E2–301 Topics in Multiuser Communication	October 09, 2007
	Due: October 16, 2007 4:00 PM

Homework 2

Instructor: Rajesh Sundaresan — *Scribe: Premkumar K.*

1. (**Interference Channel Capacity**) Consider a DM–interference channel. Show that the capacity region

$$\mathscr{C}_I = \bigcup_n \bigcup_{p_{X_1^n X_2^n} = p_{X_1^n} p_{X_2^n}} \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : R_1 \leqslant \frac{1}{n} I(X_1^n; Y_1^n), R_2 \leqslant \frac{1}{n} I(X_2^n; Y_2^n) \right\}.$$

2. (**Boundary of $\mathscr{C}_{\mathbf{BC}}$**) Consider a scalar Gaussian broadcast channel. Let $R_2(R_1)$ denote the curve representing the boundary of $\mathscr{C}_{\mathrm{BC}}$. Show that

$$-1 \leqslant \frac{dR_2}{dR_1} \leqslant 0.$$

3. (**Converse for a degraded BC**) Problem 15.11, Cover and Thomas (2nd ed.).

4. (**Capacity points**) Problem 15.12, Cover and Thomas (2nd ed.).

5. 1) Using Morton's achievability result and Körner-Morton converse, verify that $\mathscr{C}_{\mathrm{BC}}$ for a channel with a less noisy component is

$$\mathscr{C}_{\mathrm{BC}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : \begin{array}{lll} R_2 & \leqslant & I(U; Y_2) \\ R_1 & \leqslant & I(X; Y_1|U) \text{ for some } U \to X \to (Y_1 Y_2), |\mathbb{U}| < \infty \end{array} \right\}$$

   2) Using Morton's achievability result and the El-Gamal converse, verify that $\mathscr{C}_{\mathrm{BC}}$ for a channel with a more capable component is

$$\mathscr{C}_{\mathrm{BC}} = \left\{ (R_1, R_2) \in \mathbb{R}_+^2 : \begin{array}{rll} R_2 & \leqslant & I(U; Y_2) \\ R_1 + R_2 & \leqslant & I(X; Y_1) \\ R_1 + R_2 & \leqslant & I(X; Y_1|U) + I(U; Y_2) \text{ for some } U \to X \to (Y_1 Y_2), |\mathbb{U}| < \infty \end{array} \right\}$$

   3) Is Körner-Morton converse tight for the channel with a more capable component?

6. (**Extra: Slepian – Wolf**) Problem 15.17, Cover and Thomas (2nd ed.).