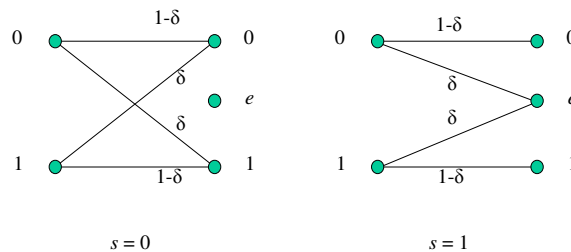


E2 203 Wireless Communication
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Problem Set 5

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1. Let $\mathbb{X} = \{0, 1\}$ be the input alphabet and $\mathbb{Y} = \{0, e, 1\}$ the output alphabet of a channel. The letter e stands for erasure. The channel is memoryless, i.e., $p(y^n | x^n, s^n) = \prod_{i=1}^n p(y_i | x_i, s_i)$, and has two states $\mathbb{S} = \{0, 1\}$ with parameters as given in the figure below.

- Show that the capacity of the channel when in state $s = 0$ is $1 - H(\delta)$ and when in state $s = 1$ is $1 - \delta$.
- The sequence of states is iid with $\Pr\{S_1 = 1\} = 1/2$. Evaluate C_{CSI} and C_{CSIR} .



2. Argue that the entropy of a discrete random variable $H(X)$ is a concave function of the probability mass function P_X . You can use derivatives, but keep in mind that the domain is a vector. (We used the analogous property for differential entropy in class).
3. Let $Y = HX + Z$, where $X \in \mathbb{C}$, and $Y \in \mathbb{C}^{n_R}$. The quantities X, Z are independent Gaussians. $X \sim \mathcal{CN}(P)$ and $Z \sim \mathcal{CN}(I_{n_R})$. Let $\tilde{Y} = c^*Y$ be the MMSE estimate of X given Y . Find c .
4. For the previous problem, argue that $I(X; Y) = I(X; \tilde{Y})$. This can be used to get an information lossless receiver architecture by MMSE and decision feedback equalisation.
5. Compare the capacity of the 2×1 MISO channel in the presence of CSIR and CSITR. What is the loss due to lack of knowledge of the channel at the transmitter?
6. Consider the fast-fading system MIMO with n_T transmit antennas and n_R receive antennas. Show that the capacity for the low SNR case is approximately $n_R \cdot \text{SNR} \cdot \log e$ bits/s/Hz. What can you conclude about multiple transmit antennas in this regime? (E.g., CDMA voice calls).
7. Consider a two-antenna V-BLAST transmission and $n_R \geq 2$ receive antennas. What diversity order does this scheme achieve?
8. What is the outage probability for the Alamouti scheme on a 2×1 MISO system, with code rate R ? Assume Rayleigh fading with $\mathbb{E}[|h_i|^2] = 1$ and give your expression in terms of SNR.
9. Exercise 5.15 of Tse and Viswanath.