Deptartment of Electrical Communication Engineering
Indian Institute of Science

E9 211 Adaptive Signal Processing

1 Oct 2019, 13:30–15:00, Mid-term exam

This exam has two questions (20 points). Question 2 is on the back side of this page.

A4 cheat sheet is allowed. No other materials will be allowed.

Question 1 (10 points): Temperature estimation

Let T_0 denote the initial temperature of a metal rod and assume that it is decreasing exponentially. We make two noisy measurements of the temperature of the rod at time instants t_1 and t_2 as

$$x_i = T_0 e^{-t_i} + v_i, \quad i = 1, 2,$$

where v_1 and v_2 are uncorrelated zero-mean random variables with variances $\sigma_1^2 = \sigma_2^2 = 1$, respectively.

(3pts) (a) Assume that T_0 is a constant (i.e., deterministic). Given x_1 and x_2, compute the *Best Linear Unbiased Estimator* (BLUE) $\hat{T}_0$.

(2pts) (b) Show that the estimator $\hat{T}_0$ is unbiased and give the resulting minimum mean-square error.

(3pts) (c) Now, suppose T_0 is a zero-mean random variable with variance $\sigma_T^2 > 0$. Assuming that T_0 and v_i are uncorrelated, compute the linear-least-mean squares estimator $\hat{T}_{0,\text{lmmse}}$.

(2pts) (d) Give the resulting minimum mean-square error for the estimator $\hat{T}_{0,\text{lmmse}}$ and compare it with the one obtained in Part (b) of this question with $\sigma_1^2 = \sigma_2^2 = 1$. Is $\hat{T}_{0,\text{lmmse}}$ unbiased?

Question 2 (10 points): Linear prediction

Suppose the signal $x(n)$ is *wide-sense stationary*. We develop a first-order linear predictor of the form

$$\hat{x}(n+1) = w(0)x(n) + w(1)x(n-1) = \mathbf{w}^H\mathbf{x}$$

where $\mathbf{w} = [w(0)\ w(1)]^T$ and $\mathbf{x} = [x(n)\ x(n-1)]^T$.

(1pts) (a) Show that the autocorrelation sequence of a wide-sense stationary random process is a conjugate symmetric function of the lag k, i.e., $r_x(k) = r_x^*(-k)$.

(4pts) (b) Derive the optimum $\mathbf{w}$ by minimizing the mean-squared error

$$J(\mathbf{w}) = E\{|\mathbf{w}^H\mathbf{x} - x(n+1)|^2\}.$$

(2pts) (c) Suppose the autocorrelation of $x(n)$ for the first three lags are $r_x(0) = 1$, $r_x(1) = 0$ and $r_x(2) = 1$. To solve the *normal equations* obtained in Part (b) of this question, we will use the *steepest gradient descent algorithm*. Will the steepest-descent algorithm converge if we choose the step size $\mu = 4$, and *why*?

(3pts) (d) To converge to $\mathbf{w}_{\text{opt}}$, how many iterations are required for the steepest-descent algorithm with $r_x(0) = 1$, $r_x(1) = 0$, $r_x(2) = 1$, $\mu = 1$, and $\mathbf{w}^{(0)} = \mathbf{0}$. To answer this question, first compute $\mathbf{w}_{\text{opt}}$.