E2–201 Information Theory Discussion: Saturday 31 January 2026

Problem Set 3

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Convergence of Cesaro mean. Let $\lim_{n\to\infty} a_n = a$. Let $b_n = \frac{1}{n}\sum_{i=1}^{n} a_i$. Then show that $\lim_{n\to\infty} b_n = a$. Identify a counterexample where $\lim_n b_n$ exists but $\lim_n a_n$ does not.
2. Let $\{s_n\}$ be a nonnegative sequence such that $0 \le s_n < \infty$ for every n. Suppose that the sequence is subadditive, i.e., for any two natural numbers k, l, we have $s_{k+l} \le s_k + s_l$. Then show that $\lim_{n\to\infty} \frac{s_n}{n}$ exists and equals $\inf_{n\ge 1} \frac{s_n}{n}$.
   [*Hint*: As a first step, group into blocks of size m and use subadditivity to show $\limsup_{n\to\infty} \frac{s_n}{n} \le \frac{s_m}{m}$.]
3. Problem 4.2 of Cover and Thomas (2nd edition).
4. Problem 4.10 of Cover and Thomas (2nd edition).
5. Problem 4.14 of Cover and Thomas (2nd edition).
6. Problem 4.30 of Cover and Thomas (2nd edition).
7. Problem 5.7 of Cover and Thomas (2nd edition).
8. Problem 5.28 of Cover and Thomas (2nd edition).
9. Problem 5.29 of Cover and Thomas (2nd edition).
10. Let P and Q be two PMFs on A. Suppose P is the true PMF, but the designer assumes Q and encodes with lengths given by $\lceil -\log_2 Q(a) \rceil$ bits for the letter $a \in A$. Give a bound on the excess from entropy for this mismatched compression, and write it in terms of relative entropy. How would your answer change for groups of n letters? In the limit?