E2 203 Wireless Communication
January-April 2011
Problem Set 3
Due: Tuesday 29 March 2011 at 1:00 PM

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Question 1 Consider N closely spaced antennas at a handset receiver so that the $h = [h_1, h_2, \cdots, h_N]^t$ is now made of correlated circular symmetric jointly Gaussian complex random variables with covariance matrix $\mathbb{E}[hh^*] = Q = Q_1 + iQ_2$, where h^* is the Hermitian conjugate of the vector h. The received vector is $Y = hX + W \in \mathbb{C}N$, where $X \in \{-1, 1\}$ is a unit energy BPSK signal. W is circular symmetric jointly Gaussian with covariance I_N.

(a) Find $\mathbb{E}[hh^t]$ and $\mathbb{E}[\Re\{h_1\}\Im\{h_1\}]$. ($\Re\{h_1\}$ is the real part of h_1 and $\Im\{h_1\}$ is its imaginary part).

(b) Assume that the realisation h is known to the receiver. What then is the optimum receiver for determining X?

(c) What is the instantaneous signal to noise ratio (SNR) for the above receiver?

(d) Noting that the instantaneous SNR is a random variable, show that its mean equals the trace of Q.

(e) Let $h = h_R + ih_I$ and write

$$\hat{h} = \begin{bmatrix} h_R \\ h_I \end{bmatrix} \in \mathbb{R}^{2N}.$$

Show that

$$\mathbb{E}\left[\hat{h}\hat{h}^t\right] = \begin{bmatrix} \frac{Q_1}{2} & -\frac{Q_2}{2} \\ \frac{Q_2}{2} & \frac{Q_1}{2} \end{bmatrix}$$

(f) Show that the standard deviation of the SNR random variable you found in (b) is the Frobenius norm of Q. (Note: If $[A, B]^t$ is a jointly Gaussian real two-dimensional zero-mean random vector with covariance matrix K, then $\mathbb{E}\left[A^4\right] = 3K_{11}^2$ and $\mathbb{E}\left[A^2B^2\right] = K_{11}K_{22} + 2K_{12}^2$.)

Question 2 A transmitter (Tx) and receiver (Rx) are placed next to two reflecting walls, as shown in Figure 1. The receiver moves with a speed of v m/s; its direction of motion is as shown in the figure. The transmitter is stationary, and the carrier frequency of transmission is f_c. Assume far field propagation conditions.

1. What is the impulse response of the channel from the transmitter to the receiver?
2. Write the above impulse response in baseband.
3. What is the delay spread of the channel?
4. What is the Doppler spread of the channel?

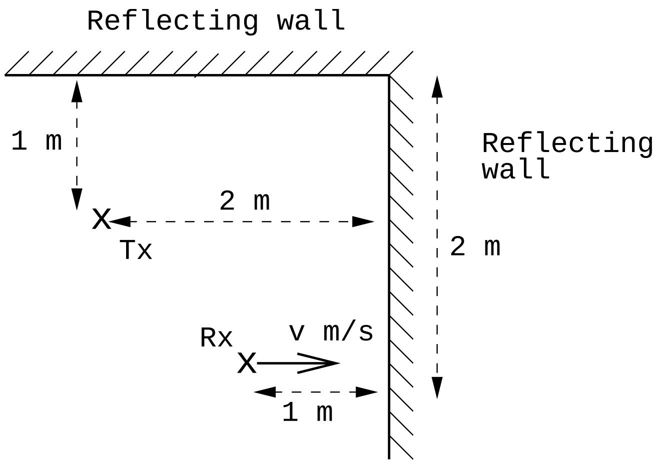

Figure 1:

Question 3 As shown in Figure 2, consider a transmitter with two antennas (1 and 2) that transmits using the Alamouti code to a receiver with one antenna. Each of the two symbols sent by the transmitter are drawn from the QPSK constellation: $\{\pm a \pm ja\}$.

Say the transmissions of the Alamouti code occur over times 1 and 2. The time-varying channel gain at time n between transmit antenna i and the receive antenna at time n is denoted by $h_i[n]$. The channel, thus, changes over the two transmissions of the Alamouti code.

Assume that the receiver knows $h_i[n]$ for $i = 1, 2$ and $n = 1, 2$, and $h_i[n]$ are i.i.d. across i and n.

1. Come up with an upper bound on the probability of error as a function of SNR.
2. What is the diversity order equal to for this system?
3. Compare the diversity order with the case where the channel does not change over the two symbol transmissions. Similarly, compare the decoding complexity.

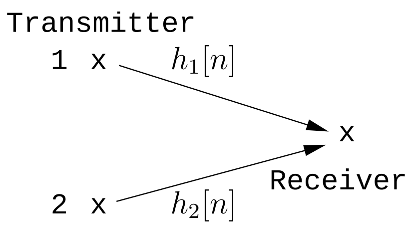

Figure 2: