**E2–201 Information Theory** Discussion: Saturday 07 February 2026

# Problem Set 4

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**Remarks:**

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

**Problems:**

1. Consider a discrete memoryless source (DMS) on $A = \{0, 1\}$ whose marginal is such that $\Pr\{X_1 = 1\} = 0.75$ or $\Pr\{X_1 = 1\} = 0.4$. The encoder and the decoder do not know which of these sources is the true one. What is $\lim_{n\to\infty} \frac{r(n,\varepsilon)}{n}$ for this 'uncertain' source?

2. Show that in a sequence of $2k$ independent tosses of an unbiased coin, the probability of getting *exactly* $k$ heads goes to 0 as $k \to \infty$.

3. Yet, show that the probability of getting exactly $k$ heads in $2k$ independent tosses of the unbiased coin is at least $1/(2k + 1)$ by showing that

$$\binom{2k}{k} \geq \binom{2k}{l} \text{ for any } l = 0, 1, \cdots, 2k.$$

   [*Hint*: Prove that $a!/b! \geq b^{a-b}$, and use it.]

4. Let $\tau$ and $\hat{\tau}$ be types that belong to $\mathcal{T}_n$. Let $A_n(\tau)$ be the set of sequences of type $\tau$. Similarly $A_n(\hat{\tau})$. Consider the DMS with marginal $\tau$ on $A$. Denote its $n$-letter PMF $\tau_n$ on $A^n$. Show that $\tau_n(A_n(\tau)) \geq \tau_n(A_n(\hat{\tau}))$. [*Hint*: Write out the multinomial probabilities and use the previous problem's hint.]

5. We proved that the upper bound for the number of types of sequences in $A^n$ is $(n + 1)^{|A|}$. Show that the exact number is

$$\binom{n + |A| - 1}{|A| - 1}.$$

6. Use Stirling's approximation $k! \simeq \sqrt{2\pi k}k^k e^{-k}$ and get an approximation for $\log |A_n(\tau)|$.

7. Problem 4.1 of Cover and Thomas (2nd edition).

8. Problem 11.3 of Cover and Thomas (2nd edition). [*Hint*: Use Lagrange multipliers to make the constrained optimisation into an unconstrained one.]
9. Problem 11.5 of Cover and Thomas (2nd edition).
10. Give an example of pair of distributions $P_1$ and $P_2$ such that $D(P_1 \| P_2) \neq D(P_2 \| P_1)$.