E2–202 Random Processes Discussion: 15 August 2026

Problem Set 2

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, practice the solution to each problem in your own words without referring to a friend, text, class notes, or AI engines.

Problems:

1. Exercise 1.2 of the reference text.
2. Show the 'if' part in the Remark following Exercise 1.2.
3. Sequences of real numbers, limsup, liminf, and all that. In a previous exercise, you would have seen this for sets. Now for real numbers.

   Consider a sequence of real numbers $\{a_n, n \geq 1\}$. The supremum of the sequence is its *least upper bound*. Similarly, the infimum of the sequence its *greatest lower bound*. These are denoted $\sup_{n\geq 1} a_n$ and $\inf_{n\geq 1} a_n$, respectively.

   Define lim sup of a sequence: $\limsup_{n\to\infty} a_n := \inf_{n\geq 0} \sup_{m\geq n} a_m$.

   The notation $a_n \leq_n b_n$ stands for the relation "a_n is less than or equals b_n, for all sufficiently large n".

   - Suppose that for each $\varepsilon > 0$, we have $a_n \leq_n a + \varepsilon$. Show $\limsup_{n\to\infty} a_n \leq a$.
   - Let $a = \limsup_{n\to\infty} a_n \in \mathbb{R}$. Show that for every $\varepsilon > 0$, the inequality $a_n > a - \varepsilon$ occurs infinitely often.
4. Let $a_n \leq_n b_n$. Show that $\limsup_{n\to\infty} a_n \leq \limsup_{n\to\infty} b_n$.
5. Define lim inf of a sequence as follows: $\liminf_{n\to\infty} a_n := \sup_{n\geq 0} \inf_{m\geq n} a_m$. Provide statements analogous to Problem 3 for lim inf?
6. (Relation between limsup and liminf.) Show that $\liminf_{n\to\infty} a_n \leq \limsup_{n\to\infty} a_n$.
7. (Limit of a sequence and a set notation.) Show that $\liminf_{n\to\infty} a_n = \limsup_{n\to\infty} a_n = a \in \mathbb{R}$ if and only if the following holds:

   for every $\varepsilon > 0$, there exists an N such that $n \geq N$ implies $|a_n - a| \leq \varepsilon$.

This establishes that the more familiar notion of the limit of a sequence and the one via limsup and liminf are equivalent.

If $E_n(\varepsilon) := \{|a_n - a| \leq \varepsilon\}$, can you write the above condition in set notation involving intersections, unions, and intersections?

8. Exercise 1.3 of the reference text.

9. Practice writing out the proofs of all five parts of Theorem 1.4 in your own words without any outside help.

10. Problem 1.5 in Problems Section (1.8) of the reference text (problems on cumulative distribution functions).