Lecture 18 : Slepian-Wolf Theorem – Converse, Generalisation

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1 SW Theorem – Converse

Converse: By providing X_2^n to encoder 1 (and decoder), we have $R_1 \geqslant H(X_1|X_2)$. Similarly, $R_2 \geqslant H(X_2|X_1)$. Joint encoding converse indicates $R_1 + R_2 \geqslant H(X_1X_2)$. □

2 Generalisation to Many Sources

Theorem 1. *(**Cover**) : Suppose $X_{[J]}^n$ is iid with generic distribution $p_{X_{[J]}}(x_{[J]})$. The achievable rate region is*

$$\left\{ R_{[J]} \in \mathbb{R}_+^J : R(S) \geqslant H(X_S|X_{S^c}), \forall S \subseteq [J] \right\}$$

- Can extend to jointly ergodic sources. Only AEP is needed with notion of typical set modified to entropy typicality.

Theorem 2. *(**Han 1979**): With $\sigma(S) := H(X_S|X_{S^c}), \forall S \subseteq [J]$, σ is a supermodular rank function, i.e.,*

(1) $\sigma(\phi) = 0$

(2) $\sigma(S) \leqslant \sigma(T)$ *if* $S \subseteq T \subseteq [J]$

(3) $\sigma(S \cup T) + \sigma(S \cap T) \geqslant \sigma(S) + \sigma(T)$

Proof. Exercise. □

Remark 1.

- *Let* $\mathscr{C}(\sigma) := \left\{ R \in \mathbb{R}_+^J : R(S) \geqslant \sigma(S), \forall S \subseteq [J] \right\}$*, a polyhedron.*
- $([J], \sigma)$ *: contrapolymatroid.* $\mathscr{C}(\sigma)$ *associated polyhedron.*
- $R \in \mathscr{C}(\sigma)$ *is a minimal extreme point iff* $R_{\pi_1} = \sigma(\{\pi_1\})$, $R_{\pi_k} = \sigma(\{\pi_1, \cdots, \pi_k\}) - \sigma(\{\pi_1, \cdots, \pi_{k-1}\})$.
- *Any* R *dominates some convex combination of these* $J!$ *extreme points.*