Problem Set 6

Instructor: Rajesh Sundaresan   *TAs: Deepthi, Prachi, Narasimha, Tejashree*

Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.

- To test your understanding, write the solution to each problem in your own words without referring to a friend, text, or class notes.

Problems:

1. Given an example of n, P_{X^n}, Q_{X^n} such that $D(P_{X^n} \parallel Q_{X^n}) \not\geq \sum_{i=1}^{n} D(P_{X_i} \parallel Q_{X_i})$.

2. Show the following:

   (a) $I(X;Y|Z) = D(P_{X|YZ} \| P_{X|Z} | P_{YZ})$.

   (b) $I(X_1, X_2, \ldots, X_n; Y) = I(X_1; Y) + \sum_{i=2}^{n} I(X_i; Y | X_1, X_2 \ldots X_{i-1})$.

3. Prove or disprove: $I(X, Y; Z) \geq I(X; Y)$.

4. Problem 2.11 of Cover and Thomas (2nd edition).

5. Problem 2.12 of Cover and Thomas (2nd edition).

6. Problem 2.15 of Cover and Thomas (2nd edition).

7. Problem 2.41 of Cover and Thomas (2nd edition).

8. Problem 2.42 of Cover and Thomas (2nd edition).

9. Problem 2.43 of Cover and Thomas (2nd edition).

10. Show that $D(P_\lambda \| P_0)$ is an increasing function of λ for $\lambda \in [0, 1]$ where

$$P_\lambda(x) = \frac{P_1(x)^\lambda P_0(x)^{1-\lambda}}{\sum_{a \in A} P_1(a)^\lambda P_0(a)^{1-\lambda}}.$$