

# E2 203 Wireless Communication

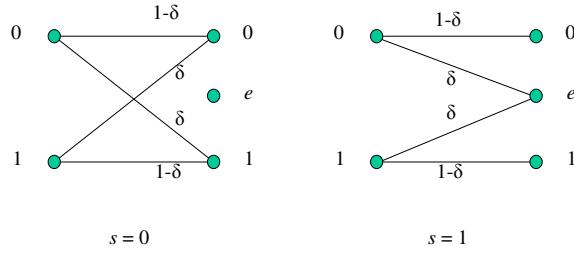
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### Problem Set 5

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1. Let  $\mathbb{X} = \{0, 1\}$  be the input alphabet and  $\mathbb{Y} = \{0, e, 1\}$  the output alphabet of a channel. The letter  $e$  stands for erasure. The channel is memoryless, i.e.,  $p(y^n | x^n, s^n) = \prod_{i=1}^n p(y_i | x_i, s_i)$ , and has two states  $\mathbb{S} = \{0, 1\}$  with parameters as given in the figure below.

- Show that the capacity of the channel when in state  $s = 0$  is  $1 - H(\delta)$  and when in state  $s = 1$  is  $1 - \delta$ .
- The sequence of states is iid with  $\Pr\{S_1 = 1\} = 1/2$ . Evaluate  $C_{\text{CSI}}$  and  $C_{\text{CSIR}}$ .



2. Argue that the entropy of a discrete random variable  $H(X)$  is a concave function of the probability mass function  $P_X$ . You can use derivatives, but keep in mind that the domain is a vector. (We used the analogous property for differential entropy in class).
3. Let  $Y = HX + Z$ , where  $X \in \mathbb{C}$ , and  $Y \in \mathbb{C}^{n_R}$ . The quantities  $X, Z$  are independent Gaussians.  $X \sim \mathcal{CN}(P)$  and  $Z \sim \mathcal{CN}(I_{n_R})$ . Let  $\tilde{Y} = c^*Y$  be the MMSE estimate of  $X$  given  $Y$ . Find  $c$ .
4. For the previous problem, argue that  $I(X; Y) = I(X; \tilde{Y})$ . This can be used to get an information lossless receiver architecture by MMSE and decision feedback equalisation.
5. Compare the capacity of the  $2 \times 1$  MISO channel in the presence of CSIR and CSITR. What is the loss due to lack of knowledge of the channel at the transmitter?
6. Consider the fast-fading system MIMO with  $n_T$  transmit antennas and  $n_R$  receive antennas. Show that the capacity for the low SNR case is approximately  $n_R \cdot \text{SNR} \cdot \log e$  bits/s/Hz. What can you conclude about multiple transmit antennas in this regime? (E.g., CDMA voice calls).
7. Consider a two-antenna V-BLAST transmission and  $n_R \geq 2$  receive antennas. What diversity order does this scheme achieve?
8. What is the outage probability for the Alamouti scheme on a  $2 \times 1$  MISO system, with code rate  $R$ ? Assume Raleigh fading with  $\mathbb{E}[|h_i|^2] = 1$  and give your expression in terms of SNR.
9. Exercise 5.15 of Tse and Viswanath.