Interference planning for multicell OFDM downlink

(Invited Paper)

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***Abstract*—In this paper we consider the downlink of an OFDM cellular system. The objective is to maximise the system utility by means of fractional frequency reuse and interference planning. The problem is a joint scheduling and power allocation problem. Using gradient scheduling scheme, the above problem is transformed to a problem of maximising weighted sum-rate at each time slot. At each slot, an iterative scheduling and power allocation algorithm is employed to address the weighted sum-rate maximisation problem. The power allocation problem in the above algorithm is a nonconvex optimisation problem. We study several algorithms that can tackle this part of the problem. We propose two modifications to the above algorithms to address practical and computational feasibility. Finally, we compare the performance of our algorithm with some existing algorithms based on certain achieved system utility metrics. We show that the practical considerations do not affect the system performance adversely.**

I. Introduction

Consider two neighbouring cells that use the same carrier and bandwidth on a downlink OFDM system. Let S_1 be a set of (let us say) half the subcarriers, and S_2 the remaining subcarriers. Frequency reuse 1/2 allocation (e.g., GSM) restricts cell 1 transmissions on the downlink to set S_1 and cell 2 transmissions to S_2. A generalisation of reuse $1/n$, one that is available in all fourth generation OFDM-based cellular systems, is termed fractional frequency reuse. To get spectral efficiencies larger than that available from reuse 1/2, mobiles close to the cell may be allocated all subcarriers, while mobiles close to the edge of the cell may be allocated subcarriers from only part of the spectrum.

Flex-band is a generalisation of frequency reuse or fractional frequency reuse [1]. In a two-band flex-band allocation, cells with even ID (say) will allocate larger powers on one half of the subcarriers and smaller powers on the other half. Cells with odd ID will do the opposite. This too is like reuse 1/2, except that nonzero powers may be allocated in the other set of subcarriers. It may enable us to operate close to best-known achievable rates on a multicell system. More importantly, these rates can be attained with simple receivers that do not need sophisticated interference cancellation. However, flex-band is a static system, with static power allocation and subcarrier assignment.

This paper will focus on a generalisation of flex-band and study approximations that get us close to the best-known achievable rates on the system. Coordination between cells is restricted to identifying transmission power levels on different subcarriers or groups of subcarriers in neighboring cells as well as scheduling of users. These power levels and scheduling constraints will be updated periodically in a dynamic fashion based on received or inferred channel quality indications at the network. We may call such a mechanism MAC-level coordination. The dynamism enables adaptation to changing mobile locations and interference scenarios; it is likely to outperform the static flex-band of [1].

We now discuss a possible application of our study. We may view the above system as one that does away with extensive frequency planning and is therefore self-organising in nature. Such a system is likely to be of use in buy and deploy femto-cell systems. Femto-cell systems are made of home-based base stations that one can buy in the market and connect to the service provider via internet or other means. They act as add-on base stations for operation within the home. For such systems it is important to ensure that similar base stations used by neighbours coordinate to plan interference and enhance coverage and data rates. The network level coordination may happen at some central controller that has (or can get) knowledge of neighbouring and interfering systems. Our approach is equally applicable to interfering wireless local area networks.

Code level (or physical layer) coordination, as in dirty-paper coding envisaged in a fully centralised MIMO-like broadcast cellular network where each cell is viewed as a different set of antennas, is out of scope of this work because such a coordination requires a drastic over-haul of the inter-cell interfaces on the infrastructure side.

This paper is organised as follows. In section II we introduce the problem and describe the objective of our work. We present prior works on related problems in section II-C. In section III, we describe the multiuser power control problem. We describe the gradient descent method and the Convex-Concave Procedure (CCP) for solving the multiuser power control problem. We then discuss techniques on solving a part of CCP. In section IV we describe the iterative scheduling and power allocation procedure (ISPA) that addresses the joint scheduling and power allocation problem. We describe two modifications to ISPA to address practical issues. In section V we study the performance of the algorithms described in section IV using simulation data.

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II. Problem Formulation

We now discuss the mathematical abstraction of our problem. Consider the downlink of a cellular system. We consider an OFDM system with single user detection (SUD) at all the receivers. Annapureddy & Veeravalli [2] showed that at low interference regimes, considering interference as noise achieves sum capacity. Further, simulation results from our work [3] show that at low interference regimes, SUD with interference planning achieves rates close to the best achievable rates. Even at high interference, the loss is marginal and hence we consider the use of SUD with interference planning over other sophisticated techniques.

J is the number of base stations in the system, M_j is the number of mobile stations associated with BS j. Let $K = \sum_{j=1}^{J} M_j$ be the total number of users in the system. N is the number of subcarriers. $P_j^n(t)$ is the power allocated by base station j on subcarrier n at time slot t. There is a total power constriant on each base station:

$$\sum_{n=1}^{N} P_j^n(t) \leq P_j \quad \forall t. \tag{1}$$

At time slot t, $H_{j,k}^n(t)$ is the channel gain seen by user k from base station j in subcarrier n. The signal to interference and noise ratio (SINR) seen by mobile station k associated with base station j on subcarrier n at time t is

$$SINR_k^n(t) = \frac{H_{j,k}^n(t) P_j^n(t)}{\sigma^2 + \sum_{j' \neq j} H_{j',k}^n(t) P_{j'}^n(t)}. \tag{2}$$

The rate user k can obtain in subcarrier n at time t is

$$R_k^n(t) = \log\left(1 + SINR_k^n(t)\right). \tag{3}$$

The actual rate obtained by user k in subcarrier n at time t is

$$c_k^n(t) = \begin{cases} R_k^n(t) & \text{if } k \text{ was scheduled at time } t \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

For ease of implementation in existing systems, we assume that each subcarrier can carry at most one user from each base station. The total rate obtained by user k at time t is

$$c_k(t) = \sum_{n=1}^{N} c_k^n(t). \tag{5}$$

The average rate obtained by user k in T time slots is

$$\bar{R}_k(T) = \frac{1}{T} \sum_{t=1}^{T} c_k(t). \tag{6}$$

Utility Functions: We consider system performance to be measured in terms of achieved utility with respect to some utility function. In this paper we consider the class of utility functions defined by:

$$U(\bar{R}_1, \bar{R}_2, \ldots, \bar{R}_K) := \begin{cases} \frac{\sum_k (\bar{R}_k^{1-\alpha} - 1)}{1-\alpha} & \alpha \in (0, \infty),\ \alpha \neq 1 \\ \sum_k \log(\bar{R}_k) & \alpha = 1 \end{cases} \tag{7}$$

where $\bar{R}_k$ is the average rate obtained by user k. When $\alpha = 0$, U is sum rate minus a constant, and so the arithmetic mean of average rates is maximised. When $\alpha = 1$, U is $\sum_k \log(\bar{R}_k)$, and so the geometric mean of average rates is maximised. The case $\alpha = 2$ leads to maximisation of the harmonic mean of the average rates, and the case $\alpha = \infty$ results in maximisation of the minimum average rate (the so-called max-min fairness objective). By varying α, we can adjust the utility function to balance throughput and fairness. The theory however is applicable to any strictly increasing concave utility function.

A. Objective

Our objective is to *maximise system utility subject to power and scheduling constraints at each base station*. Observe that the problem spans across time, subcarriers and users. Given that channel estimates are not available beforehand, an exact solution becomes intractable or impossible. As a first simplification we break the problem into smaller optimisation problems at each time slot. Kushner & Whiting [4] and Stolyar [5] showed that gradient scheduling algorithm at each time slot asymptotically converges to the optimal allocation. Further, gradient scheduling boils down to a weighted sum-rate maximisation problem with power constraint and scheduling constraint at each base station of the form:

Problem 2.1: [1]

$$\begin{aligned} Minimise\ -R &= -\sum_{k=1}^{K} w_k(t)\ .\ \left(\sum_{n=1}^{N} x_k^n\ R_k^n(t)\right) \quad (8) \\ \text{where} \quad w_k(t) &= U^{'}(\bar{R}_k(t-1)) \end{aligned}$$

subject to

$$\begin{aligned} \sum_{n=1}^{N} P_j^n(t) &\leq P_j \quad \forall j && (9) \\ x_k^n &\in \{0,1\} && (10) \\ \sum_{k \in S_j} x_k^n &\leq 1 \quad \forall j, n && (11) \\ P_j^n &\geq 0 \quad \forall j, n && (12) \end{aligned}$$

where $S_j = \{k : k \text{ associated with BS } j\}$.

We make the following observations.

- The objective function (8) is not a convex function in the variables x_k^n's and P_j^n's.
- The constraint set specified by (11) is a nonconvex set, because of the discrete nature of x_k^n's.
- Scheduling and power allocation are interlinked. Tackling this joint problem together is a difficult computational problem. We simplify it via an iterative procedure described below.

[1]Throughout this paper all optimisation problems will be represented as minimisation problems.

B. Iterative scheduling and power allocation

The procedure involves a multiround algorithm which goes back and forth between power allocation and scheduling until convergence of the weighted sum rate is attained.

Algorithm : Iterative Scheduling and Power Allocation (ISPA)
At each time t,

- initialise P = a random feasible power allocation.
- repeat
  - Fixing power allocation at P, find an appropriate schedule.
  - Using the above schedule, update power allocation P.
- until convergence of weighted sum rate.

The algorithm generates a nondecreasing sequence of weighted sum rates. Since the maximum weighted sum rate is bounded from above, value of the objective function converges. Given a power allocation, computing the optimum schedule is easy. Given a schedule, the optimum power allocation is a solution to a weighted sum-rate maximisation problem. We address this problem in section III.

C. Prior Work

Kushner & Whiting [4] and Stolyar [5] showed that maximising

$$\sum_k U^{'}(\bar{R}_k(t-1)) \cdot R_k(t)$$

at each time slot t achieves the optimal rate allocation that maximises the system utility $\sum_k U(\bar{R}_k)$ asymptotically. In our problem this translates to maximising a weighted sum rate at each time slot for optimal performance, where $U^{'}(\bar{R}_k(t-1))$ is taken to be the weights. Hence at each time slot the optimal power allocation and scheduling boils down to maximising a weighted sum rate problem as considered by Yu & Lui in [6].

Yu et al. [7] modelled the multiuser power control problem as a noncooperative game between the different base stations. They obtained conditions for the existence and uniqueness of a Nash equilibrium for a two-user case. They further showed that if the conditions for existence and uniqueness hold, then the iterative waterfilling algorithm, where each base station does waterfilling allocation considering signal from the other base stations as noise, converges to the unique Nash equilibrium.

Cendrillion et al. [8] studied the power control problem using duality methods. They proposed OSB (Optimal Spectrum Balancing) algorithm with linear complexity in number of subcarriers but still exponential complexity in the number of base stations. They showed that their algorithm performed better than the iterative waterfilling scheme of [7].

Yu & Lui [6] proposed an extension to OSB algorithm called ISB (Iterative Spectrum Balancing). This has linear complexity in the number of base stations, but at the cost of optimality. They further showed that as the number of subcarriers goes to infinity the duality gap between the primal solution and the dual solution goes to zero.

Tsiaflakis et al. [9] proposed the convex concave procedure (CCP) for solving the weighted rate maximisation problem. CCP involves solving a sequence of convex optimisation problems, which yields a lower bound on the original problem. The procedure yields a nondecreasing sequence of function values.

Stolyar & Viswanathan [10] proposed a distributed procedure. At each base station power allocation and scheduling is done in a two-step procedure. Assuming a fixed schedule, power in each band is increased or decreased by a small quantity depending on the change it will bring to the entire system utility. Similarly for scheduling, for each band a user who maximises the weighted rate is selected.

Son et al. [11] considered the same problem of maximising system utility in a multicell OFDM downlink scenario. As in this paper and in the dissertation of the first author [3], they used gradient scheduling algorithm to transform the problem into a weighted sum-rate maximisation problem at each time slot. Further for the weighted sum-rate maximisation problem Son et al. [11] proposed iterative scheduling and power allocation algorithm. But differing from this work and the dissertation [3], for the power allocation problem, they proposed a distributed scheme where each BS tries to maximise the weighted sum rate for its users and that of its worst victims in its neighbouring cells.

D. Our Contribution

Building on the two-cell results of the first author's dissertation [3], we do the following in this paper.

- We compare the performance of CCP based algorithms with that of plain gradient descent algorithm. We show that the gradient descent algorithm yields similar system performance at lower computational costs, even in a 19-cell cluster.
- We propose a two time-scale scheduling and power allocation scheme, where the radio network controller (RNC) computes a schedule and power allocation for the base stations at a slower pace, and the base stations perform fast local scheduling based on the RNC power allocation.
- We propose the top-two interferers scheme, where instead of considering interference from all the base stations in the network, we consider only the top two interferers per user. We show that this consideration does not affect the system performance adversely. We chose only the top "two" interferers because in a sectorised system on a hexagonal lattice a cell edge user sees interference from two immediate neighbouring cells.
- We propose a further simplification where, during an RNC scheduling/power allocation interval, each base station schedules only a subset of users instead of considering all the associated users. Empirically, we show that even a subset size of five achieves system performance close to the case when all the users are eligible for scheduling.

III. Multiuser Power Control Problem

We now consider the problem of finding the optimum power allocation, given a schedule.

Problem 3.1:

$$\begin{aligned} &Minimise \quad -R = -\sum_{j=1}^{J} R^j \\ &\text{subject to} \quad \sum_{n=1}^{N} P_j^n \leq P_j \quad \forall j \\ &\qquad\qquad P_j^n \geq 0 \quad \forall j, n \end{aligned} \tag{13}$$

where R^j can be expanded as

$$R^j = \sum_{n=1}^{N} w_j^n \log\left(1 + \frac{H_{j,j}^n P_j^n}{\sigma^2 + \sum_{j' \neq j} H_{j',j}^n P_{j'}^n}\right). \tag{14}$$

Note that the objective function of Problem 3.1 is a difference of two convex functions and hence the function may not be convex. Observe that the objective function can be written as

$$-R = g(p) - h(p) \tag{15}$$

where

$$g(p) = -\sum_{n=1}^{N}\sum_{j=1}^{J} w_j^n \log(\sigma^2 + \sum_{j'} H_{j',j}^n P_{j'}^n) \tag{16}$$

$$h(p) = -\sum_{n=1}^{N}\sum_{j=1}^{J} w_j^n \log(\sigma^2 + \sum_{j' \neq j} H_{j',j}^n P_{j'}^n). \tag{17}$$

In general it is difficult to verify the following necessary and sufficient condition on the Hessian for convexity:

$$\nabla^2 g(p) - \nabla^2 h(p) \succeq 0 \tag{18}$$

for every feasible p. Hence we explore numerical methods to solve Problem 3.1. Since the problem is nonconvex the convergence of numerical methods to a global minimum is not assured. We implemented the following algorithms to solve Problem 3.1.

A. Gradient Descent Algorithm

Gradient descent algorithm attacks the problem variables directly. From any point inside the feasible set, the algorithm moves in the direction opposite to the gradient at that point. If the point goes outside the feasible set, it is projected back onto the feasible set and the algorithm continues until a termination criterion is reached. This method was found to be the fastest among the other methods discussed in this paper. Since the algorithm yields a sequence of decreasing function values, the algorithm has to converge to possibly a local minimum.

Algorithm : Gradient Descent

- initialise x^0 to a random feasible vector
- repeat
  - d^k = gradient(x^k)
  - repeat
    - $x^{k+1} = x^k - \beta d^k$ where $\beta > 0$ is a parameter known as step size.
    - if x^{k+1} is outside the feasible set, then x^{k+1} = projection(x^{k+1})
    - if $f(x^{k+1}) > f(x^k)$ then $d^k = \frac{d^k}{2}$
  - until $f(x^{k+1}) \leq f(x^k)$
- until convergence of $f(x^k)$ or maximum number of iterations.

B. CCP on Multiuser power control problem

We now describe the convex-concave procedure (CCP) and how it can be used in the multiuser power control problem.

1) General CCP: Sriperumbudur et al. [12] described the convex-concave procedure (CCP) procedure for addressing minimisation of a difference of two convex functions. CCP yields an upper bound on the minimum value of a difference of two convex functions. The procedure involves solving a sequence of convex optimisation problems. The following properties of CCP are taken from [12]. Consider a nonconvex optimisation problem of the form

Problem 3.2:

$$\begin{aligned} &Minimise\; f(x) = g(x) - h(x) \\ &\text{subject to} \quad f_i(x) \leq 0 \qquad i = 1, 2, \ldots, m \end{aligned} \tag{19}$$

where $g(x)$, $h(x)$ and $f_i(x)$ are convex functions.

Since $h(x)$ is a convex function, it can be lower bounded by any of its supporting hyperplanes. The particular supporting hyperplane at an arbitrary y yields the following result.

Proposition 3.3: Let $h(x)$ be a convex function. Define

$$j(x, y) = g(x) - h(y) - \nabla h(y)^T (x - y). \tag{20}$$

Then

$$f(x) = j(x, x) \leq j(x, y) \quad \forall y. \tag{21}$$

It is immediate that if x^* is a solution to Problem 3.2, then

$$f(x^*) \leq f(x) \leq j(x, y) \quad \forall x, y. \tag{22}$$

Consider the new problem:

Problem 3.4:

$$\begin{aligned} &\text{Minimise}_x \quad j(x, y) = g(x) - h(y) - \nabla h(y)^T (x - y) \\ &\text{subject to} \quad f_i(x) \leq 0 \qquad i = 1, 2, \ldots, m. \end{aligned} \tag{23}$$

This is a convex optimisation problem and the solution can be obtained by using convex optimisation techniques. Let x_y^* be the solution to Problem 3.4 for a given y, then

$$f(x_y^*) \leq j(x_y^*, y) \quad \forall y \tag{24}$$

$$f(x^*) \leq \min_y f(x_y^*) \leq \min_y j(x_y^*, y) \tag{25}$$

This motivates us to use Problem 3.4 to obtain an upper bound on Problem 3.2. This is called the convex-concave procedure or CCP.

Algorithm: CCP

- initialise y^0 = a random feasible vector
- repeat
  - $y^k = \arg\min_x j(x, y^{k-1})$
- until convergence of $f(y^k) = j(y^k, y^k)$.

Lemma 3.5: The sequence of function values $\{f(y^k)\}_{k \geq 1}$ is a monotonically decreasing sequence and converges if f is bounded from below.

Proof: The algorithm is such that

$$j(y^{k+1}, y^k) = \min_x j(x, y^k).$$

We therefore have the sequence of inequalities:

$$j(y^{k+1}, y^{k+1}) \overset{a}{\leq} j(y^{k+1}, y^k) \overset{b}{\leq} j(y^k, y^k) \qquad (26)$$

where (a) follows from Proposition 3.3 and (b) follows from the definition of $j(y^{k+1}, y^k)$. ∎

Lemma 3.6: If $\{y^k\}_{k=0}^{\infty}$ is a sequence of vectors generated by the CCP procedure, then all limit points are stationary points of the original problem.

Proof: See [3, Appendix A]. ∎

2) CCP on multiuser power control problem: Tsiaflakis et al. [9] suggested the use of CCP for the multiuser power control problem. We have seen that the weighted sum rate function (15) can be written as the difference of two convex functions:

$$-R \quad = \quad g(p) - h(p),$$

thereby qualifying for the CCP procedure, with

$$j(p, y) = g(p) - h(y) - \nabla h(y)^T (p - y).$$

The CCP procedure for multiuser power control problem is:

Algorithm : CCP Procedure for multiuser power control

- initialise y^0 = random feasible power allocation.
- repeat

$$\begin{aligned} y^{k+1} &= \arg\min_{p \in \mathcal{P}} j(p, y^k) \\ &= \arg\min_{p \in \mathcal{P}} g(p) - h(y^k) - \nabla h(y^k)^T (p - y^k) \end{aligned}$$

  where $\mathcal{P}$ is the feasible set.
- until convergence of $j(y^{k+1}, y^{k+1})$.

3) Convex optimisation part of CCP: The convex optimisation part of CCP involves solving the problem:

$$z = \arg\min_{p \in \mathcal{P}} j(p, y) = g(p) - h(y) - \nabla h(y)^T (p - y). \quad (27)$$

We present three methods that can solve the above problem.

a) CCP with Gradient Descent: This is a straight forward method wherein we start from a random feasible point and move along the direction opposite to the gradient, but making sure that the step size is chosen such that at each step the function value decreases. Since the problem is a convex optimisation problem, this method converges to a global minimum.

Algorithm : CCP_GD

- initialise p^0 = a random feasible power vector
- repeat
  - d^k =gradient(p^k) = $\nabla g(p^k) - \nabla h(y)$
  - repeat
    - $p^{k+1} = p^k - \alpha^k d^k$.
    - if $j(p^{k+1}, y) > j(p^k, y)$ then $d^k = d^k/2$.
  - until $j(p^{k+1}, y) < j(p^k, y)$
- until convergence of $j(p^k, y)$

The algorithm has the following pros and cons.

- **Pros:** Since gradient can be explicitly computed, this method is easily implemented. It is the fastest among the three algorithms. It scales well with the number of base stations and subcarriers.
- **Cons:** We do not know the rate of convergence of this method.

b) Dual Method and Block Descent Method: Two other techniques for addressing the convex optimisation part of CCP, 1) Dual method and 2) Block descent, are explored in [3]. Dual method addresses the dual of the problem. Given a price vector, the Lagrangian minimiser can be obtained by solving two systems of linear equations. But finding the optimal price vector is a bottle-neck in the algorithm. Block descent tries to solve the problem in an iterative fashion wherein, at each step, we optimise the function with respect to a subset of variables, while keeping the values of other variables constant. In our case, a subset of users corespond to the power allocation at a particular base station. We iterate this across base stations. It is shown that given a price value, the Lagrangian minimiser can be obtained by solving polynomials of degree equal to the number of base stations. This results in an easily implementable algorithm for a two base station scenario, which consists of solving a quadratic equation. But for more number of base stations other methods have to be used.

Based on simulation study [3], we propose the use of gradient descent method over the other two variants of CCP for its speed and scalability in number of base stations.

IV. Scheduling and Interference Planning

In this section we address the problem of joint scheduling and power allocation. The aim is to find a suitable joint scheduling and power allocation scheme to maximise the system utility, in our case α-utility. Son et al. [11] also proposed an iterative scheduling and power allocation algorithm (ISPA) for the joint scheduling and power allocation problem. (We remark that our power allocation algorithm part is different from Son et al. [11] as indicated in Section II-C). For the scheduling problem we use gradient scheduling ideas analysed by Kushner & Whiting [4] and Stolyar [5]. For the power allocation problem we propose the use of gradient descent based algorithms. Taking practical considerations into account, we propose two modifications to ISPA algorithm.

A. Iterative joint scheduling and power allocation

The procedure involves a multiround algorithm which goes back and forth between power allocation and scheduling until convergence of the weighted sum rate.

- At time slot t, $w_k = \bar{R}_k(t-1))^{-\alpha}$.
- $R_k^n := R_k^n(t)$
- WR := weighted rate as defined in Problem 2.1

It can be seen that the algorithm generates a nondecreasing sequence of WRs after each iteration. Since the maximum weighted sum rate is bounded from above, the algorithm converges. In this work, we study two variants of ISPA: 1) ISPA_GD where the power allocation algorithm is based on plain gradient descent algorithm, and 2) ISPA_CCP_GD, where the power allocation algorithm is based on CCP procedure with gradient descent used in the convex optimisation part of CCP.

Algorithm : Iterative Scheduling and Power Allocation (ISPA)
At each time slot t,

- initialise P = a random feasible power allocation (or the previous slots value).
- repeat
  - Fixing power allocation at P, at each base station j and subcarrier n, schedule user $s_j^n = \arg\max_k \ w_k R_k^n$, i.e, select the user with the maximum weighted rate.
    Assign $w_j^n = w_{s_j^n}$ and $G_{j',j}^n = H_{j',s_j^n}^n \quad \forall j = \{1, 2, \ldots, J\}$. (Gradient scheduling)
  - Fix **w** and **G** to the ones obtained in the previous step. Update P by solving the multiuser power control problem 3.1, with arguments **w** for weights and **G** for channel gains.
- until convergence of weighted rate WR.

B. Stolyar & Viswanathan Algorithm

Stolyar & Viswanathan [10] proposed a distributed procedure, Multicell Gradient (MGR), where at each base station power allocation and scheduling is done in a two-step procedure. Assuming a fixed schedule, power in each band is increased or decreased by a small quantity depending on the change it will bring to the entire system utility. Similarly for scheduling, in each band a user who maximises the weighted rate is selected. See [10] for details.

C. Practical Considerations

The scenario under consideration till now involved scheduling and power allocation on a per time-slot basis. But such a consideration has the following practical issues:

- Every user has to estimate the channel gains from all of the other base stations in all the subcarriers, i.e., if there are M users per BS, J BSs and N subcarriers, a total of MJ^2N values have to be estimated.
- The estimated values have to be communicated to the corresponding base stations and from there to the Radio Network Controller (RNC).
- Before every time slot, the RNC has to communicate the power allocation and schedule for that time slot to all the base stations.

The above issues can result in huge amounts of control signaling in the network.

- RNC has to compute the power allocation and scheduling algorithms every time slot. This may require significant computing capabilities.

We propose a modified scheme which is low on control signaling and computational requirements, but at the cost of performance.

1) Top Two Interferers: Here the idea is to restrict our attention to only the top two interferers for each user. This reduces the signaling overhead required to communicate the channel gains from the base stations to RNC. Such a consideration is justified by the fact that a cell-edge user will have atmost two first level interfering base stations.

- **Algorithm : Top-Two Interferers**
  - At each RNC scheduling instant, determine the top two interferers for each mobile. (Averaged over channel gains in all the subcarriers)
  - Run previous algorithms with the channel gains from other BSs set as zero.

2) Slow scheduling and power allocation at RNC and fast local scheduling at base stations: As before, the RNC does a power allocation and scheduling, but at a slower pace than before (say 10 Hz) and the base stations do local scheduling at a faster rate (say 500Hz) without adjusting the power allocation. Scheduling at the RNC provides a subset of users, among whom local scheduling is done at the base stations. Such an approach reduces the computational requirements and also reduces the frequency of transmission of control signaling between the base stations and RNC.

- **Algorithm : Subset Schedules**
  - Step 1 : At each RNC scheduling instant, allow ISPA_GD / ISPA_CCP_GD to converge. After convergence, for each subcarrier, determine top N_s users that maximise the weighted rate. These N_s users form the subset on which the local schedulers work.
  - Step 2 : Communicate power allocation P and subsets of users to the base stations.
  - Step 3 : At each BS scheduling instant, determine the user from among the N_s users who has the highest weighted rate. Schedule this user for that time slot.
  - Step 4 : Update the cumulative rate obtained by the scheduled user.
  - If it is the next RNC scheduling instant, then jump to Step 1, else repeat Steps 2 - Step 4.

V. Simulation Study

A. 19-BS Hexagonal Cluster

We consider a 19-BS hexagonal cluster. M mobiles are uniformly deployed in each hexagon. For simplicity, mobiles are associated with the nearest BS, irrespective of their channel gains. Path losses and channel fading factors (K-factors) are dependent on the respective BS-user distances. Over and above path loss, the channel fades slowly as per the SUI model [13]. A few of the parameters used for the simulation were:

Cell size (side of a hexagon)	1 km
Carrier frequency	2.4 GHz
Bandwidth allocated	5 MHz
Number of subcarriers	128
Variance of shadow fading	8dB
SUI-model	1
RNC scheduling rate	10 Hz
Channel observation rate	100 Hz
Base station scheduling rate	500 Hz
Simulation length	5000 time slots
Number of deployments	10

1) Separated Power Allocation And Scheduling With Top Two Interferers: We now describe the performance curves in Figures 1 - 5. In these figures, three different algorithms – ISPA_GD, ISPA_CCP_GD, and Full Reuse (FR) – are compared with each other and with variants as described below. (Full reuse (FR) corresponds to all the base stations using all the subcarriers and with equal power allocation in each subcarrier). For these algorithms power control and scheduling are adapted at different rates. Power control is done at the RNC at a much slower rate since this requires signaling of channel states to the RNC. This is done at a rate of 10 Hz (once every 100 ms), as indicated in the simulation parameters above. Scheduling is done at a much faster rate of 500 Hz (once every 2 ms). The channel itself is observed at rate 100 Hz (once every 10 ms).

We also show performances of approximations of the power control algorithm where only the *top two interferers* are assumed to cause interference. Other channel gains are not signaled and assumed 0 during the optimisation. This is done to alleviate both computation and signaling requirements for the algorithms. The corresponding curves are denoted ISPA_GD_TopTwo, ISPA_CCP_GD_TopTwo, and Full Reuse_TopTwo. Here too power allocation is done at rate 10 Hz, scheduling at 500 Hz, and channel is sampled at 100 Hz.

Finally, these are compared with the algorithm of Stolyar-Viswanathan (see section IV-B). This last algorithm is given the advantage of power and scheduling adaptation at the much faster rate of 500 Hz (once every 2 ms), since their algorithm is a joint power allocation and scheduling algorithm.

We make the following remarks:

- A quick scan of the Figures 1-5 indicates that our proposed algorithms outperform the Stolyar-Viswanathan algorithm and the Full Reuse algorithm in terms of sum utilities. (We suggest that the plots be viewed in colour.)
- Despite the aforementioned advantage for the Stolyar-Viswanathan algorithm, the only α for which their algorithm gives a better sum rate is the case when $\alpha = 1$. This is at the expense of fairness, as seen from the CDF plots. Our algorithm gives better rates to the low rate (edge-cell) users. We expect that the sum rate of the Stolyar-Viswanathan algorithm will suffer a worse degradation compared to ours when their algorithm is modified to adapt powers at a slower rate.
- ISPA_GD has nearly the same utility as ISPA_CCP_GD, but fared better in terms of sum rate. This makes ISPA_GD a better choice.
- The top-two interferers' approximation suffers from a marginal loss in sum rate, but otherwise has all the qualitative features of the more accurate case.
- The CDF plots suggest that our proposed algorithms provide a fairer solution to the low rate (edge-cell) users than the Stolyar-Viswanathan algorithm.

  Table I gives the rates obtained by the 10^{th} percentile user, the median user and the average rate for the various schemes at $\alpha = 1$ and N_s = all users. The data corresponds to 48 users per base station.

2) Local Subset Scheduling: Simulation Results: We next refer the reader to Figure 3, and Figures 6 - 8. These are for the case of proportional fair scheduler when $\alpha = 1$. Note that in all these curves, the Stolyar-Viswanathan algorithm retains the advantage of fast adaptation of powers and schedules (500 Hz), while our algorithms adapt powers at 10 Hz. In addition, the Stolyar-Viswanathan algorithm retains another advantage of being able to schedule to any mobile among those associated with a base station. In Figure 3, our algorithms can schedule to any user, while in Figures 6, 7, and 8 our algorithms can schedule a subcarrier to one among a subset of size N_s = 10, 5, and 1 users, respectively.

We make the following observations:

- When we compare utilities, our algorithms fare better, except for the case when $N_s = 1$, among the cases considered. This suggests that it is sufficient to indicate a maximum of 5 users to each base station (per subcarrier) for scheduling in-between the RNC power updates.
- When we compare rates, Stolyar-Viswanathan fares better, then comes the plain gradient based algorithms, and then the CCP-based algorithms. This is expected because fairness is exactly in the reverse order, as evidenced in the CDF plots.

TABLE I
PROPERTIES OF CDF OF RATES. $\alpha = 1.0$. N_s = ALL USERS

Scheme	Rate of 10^{th} percentile user	Rate of median user	Average rate
ISPA_GD	1.040	3.096	1.536
ISPA_GD Top-Two Intf.	0.998	3.124	1.500
ISPA_CCP_GD	1.218	2.536	1.440
ISPA_CCP_GD Top-Two Intf.	1.164	2.518	1.463
Full Reuse	0.184	2.406	1.259
Stolyar-Viswanathan	0.642	3.146	1.645

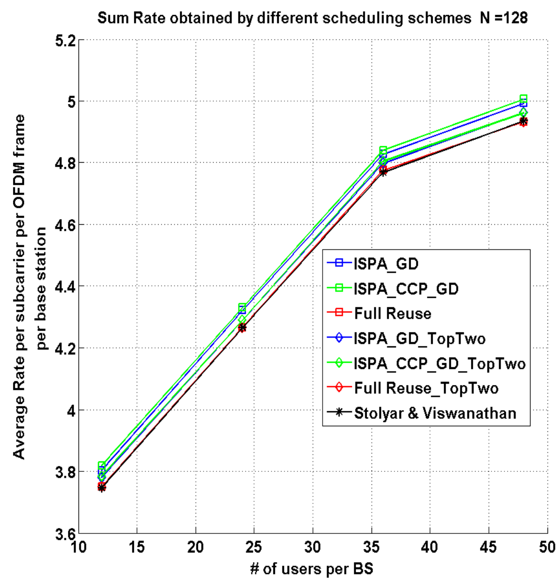

Fig. 1. α-Utility at $\alpha = 0$ (sum rate) for different algorithms. N_s = all users

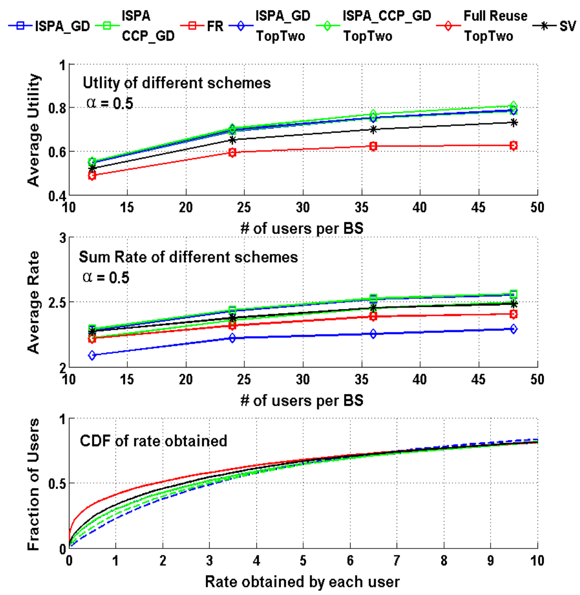

Fig. 2. α-Utility at $\alpha = 0.5$ for different algorithms. N_s = all users. Top: Achieved sum utility for $\alpha = 0.5$. Middle: Corresponding sum rate obtained. Bottom : CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

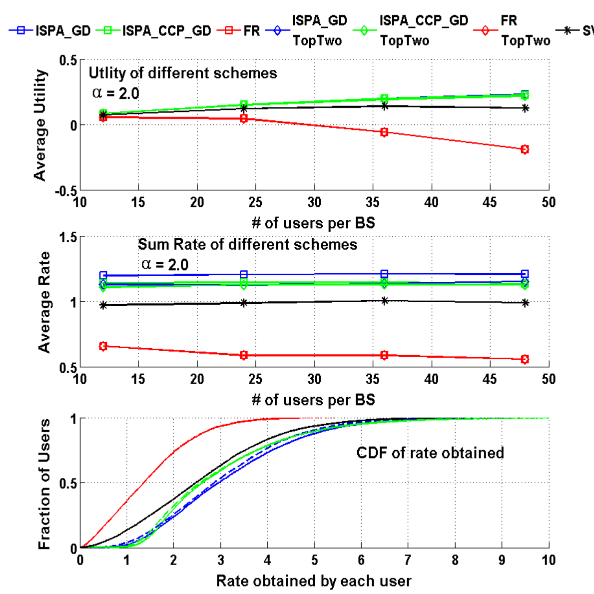

Fig. 4. α-Utility at $\alpha = 2.0$ for different algorithms. N_s = all users. Top: Achieved sum utility for $\alpha = 2.0$. Middle: Corresponding sum rate obtained. Bottom : CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

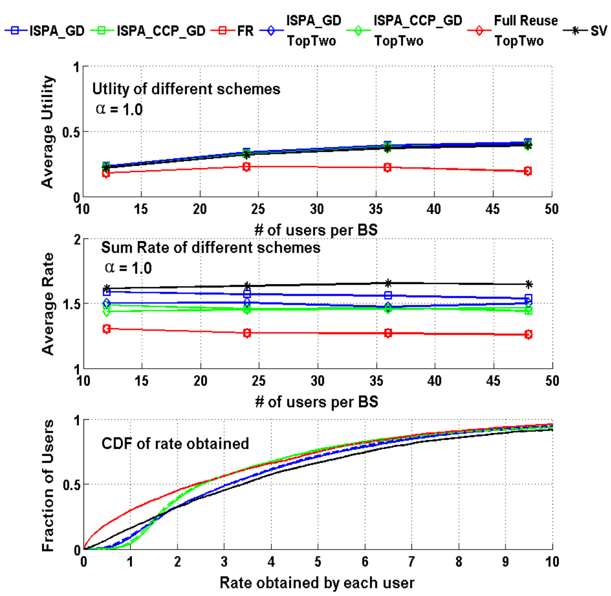

Fig. 3. α-Utility at $\alpha = 1.0$ for different algorithms. N_s = all users. Top: Achieved sum utility for $\alpha = 1.0$. Middle: Corresponding sum rate obtained. Bottom : CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

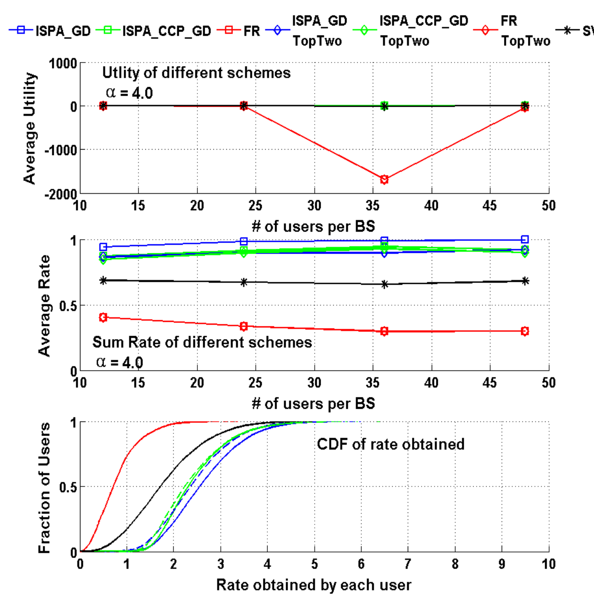

Fig. 5. α-Utility at $\alpha = 4.0$ for different algorithms. N_s = all users. Top: Achieved sum utility for $\alpha = 4.0$. Middle: Corresponding sum rate obtained. Bottom : CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

VI. Conclusion

We accomplished the following in this paper.

- Based on the idea of gradient scheduling, the system utility maximisation problem was broken down into weighted sum-rate maximisation problems at each time step, with the weights evolving with time.
- We considered the iterative scheduling and power allocation procedure (ISPA) to address the weighted sum-rate maximisation problem.
- We formulated the power control problem as a nonconvex optimisation problem. We implemented two algorithms, ISPA_GD, ISPA_CC_GD which can be used as possible solution methods for Problem 3.1.

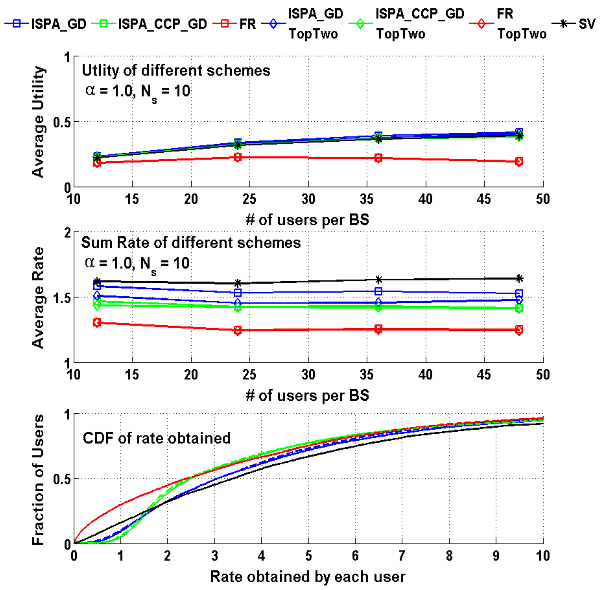

Fig. 6. α-Utility at $\alpha = 1$ for different algorithms. $N_s = 10$ users. Top: Achieved sum utility for $\alpha = 1.0$. Middle: Corresponding sum rate obtained. Bottom: CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

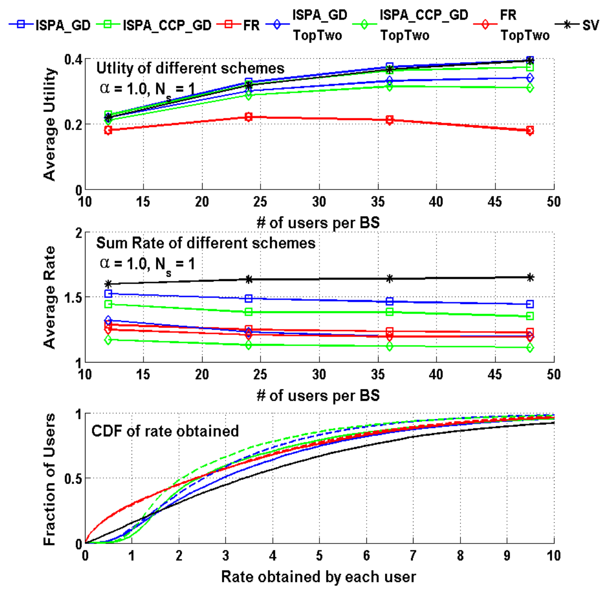

Fig. 8. α-Utility at $\alpha = 1$ for different algorithms. $N_s = 1$ users. Top: Achieved sum utility for $\alpha = 1.0$. Middle: Corresponding sum rate obtained. Bottom: CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

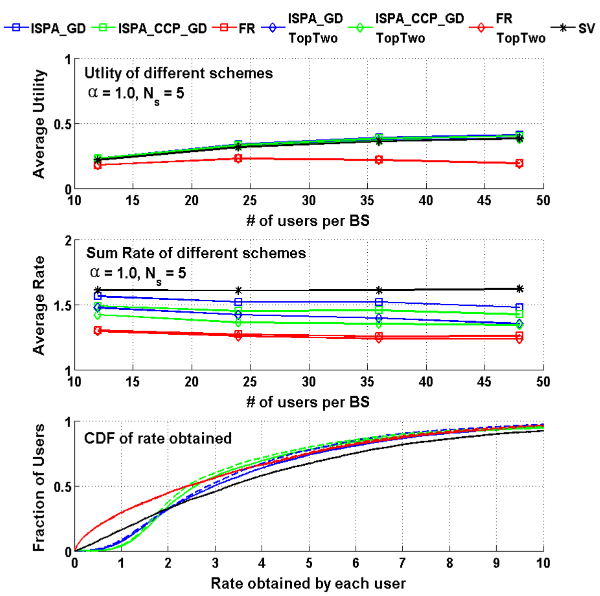

Fig. 7. α-Utility at $\alpha = 1$ for different algorithms. $N_s = 5$ users. Top: Achieved sum utility for $\alpha = 1.0$. Middle: Corresponding sum rate obtained. Bottom : CDF of rates obtained by a user. The dashed lines correspond to the TopTwo variants with the same colour coding.

- We proposed the use of plain gradient descent method over CCP based algorithms because it yielded nearly the same performance and was also faster in computation.
- For the joint scheduling and power allocation algorithm, we established the superiority of the proposed iterative scheduling and power allocation (ISPA) algorithm over equal power allocation and over that proposed by Stolyar & Viswanathan for the joint scheduling and power allocation problem. (Note that the better performance of the Stolyar & Viswanathan algorithm in $\alpha = 1$ case is with the advantage of power and scheduling adaptation at the faster rate of 500Hz(once every 2ms)).
- We studied the top-two interferers algorithm, where for each user we consider only the top two interfering base stations. We showed that such an approximation does not affect the system performance adversely. We chose the top two interferers keeping the geometry of the hexagonal lattice in mind.
- We studied the two time-scale scheduling and power allocation algorithm. Using simulation data we showed that the system performance is not adversely affected by the simplification.
- We studied scheduling from only a subset of users instead of considering the entire set of users within a BS during an RNC scheduling and power allocation interval. We showed that a subset of five users ($N_s = 5$) achieves system performance very close to the case where all users are eligible for scheduling.

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