E2–202 Random Processes Discussion: 08 August 2026

Problem Set 1

Instructors: Rajesh Sundaresan
TAs: Sowmya, Ram Rathan, Tithi, Yash, Dwaipayan

Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, practice the solution to each problem in your own words without referring to a friend, text, class notes, or AI engines.

Problems:

1. Let $\{A_i, i \geq 1\}$ be a collection of sets. Explain in plain language what we mean by ω belongs to the set ...

$$B_n = \bigcap_{i=1}^{n} A_i, \quad B = \bigcap_{i=1}^{\infty} A_i := \lim_{n\to\infty} \left(\bigcap_{i=1}^{n} A_i \right).$$

   Similarly, explain the above when unions are involved instead of intersections.

2. Let $\{A_i, i \geq 1\}$ be a collection of sets. The index set is $\{1, 2, \ldots\}$. (Often, we write $\cap_i A_i$, when the index set is understood.) Prove the following:

$$\text{de Morgan's laws: (a)} \left(\bigcup_i A_i \right)^c = \bigcap_i A_i^c, \quad \text{(b)} \left(\bigcap_i A_i \right)^c = \bigcup_i A_i^c.$$

   While our attention will be on countably infinite unions and intersections, do de Morgan's laws hold for arbitrary index sets?

3. Problem 1.1 of the reference text.

4. Problem 1.2 of the reference text.

5. Problem 1.3 of the reference text.

6. Exercise 1.1 of the reference text. Write the solution in your own words, highlighting the places where the probability axioms are used.

7. In Example 1.2, what is the probability of the sequence 10000...? What is the set corresponding to $\{w_i = 1\}$? This should be a union of intervals. What is the total length of the union? Interpret.

8. (Finite, countable, and all that). We are given the set of natural numbers $\mathcal{N}$. A set is called *finite* if it is either empty or is the range of $\{1, 2, \ldots, n\}$ for some $n \in \mathcal{N}$. A set is called *countable* (or *denumerable*) if it is either empty or is the range of $\mathcal{N}$. Try to show the following.
   - The set of whole numbers, the set of integers, the set of even numbers, the set of odd numbers, are all countable.
   - Every subset of a countable set is countable.
9. (Some additional questions on the same theme for you to think about. The TA will help you with the proofs.)
   - Let A be a countable set. Then set E of all *finite* sequences from A is also countable.
   - The set of all rational numbers $\mathcal{Q}$ is countable.
   - The union of a countable collection of countable sets is countable.
   - Let $A = \{0, 1\}$, binary. The set E of infinite sequences from A is not countable. (You will be introduced to Cantor's diagonal process for this question.)
   - The set $[0, 1]$ is not countable. Hence the set of real numbers is not countable.
10. Let A and B be events with probabilities $P(A) = 3/4$ and $P(B) = 1/3$. Can you find the possible minimum and maximum values for $P(A \cap B)$? (From Grimmett and Stirzaker).