E2–201 Information Theory Discussion: Saturday 17 January 2025

Problem Set 2

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Consider a discrete memoryless source DMS on A with PMF p. Recall that $s_q(n, \varepsilon)$ is the minimum q-weight of sets whose p-probability is at least $1 - \varepsilon$. Prove the following theorem which was discussed in class: For any ε satisfying $0 < \varepsilon < 1$, we have $\lim_{n\to\infty} \frac{\log s_q(n,\varepsilon)}{n} = -\sum_{x\in A} p(x) \log \frac{p(x)}{q(x)}$.
2. Suppose a_i and b_i are positive numbers for $i = 1, \ldots, n$. Prove the log-sum inequality:
$$\sum_i a_i \log \frac{a_i}{b_i} \geq \left(\sum_i a_i \right) \frac{\sum_i a_i}{\sum_i b_i}.$$
3. Find the derivative of $f(p) = -p \log p - (1-p)\log(1-p)$ at $p = 0$ and at $p = 1$.
4. Show that instantaneous codes are uniquely decodable.
5. Prove or disprove: "$H(X|Y = y)$ can be strictly larger than $H(X)$."
6. Problem 2.4 of Cover and Thomas (2nd edition).
7. If $g : A \to B$, show that $H(X) = H(g(X))$ if g is invertible.
8. Problem 2.5 of Cover and Thomas (2nd edition).
9. Problem 2.14 of Cover and Thomas (2nd edition).
10. Problem 2.19 of Cover and Thomas (2nd edition).