E2–201 Information Theory Discussion: Saturday/Tuesday 16/18 March 2025

Problem Set 7

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1. Prove the "joint" asymptotic equipartition property for an $A(n, \delta)$ that consists of sequences whose empirical and both marginal frequencies (normalised histograms) are within a small error around the true PMFs.

2. Show that capacity does not change if $P_e^{(n)}(c)$, the average probability of error, is replaced by $\overline{P}_e^{(n)}(c)$, the maximum probability of error.

3. Can you apply the source-channel separation principle for a two-terminal system with feedback? In other words, is a stationary and ergodic source with $H > C$ transmissible over a DMC with perfect feedback?

4. Find the capacity of the noisy typewriter with five input symbols.

5. Find a code for the noisy typewriter with five input symbols such that the probability of error is zero, and the rate is strictly larger than 1 bit per transmission.

6. Consider a channel whose input alphabet is $\{0, 1\}^7$, i.e., 7 bits. The channel is such that either there is no error, or exactly one of the seven bits is flipped. These eight possibilities have equal probabilities. What is the capacity of this channel? Can you come up with a code operating at that capacity? (If you can, find your code's probability of error).

7. Suppose $F_0 = 1, F_1 = 2$, and define $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$. Show that F_n grows exponentially with n, i.e., there is a $\gamma > 1$ such that $n^{-1} \log F(n) \to \log \gamma$, so that $F_n \sim \gamma^n$ for large n. What if $F_n = \sum_{i=1}^{k} a_i F_{n-i}$ with some initial condition?

8. Consider the following telegraph channel where dots and dashes are used as input symbols. The channel is noiseless. Every dot lasts one unit of time and every dash lasts two units of time. Get a recurrence relation for the total number of strings that last n units of time. Hence deduce the capacity of this channel in bits per unit time.

9. Consider the binary symmetric channel with cross-over probability $\delta > 0$. For an arbitrary code word, what is the support set of the output string? If your code must have zero probability of error, what is the (zero-error) capacity?

10. Consider a binary input binary output channel with $W(0|0) = 1$ and $W(1|1) = \delta$, where $0 < \delta < 1$. This is called a Z-channel for reasons that will be obvious when you draw the channel picture. Call the capacity of this channel $C(\delta)$. Show that $\lim_{\delta \downarrow 0} C(\delta)/\delta = e^{-1}$.