\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n = a \in \mathbb{R}$ if and only if the following holds: for every $\varepsilon > 0$, there exists an N such that $n \geq N$ implies $|a_n - a| \leq \varepsilon$. This establishes that the usual notion of a limit and the one via limsup and liminf are equivalent.
2. Problem 3.1 of Cover and Thomas (2nd edition).
3. Problem 3.3 of Cover and Thomas (2nd edition).
4. Problem 3.7 of Cover and Thomas (2nd edition).
5. Problem 3.13 of Cover and Thomas (2nd edition).