E2–202 Random Processes Due: 03 September 2026

Problem Set 4

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Remarks:

- Collaboration, discussion, and working in teams to solve problems is strongly encouraged.
- To test your understanding, practice the solution to each problem in your own words without referring to a friend, text, class notes, or AI engines.

Problems:

1. Suppose X_1, X_2 are independent exponentially distributed random variables with parameter λ. What are the distributions of the following?
   - $Y = \min\{X_1, X_2\}$
   - $Z = X_1 + X_2$.
2. Characteristic functions.
   - Write the characteristic functions of $U = X + \mu$ and $V = aX$ in terms of the characteristic function of X.
   - Suppose X and Y are independent. Write the characteristic function of $Z = X + Y$ in terms of the characteristic functions of X and Y.
   - Compute the characteristic functions of the following random variables and vectors: (a) $\text{Exp}(\lambda)$, (b) $N(0, 1)$, (c) multivariate $N(\mathbf{0}, I_d)$, (d) multivariate $N(\mathbf{0}, K)$ (jointly Gaussian with zero means and covariance K.
3. Suppose X is a jointly Gaussian random vector of dimension d with mean μ and covariance K where K is positive definite. What are the distributions of the following?
   - $Y = CX$, where C is full rank.
   - $Z = X_1^2 + X_2^2$.
4. Let X and Y be jointly Gaussian with zero means, unit variances, and correlation ρ. Write down the K. Find the joint density of $(U, V) = (X + Y, X - Y)$.
5. Let X_1, X_2, and X_3 be three independent and identically uniformly distributed on $[0, 1]$ random variables. What is the chance that these lengths will form a triangle?
6. Lower bounds on probabilities are hard to come by. But here is one. Suppose $0 \leq X \leq 1$. Show that $P(X \geq a) \geq \frac{E[X]-a}{1-a}$. (Hint: Use $X \leq aI\{X < a\} + 1I\{X \geq a\}$.)