Wireless synchronization and localization of sensors on a platform

Sundeep Prabhakar Chepuri

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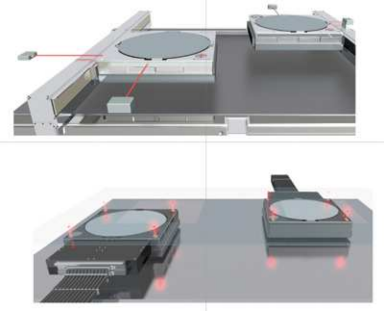
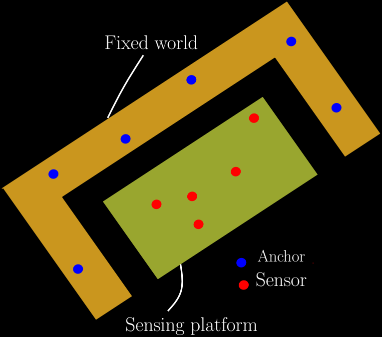
TU Delft
Delft University of Technology

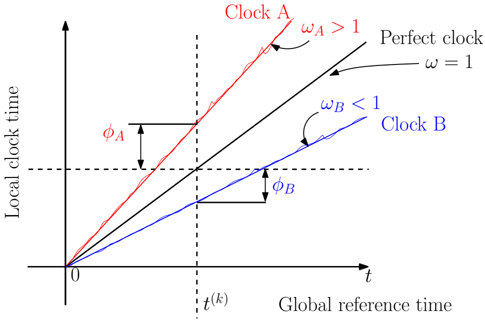


(Twin scan lithography machine)



How to distribute the clock signals over wireless links?

Clock model



Local time and global time:

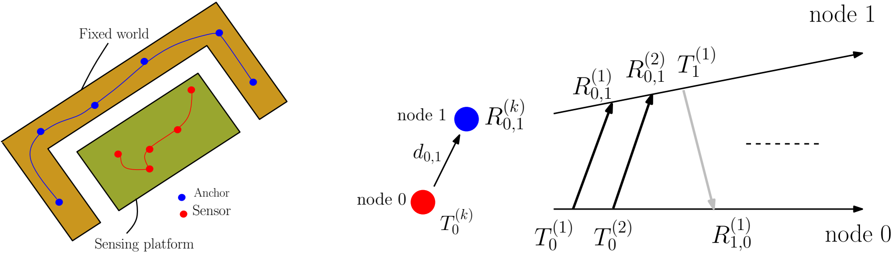
$$t_i = \omega_i t + \phi_i \quad \Leftrightarrow \quad t = \alpha_i t_i + \beta_i$$

ω_i: clock-skew and ϕ_i: clock-offset

For a perfect (reference) clock: $\alpha = 1$ and $\beta = 0$.

Time stamping: point-to-point link

Time and distance are related to each other.



Time-of-flight for an LOS transmission:

$$\tau_{i,j} = (\alpha_j R_{i,j}^{(k)} + \beta_j) - (\alpha_i T_i^{(k)} + \beta_i) + n_{ij}^{(k)}$$

$\tau_{i,j} = v^{-1} d_{i,j}$ (v is speed of wave)

Least-squares solution

Collect clock parameters in $\mathbf{c}_i = [\alpha_i, \beta_i]^T$ for $i = 0$ and $i = 1$.

$$\begin{array}{c} \text{forward link} \\ \text{reverse link} \end{array} \overbrace{\left[\begin{array}{cc|cc|c} \mathbf{t}_0 & \mathbf{1}_{K_0} & -\mathbf{r}_{0,1} & -\mathbf{1}_{K_0} & \mathbf{1}_{K_0} \\ -\mathbf{r}_{1,0} & -\mathbf{1}_{K_1} & \mathbf{t}_1 & \mathbf{1}_{K_1} & \mathbf{1}_{K_1} \end{array}\right]}^{\mathbf{A} \in \mathbb{R}^{K \times 5},\ K = K_0 + K_1} \begin{bmatrix} \mathbf{c}_0 \\ \mathbf{c}_1 \\ \tau_{0,1} \end{bmatrix} = \overbrace{\begin{bmatrix} \mathbf{n}_{0,1} \\ \mathbf{n}_{1,0} \end{bmatrix}}^{\mathbf{n}\ \in \mathbb{R}^{K \times 1}}.$$

Anchor node is the clock reference: $\mathbf{c}_1 = [\alpha_1, \beta_1]^T = [1, 0]^T$.

$$\begin{array}{c} \text{forward link} \\ \text{reverse link} \end{array} \overbrace{\left[\begin{array}{cc|c} \mathbf{t}_0 & \mathbf{1}_{K_0} & \mathbf{1}_{K_0} \\ -\mathbf{r}_{1,0} & -\mathbf{1}_{K_1} & \mathbf{1}_{K_1} \end{array}\right]}^{\bar{\mathbf{A}} \in \mathbb{R}^{K \times 3},\ K = K_0 + K_1} \overbrace{\begin{bmatrix} \mathbf{c}_0 \\ \tau_{0,1} \end{bmatrix}}^{\boldsymbol{\theta}} = \overbrace{\begin{bmatrix} \mathbf{r}_{0,1} & \mathbf{1}_{K_0} \\ -\mathbf{t}_1 & -\mathbf{1}_{K_1} \end{bmatrix}}^{\bar{\mathbf{x}}\ \in \mathbb{R}^{K \times 1}} \mathbf{c}_1 + \quad \mathbf{n}.$$

Solve linear least-squares for clock parameters and range:

$$\boxed{\widehat{\boldsymbol{\theta}} = (\bar{\mathbf{A}}^T \bar{\mathbf{A}})^{-1} \bar{\mathbf{A}}^T \bar{\mathbf{x}}}$$

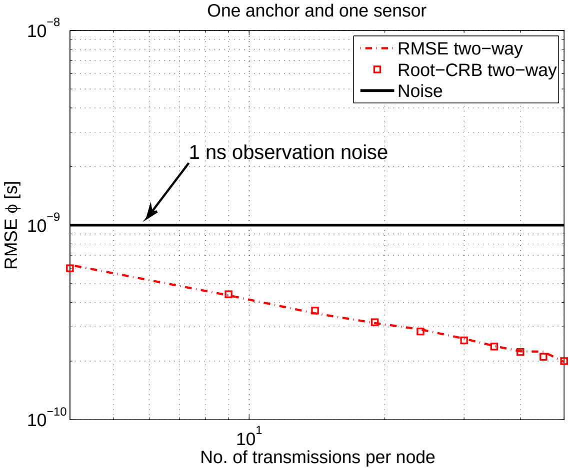
$\widehat{\boldsymbol{\theta}} = [\widehat{\mathbf{c}}_0^T, \widehat{\tau}_{0,1}]^T$, and $\widehat{d}_{0,1} = v\widehat{\tau}_{0,1}$

Synchronization accuracy: point-to-point link

- Observation window: 100 s

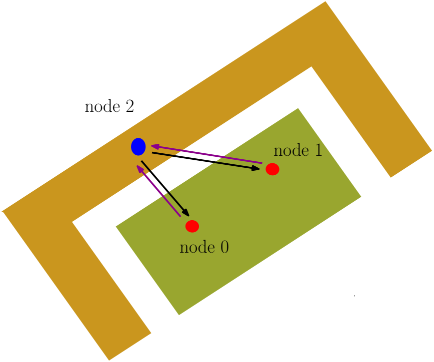
  Allan deviation $\rightarrow \sigma(\tau = 100 \text{ s}) \sim O(10^{-12})$

- max. range: 300 cm (30 cm square platform)



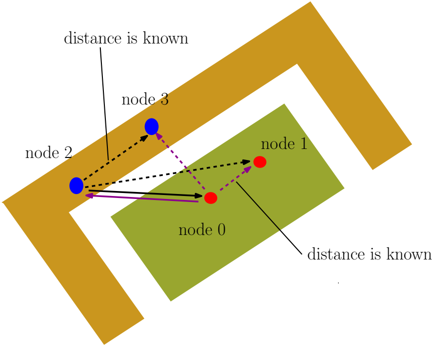
Extension - multiple nodes

Two-way comm.



- $O(N)$ equations
- sensors do not talk to each other

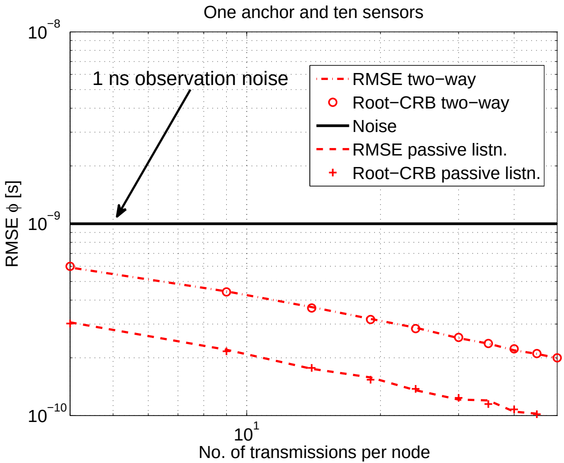
Passive listening



- $O(N^2)$ equations
- Exploit known inter-sensor distance

Synchronization accuracy: multiple nodes

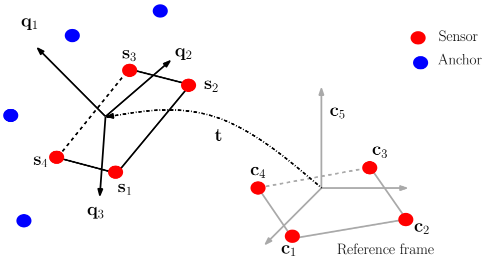
- Observation window: 100 s
- max. range: 300 cm (30 cm square platform)



10 nodes, 10 msgs → factor 2 improvement

Localization - rigid platform

Range estimates can then be used for localization



Rigid body transformation:

$$\mathbf{s}_n = \mathbf{Q}\mathbf{c}_n + \mathbf{t} \quad \Rightarrow \quad \mathbf{S} = \mathbf{Q}\mathbf{C} + \mathbf{t}\mathbf{1}_N^T = \overbrace{\left[\, \mathbf{Q} \mid \mathbf{t} \,\right]}^{\Theta} \overbrace{\begin{bmatrix} \mathbf{C} \\ \hline \mathbf{1}_N^T \end{bmatrix}}^{\mathbf{C}_e}$$

Unknown parameters

Rotation matrix $\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3] \in \mathcal{V}_{3,3}$, translation vector $\mathbf{t} \in \mathbb{R}^{3\times 1}$, and absolute sensor positions $\mathbf{S} \in \mathbb{R}^{3\times N}$

Stiefel manifold: $\mathcal{V}_{3,3} = \{\mathbf{Q} \in \mathbb{R}^{3\times 3} : \mathbf{Q}^T\mathbf{Q} = \mathbf{I}_3\}$

Localization - rigid platform

Noisy range between the m-th anchor and n-th sensor

$$
\begin{aligned}
\widehat{d}_{mn} &= \overbrace{\|\mathbf{a}_m - \mathbf{s}_n\|_2}^{d_{mn}} + \overbrace{w_{mn}}^{\mathcal{N}(0,\sigma_{mn}^2)} \\
&= \|\mathbf{a}_m - (\mathbf{Q}\mathbf{c}_n + \mathbf{t})\|_2 + w_{mn},
\end{aligned}
$$

$\mathbf{s}_n$: coordinates of the nth sensor
$\mathbf{a}_m$: coordinates of the mth anchor

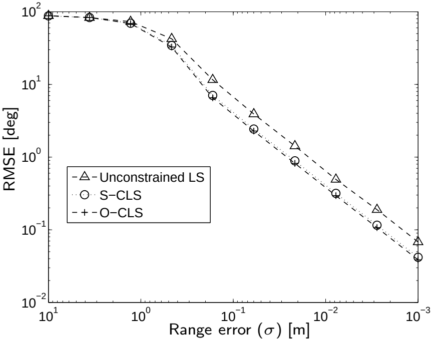
ML estimator

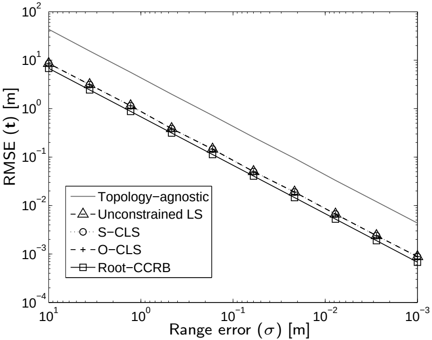
$$
\begin{aligned}
\arg\min_{\mathbf{Q},\mathbf{t}} \sum_{m=1}^{M}\sum_{n=1}^{N} \sigma_{mn}^{-2}\left(\widehat{d}_{mn} - \|\mathbf{a}_m - (\mathbf{Q}\mathbf{c}_n + \mathbf{t})\|_2\right)^2 \\
\text{s.t.} \quad \mathbf{Q}^T\mathbf{Q} = \mathbf{I}_3.
\end{aligned}
$$

Non-linear non-convex, but can be linearized via squaring

POSE accuracy using range measurements

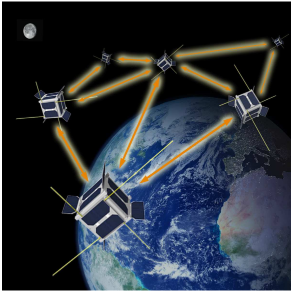
- $M = 4$ anchors, $N = 5$ sensors





$O(10^{-2})$ m positioning accuracy corresponds to $O(0.1)$ deg orientation est. accuracy

Conclusions



- Proposed frameworks are very general - applicability beyond FASTCOM.
- 10 msgs, 10 nodes we can achieve a factor 2 improvement in the sample clock accuracy.
- $O(10^{-2})$ m positioning accuracy corresponds to $O(0.1)$ deg orientation accuracy.