On Sequence Design for a Synchronous CDMA Channel

Rajesh Sundaresan[1]
Qualcomm Inc.
675 Campbell Technology Parkway,
Campbell, CA 95131, USA
e-mail: rajeshs@qualcomm.com

Abstract — **The sum capacity on a symbol-synchronous CDMA system having processing gain N and supporting K power constrained users can be achieved by employing at most $2N-1$ sequences. Analogously, the minimum received power (energy-per-chip) on the symbol-synchronous CDMA system supporting K users that demand specified data rates can be attained by employing at most $2N-1$ sequences. If there are L *oversized* users in the system, we need at most $2N-L-1$ sequences. We show the above results by proving a converse to a well-known result of Weyl on the interlacing eigenvalues of the sum of two Hermitian matrices, one of which is of rank 1. The converse is analogous to a known converse to the interlacing eigenvalues theorem for bordering matrices.**

Consider a symbol-synchronous code-division multiple access (CDMA) system. The kth user is assigned an N-sequence $s_k \in \mathcal{R}^N$ of unit energy, i.e., $s_k^t s_k = 1$. The processing gain is N chips, and the number of users is K, with $K > N$. User k modulates the vector s_k by its data symbol $X_k \in \mathcal{R}$ and transmits $X_k s_k$ over N chips. This transmission interferes with other users' transmissions and is corrupted by noise. The received signal is modeled by $Y = \sum_{k=1}^{K} s_k X_k + Z$, where Z is a zero-mean Gaussian random vector with covariance I_N, the $N \times N$ identity matrix.

We consider two problems already studied in the literature ([7], [3], and references therein). In **Problem I**, user k has a power contraint $E[X_k^2] \leq p_k$. The goal then is to assign sequences and data rates to users so that the sum of the individual rates at which the users can transmit data reliably (in an asymptotic sense) is maximized. The maximum value C_{sum} is called the sum capacity. Reference [7] studies this problem and shows the following. Let $p_{tot} = \sum_{k=1}^{K} p_k$.

- *Oversized* users, i.e., those capable of transmitting at large powers relative to other users' power contraints, are best allocated non-interfering sequences;
- others are allocated generalized Welch-bound equality (WBE) sequences;
- $C_{sum} \leq (1/2)\log(1+p_{tot})$, with equality if, and only if, no user is oversized;
- no user is oversized if $Np_k \leq p_{tot}$ for every user k.

Problem II, a dual to Problem I, is one where user k demands reliable transmission at a minimum rate r_k bits/chip. The goal is to assign sequences and powers to users so that despite their mutual interference and noise, each of the users can transmit reliably at or greater than their required rates, and the sum of the received powers (energy/chip) at the base-station is minimized. Reference [3] shows results analogous to Problem I. Let $r_{tot} = \sum_{k=1}^{K} r_k$.

- *Oversized* users, i.e., those that demand large rates relative to other users' requirements, are best allocated non-interfering sequences;
- others are allocated WBE sequences;
- the minimum received sum power is $\exp(2r_{tot}) - 1$, if, and only if, no user is oversized;
- no user is oversized if $Nr_k \leq r_{tot}$ for every user k.

Problems of similar flavor are considered in [5], [6] and [2]. Reference [6] focuses on linear MMSE receivers and identifies sequences and powers to meet a per-user signal to interference ratio (SIR) requirement. Reference [2] extends the results to decision-feedback receivers.

The first contribution of this paper is the following. For both Problem I and Problem II, we come up with a generalized WBE sequence allocation for the non-oversized users that employs at most $2N-L-1$ sequences, where L is the number of oversized users. To do this, we only need to restrict our attention to the case when no user is oversized and show that we need at most $2N-1$ sequences. The $2N-L-1$ requirement in the presence of oversized users follows immediately.

The solution to the above problem draws from an interesting result in matrix theory. For a matrix A, let A^* denote its Hermitian conjugate. If A is Hermitian, let $\sigma(A)$ denote its eigenvalues in decreasing order. The following is a well-known result due to Weyl (see for e.g., [4, Section 4.3]). If A is an $N \times N$ Hermitian matrix with $\sigma(A) = (\lambda_1, \cdots, \lambda_N) = \lambda$, and $c \in \mathcal{C}^N$, then $\sigma(A + cc^*) = (\hat{\lambda}_1, \cdots, \hat{\lambda}_N) = \hat{\lambda}$ satisfies the interlacing property

$$\hat{\lambda}_1 \geq \lambda_1 \geq \hat{\lambda}_2 \geq \lambda_2 \geq \cdots \geq \hat{\lambda}_N \geq \lambda_N. \qquad (1)$$

The second contribution of this paper is a converse to Weyl's result. We show that for any Hermitian A with $\sigma(A) = \lambda$, and any real $\hat{\lambda}$ such that λ and $\hat{\lambda}$ satisfy (1), there exists c such that $\sigma(A + cc^*) = \hat{\lambda}$. This converse facilitates a sequence allocation algorithm that, for both Problems I and II, satisfies the declared properties.

To get the fuller version of this paper [1], please contact the author.

References

[1] R.Sundaresan, "On sequence design for a synchronous CDMA channel", draft, Apr. 2004.

[2] T.Guess, "Optimal sequences for CDMA with decision-feedback receivers," *IEEE Trans. Inform. Theory*, vol.IT-49, no.4, pp.886-900, Apr. 2003.

[3] T.Guess, "CDMA with power control and sequence design: the capacity region with and without multidimensional signaling," preprint, Jun. 2003.

[4] R.A.Horn and C.R.Johnson, *Matrix Analysis*, Cambridge University Press, 1999.

[5] M.Rupf and J.L.Massey, "Optimum sequence multisets for synchronous code-division multi-access channels" *IEEE Trans. Inform. Theory*, vol.IT-40, no.4, pp.1261-1266, Jul. 1994

[6] P.Vishwanath, V.Anantharam, and D.Tse, "Optimal sequences, power control, and user capacity of synchronous CDMA systems with linear MMSE multiuser receivers," *IEEE Trans. Inform. Theory*, vol.IT-45, no.6, pp.1968-1983, Sept. 1999.

[7] P.Vishwanath and V.Anantharam, "Optimal sequences and sum capacity of synchronous CDMA systems," *IEEE Trans. Inform. Theory*, vol.IT-45, no.6, pp.1984-1991, Sept. 1999.

[1] This work was supported in part by Qualcomm Inc.