Robust Decoding for Non-Gaussian CDMA and Non-Exponential Timing Channels

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Abstract — **The capacity region of the two-user Gaussian CDMA channel is shown to be achievable when the (stationary, ergodic) noise is not Gaussian, using a decoder matched to the Gaussian distribution. Similarly, the capacity of the exponential server timing channel is shown to be achievable when the service distribution is not exponential, using a decoder matched to the exponential distribution. The codebook in either case will depend on the true channel.**

I. Introduction

Consider the problem of reliable transmission of information over a channel, where the receiver performs decoding assuming a different channel (receiver mismatch). Such an implementation, although suboptimal, might be desired because the mismatched receiver might have some structure which makes it easier to implement. We consider two such mismatched receivers (for CDMA and Timing Channels) and show that the receivers are robust to channel statistics, in a sense made precise below.

Similar results were shown for the single-user Non-Gaussian Channel and for the two-user Non-Gaussian Channel where the two users share the same signature waveform [2]. Here we extend them to the two-user symbol-asynchronous CDMA channel with possibly different signature waveforms [3] and to Timing Channels [1].

II. CDMA Channel

The discrete-time model for the two-user CDMA channel can be written as $\mathbf{y}(k) = \mathbf{H}\mathbf{x}(k) + \mathbf{z}(k)$, $1 \le k \le n$, for both the symbol-synchronous and the symbol-asynchronous cases [3]. In the former, $\mathbf{y}, \mathbf{x}$, and $\mathbf{z}$ are vectors of size 2, and in the latter of size $2(m+2)$, with a zero appended before and after a "letter" of length m. The length of the codewords are n and $n(m+2)$, respectively. $\mathbf{H}$ is the appropriate cross-correlation matrix of the signature waveforms. The only assumption we make on the noise is that it is stationary and ergodic, with zero mean and covariance $E[\mathbf{z}(k)\mathbf{z}^T(k)] = \sigma^2\mathbf{H}$, for $1 \le k \le n$.

The receiver assumes that the noise $\{\mathbf{z}(k) : 1 \le k \le n\}$, is independent and Gaussian with the said covariance matrix. Consequently, the receiver outputs that codeword which minimizes $\sum(\mathbf{y}(k) - \mathbf{H}\mathbf{x}(k))^T\ \mathbf{H}^{-1}\ (\mathbf{y}(k) - \mathbf{H}\mathbf{x}(k))$. Let $\mathcal{R}_G$ be the capacity region when the noise is Gaussian (see [3] for an expression). We follow the usual definition of achievability, except that the receiver is now fixed as mentioned above.

Theorem 1 *For every stationary and ergodic additive noise with zero mean and covariance* $\mathbf{H}$, $\mathcal{R}_G$ *is an achievable rate region using the decoder that is optimal for Gaussian noise.*

This result holds for both the symbol-synchronous and the symbol-asynchronous cases. We remark that the codebook will depend on the true noise distribution. Further, the probability of error might go to zero at different rates for different channels. In this sense the robustness is not uniform. Theorem 1 can also be shown using the technique of [2] which has the added advantage of showing a "random-coding" converse. However, we prove Theorem 1 and the following result for a timing channel in a unifying manner, where the mutual information saddle-point inequality plays a crucial role.

III. Timing Channel

Consider the problem of transmitting information through arrival epochs of packets to a single-server queueing system [1]. The queue is work conserving, follows a first-in-first-out service discipline and the service times are stationary and ergodic with mean $1/\mu$ seconds.

An (n, M, T, ϵ)-code consists of a codebook of M codewords, each of which is a vector of n nonnegative interarrival times $(x_1, \cdots, x_n)$ such that the kth arrival occurs at time $(x_1 + \cdots + x_k)$; a decoder, upon observation of all n depatures, selects the correct codeword with probability greater than $1-\epsilon$; and the nth departure occurs on the average (over equiprobable codewords and queue distributions) no later than T. The rate of the code is $(\log M)/T$. R is an achievable rate with a specified decoder if using this specified decoder, for every $\gamma > 0$, there exists a sequence of (n, M, T, ϵ_n)-codes that satisfy $(\log M)/T > R - \gamma$ for all sufficiently large n and $\epsilon_n \to 0$.

The decoder assumes that service times are independent and exponentially distributed with mean $1/\mu$ seconds. Consequently, the receiver outputs that codeword (among the compatible ones) which corresponds to the smallest sum of the service times. It is known that when the service times are indeed independent and exponentially distributed, the capacity of the channel is $e^{-1}\mu$ nats/second [1]. Further, $e^{-1}\mu$ nats/second is an achievable rate when the service times are i.i.d with mean $1/\mu$ seconds, but not exponentially distributed [1]. However, in the second situation, the receiver needs to know the service distribution. The following is a sharpening of that result.

Theorem 2 *For any server with stationary and ergodic service times with mean* $1/\mu$ *seconds,* $e^{-1}\mu$ *nats/second is an achievable rate using the decoder that is optimal for the exponential server.*

References

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