E2–201 Information Theory Discussion: Saturday 01 February 2025

Problem Set 4

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Consider a discrete memoryless source (DMS) on $A = \{0, 1\}$ whose marginal is such that $\Pr\{X_1 = 1\} = 0.75$ or $\Pr\{X_1 = 1\} = 0.4$. The encoder and the decoder do not know which of these sources is the true one. What is $\lim_{n\to\infty} \frac{r(n,\varepsilon)}{n}$ for this 'uncertain' source?

2. Show that in a sequence of $2k$ independent tosses of an unbiased coin, the probability of getting *exactly* k heads goes to 0 as $k \to \infty$.

3. Yet, show that the probability of getting exactly k heads in $2k$ independent tosses of the unbiased coin is at least $1/(2k + 1)$ by showing that
$$\binom{2k}{k} \geq \binom{2k}{l} \text{ for any } l = 0, 1, \cdots, 2k.$$
[*Hint*: Prove that $a!/b! \geq b^{a-b}$, and use it.]

4. Let τ and $\hat{\tau}$ be types that belong to $\mathcal{T}_n$. Let $A_n(\tau)$ be the set of sequences of type τ. Similarly $A_n(\hat{\tau})$. Consider the DMS with marginal τ on A. Denote its n-letter PMF τ_n on A^n. Show that $\tau_n(A_n(\tau)) \geq \tau_n(A_n(\hat{\tau}))$. [*Hint*: Write out the multinomial probabilities and use the previous problem's hint.]

5. We proved that the upper bound for the number of types of sequences in A^n is $(n + 1)^{|A|}$. Show that the exact number is
$$\binom{n + |A| - 1}{|A| - 1}.$$

6. Use Stirling's approximation $k! \simeq \sqrt{2\pi k}k^k e^{-k}$ and get an approximation for $\log |A_n(\tau)|$.

7. Problem 4.1 of Cover and Thomas (2nd edition).

8. Problem 4.2 of Cover and Thomas (2nd edition).

9. Problem 11.3 of Cover and Thomas (2nd edition). [*Hint*: Use Lagrange multipliers to make the constrained optimisation into an unconstrained one.]

10. Give an example of pair of distributions P_1 and P_2 such that $D(P_1 \| P_2) \neq D(P_2 \| P_1)$.