E2 203 Wireless Communications (Jan.-Apr. 2011) Problem Set 2

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Due: March 1, 2011, 1:00 PM

Remarks:

- You may collaborate, discuss, work in teams to solve problems.
- Each student must write the solution in his/her own words.

I. Problems to work on and submit

1) (ML rule practice) Consider a receiver that observes n samples $y_1, \ldots, y_n$, which are given as follows:

$$y_n = e^{jn\theta} + w_n, \quad n = 1, \ldots, N, \tag{1}$$

where $w_1, \ldots, w_n$ are circular symmetric white Gaussian noise with unit variance and $\theta \in [-\pi, \pi]$. Find the maximum likelihood estimate of θ and interpret your answer.

2) (Maximal ratio combining) In an L-branch diversity system the received signal is given by

$$y[i] = h[i]x + n[i], \quad i = 1, \ldots, L, \tag{2}$$

where x is the transmitted symbol and $n[1], \ldots, n[L]$ are i.i.d. circular symmetric additive white Gaussian noise with variance σ^2.

Consider a coherent receiver that linearly weights the received signals as $w_i y[i]$ and adds them to get $Z = \sum_{i=1}^{L} w_i y[i]$. Show that the optimal weights that maximize the instantaneous (i.e., not averaged over fading) SNR of Z are given by

$$w_i = h[i]^*. \tag{3}$$

What is the final expression for the instantaneous SNR? Interpret it.

Hint: Use the Cauchy-Schwartz inequality.

3) Exercise 3.37 (first part).

4) The Nakagami-m fading distribution is given by

$$f_X(x) = \frac{2m^m}{\Gamma(m)\Omega^m} x^{2m-1} \exp\left(-\frac{mx^2}{\Omega}\right), \quad x \geq 0 \text{ and } m > 0.5. \tag{4}$$

- Verify that this is a valid distribution and show that $m = 1$ corresponds to Rayleigh fading.
- What is the value of $\mathbf{E}\left[X^2\right]$?
- What is the probability of a deep fade if the fading follows the Nakagami-m distribution?

5) Exercise 3.19 of [Tse & Viswanath]

II. Problems to work on but not submit

1) Prove that $Q^2(x) = \frac{1}{\pi} \int_0^{\frac{\pi}{4}} \exp\left(-\frac{x^2}{\sin^2\theta}\right) d\theta$.

2) Exercise 3.12 of [Tse & Viswanath]