E2–201 Information Theory	Discussion: Saturday 11 January 2025

Problem Set 2

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Consider a discrete memoryless source DMS on A with PMF p. Recall that $s_q(n, \varepsilon)$ is the minimum q-weight of sets whose p-probability is at least $1 - \varepsilon$. Prove the following theorem which was discussed in class: For any ε satisfying $0 < \varepsilon < 1$, we have $\lim_{n \to \infty} \frac{\log s_q(n, \varepsilon)}{n} = -\sum_{x \in A} p(x) \log \frac{p(x)}{q(x)}$.
2. Suppose a_i and b_i are positive numbers for $i = 1, \ldots, n$. Prove the log-sum inequality:
$$\sum_i a_i \log \frac{a_i}{b_i} \geq \left(\sum_i a_i \right) \frac{\sum_i a_i}{\sum_i b_i}.$$
3. Find the derivative of $f(p) = -p \log p - (1-p) \log(1-p)$ at $p = 0$ and at $p = 1$.
4. Show that instantaneous codes are uniquely decodable.
5. If $g : A \to B$, show that $H(X) = H(g(X))$ if g is invertible.
6. Prove or disprove: "$H(X|Y = y)$ can be strictly larger than $H(X)$."
7. Problem 2.4 of Cover and Thomas (2nd edition).
8. Problem 2.5 of Cover and Thomas (2nd edition).
9. Problem 2.14 of Cover and Thomas (2nd edition).
10. Problem 2.19 of Cover and Thomas (2nd edition).