

# A Low Complexity Detector with Near-ML Performance for Generalized Differential Spatial Modulation

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**Abstract**—Detection of most differential modulation schemes involves the past and present received symbols only. Differential spatial modulation (DSM) is a scheme where an extra degree of freedom for transmitting information is available in the form of transmit antennas. A variant of this scheme called generalized differential scheme for spatial modulation (GD-SM) has a power allocation strategy to improve the error performance as compared to DSM. But the maximum likelihood (ML) detector for this scheme is computationally intensive for higher order modulation. We propose a low complexity detection strategy that makes use of the correlation between the channel coefficients during successive time slots to detect the activated antenna followed by the decoding of the M-ary phase shift keying (MPSK) symbol transmitted through that antenna. The proposed detector achieves a tremendous reduction in complexity close to 83% compared to the ML detector, but with a negligible penalty in error performance.

**Index Terms**—Detection, Green communication, Differential spatial modulation, Maximum-likelihood, Low complexity

## I. INTRODUCTION

The devices of the future have to integrate information of both scientific and commercial value. For most of the applications, this can be achieved using devices supporting low to medium data rates working at low power [1]. Existing multiple input multiple output (MIMO) based communication systems are designed for high data rates where the transmitter has certain redundant hardware components like radio frequency (RF) chains [2] consisting of power amplifiers, couplers, mixers and local oscillators which are required to drive a transmit antenna. In a 4G-Advanced base station (BS), these RF chains consume 65% of the total transmit circuitry power [3]. Each transmit antenna at the BS requires a dedicated RF chain to activate it in the present MIMO architecture. There are over 5 million BSs [4] around the globe serving mobile users and each of them consumes an average of 25 MWh [5] of power per year. 80% of the cellular operator's energy is consumed by the radio network. As the information and communication technology (ICT) sector contributes to 2% of the global carbon emissions [6], and due to the restrictions on capital expenditure for the cellular operators, there is an increasing pressure to adopt greener practices such as reducing the hardware cost as well as the energy consumption for the communication devices.

Spatial modulation (SM) helps in minimizing the hardware resources by sharing a single RF chain among all the transmit antennas. Thus a single transmit antenna is

only active at a time for transmitting an M-ary phase shift keying (MPSK) or M-ary quadrature amplitude modulation (MQAM) symbol. Hence SM dispenses with the requirement for inter-antenna synchronization and avoids inter-antenna interference as well, which is common in conventional MIMO schemes [2]. The decoding stage in the receiver has to detect the active antenna along with the MQAM or MPSK symbol transmitted through this antenna. The receiver should be able to find the unique antenna which is activated, based on the unique spatial signature of each of the transmit antennas. Channel estimation (CE) is done to find the channel coefficients which correspond to the unique spatial signature. Thus the receiver has to perform the CE operation at periodic intervals along with the normal symbol detection, which is also a resource intensive operation and thus it is imperative for energy constrained devices to do away with frequent CE at the receiver. Apart from this, the training symbol overhead due to the pilot transmission reduces the throughput of the system as well.

Differential schemes for SM retain almost all the benefits of the conventional SM, in addition to the increase in throughput due to the absence of pilot symbols. One of the most popular types of differential scheme is the differential SM (DSM) [7], where a single DSM symbol is encoded in  $M_t$  consecutive time slots, where  $M_t$  is the number of transmit antennas. Here each of the transmit antenna is activated without repetitions to create a unique pattern and MPSK symbols are transmitted in each of those activated antennas. Some low complexity detectors were implemented for this scheme in [8] and [9] based on the column by column approach and it reduced the number of computations significantly at the decoding stage. Later the amplitude information was also incorporated through amplitude phase shift keying-DSM (APSK-DSM) schemes of [10] and [11] to achieve higher spectral efficiency. The detectors for these schemes were also ML-based, thus making the scheme computationally intensive at the receiver for decoding higher order constellations. A new differential scheme called generalized differential scheme for SM (GD-SM) is presented in [12] where a power allocation strategy implemented for the transmitted symbol helped in improving the error performance by 2.5dB and brought it closer to the coherent SM [12]. Unlike the conventional DSM schemes, GD-SM involves transmitting reference symbols at periodic intervals at a much lesser frequency than the pilot symbols of conventional SM.

Even though the ML detector for this scheme is computationally less intensive than the optimal detectors for other DSM schemes, it is still not simple enough for devices having limited processing power and energy.

In this paper, we propose a low complexity detector for GD-SM based on inner product and  $\ell_\infty$ -norm approach. The results show that the error performance of the proposed detector is comparable with that of the ML detector. Also, the computational complexity of the detector is found to be much lower than the ML detector, thereby making it a suitable candidate for communication devices working with a limited power budget. The rest of the paper is organized as follows: Section II introduces the system model of GD-SM and the existing optimal detector is discussed followed by the proposed detector for the scheme. Computational complexity for the detection operation is presented in Section III. Followed by the numerical studies and the corresponding discussions in Section IV, we conclude the paper in Section V

*Notations:* Vectors and matrices are represented by boldface lower and upper case letters. When  $H$ ,  $T$  and  $\dagger$  are used as superscripts, they denote Hermitian operator, transpose and pseudo-inverse respectively. The notation  $|\cdot|$ ,  $[\cdot]$  and  $\|\cdot\|_p$  stands for the magnitude of the complex number, floor operation and the  $\ell_p$  norm in order. The symbol  $!$  is the factorial operation when it succeeds a variable or a number. The number of transmitter and receiver antennas are denoted by  $M_t$  and  $M_r$ .  $\mathbb{R}$  and  $\mathbb{C}$  denotes real and complex numbers.

## II. SYSTEM MODEL

In a GD-SM transmitter with  $M_t$  antennas, symbols are transmitted in frames containing  $K = M_t + L$  symbols each, where the reference symbols are transmitted during the first  $M_t$  time slots of every frame. Even though these reference symbols are not information carrying symbols, they are used for encoding the normal symbols and the former is also used at the decoder. The remaining  $L$  normal symbols transmit the actual information.

At first, the reference symbol  $s_r = 1$  is transmitted through each transmit antenna over the first  $M_t$  consecutive time slots. So the received reference signal during the  $j^{th}$  ( $j \in [1, M_t]$ ) reference time slot is

$$y_{rj} = h_{rj}s_r + n_{rj} \quad (1)$$

where  $y_{rj} \in \mathbb{C}^{M_r \times 1}$  and  $h_{rj} \in \mathbb{C}^{M_r \times 1}$  is the  $j^{th}$  column vector of the channel fading matrix  $\mathbf{H}_r \in \mathbb{C}^{M_r \times M_t}$ , whereas  $n_{rj} \in \mathbb{C}^{M_r \times 1}$  is the column vector of the noise matrix  $\mathbf{N}_r \in \mathbb{C}^{M_r \times M_t}$ , whose elements follow the complex Gaussian distribution  $\mathcal{CN}(0, \sigma_r^2)$ .

While transmitting the information via the remaining  $L$  symbols, if the  $j^{th}$  transmit antenna is activated during a time slot, so as to transmit the modulation symbol  $x$  from the MPSK constellation, then that symbol is represented as  $\mathbf{x}^{(t)} = \{0 \dots x \dots\}^T$ , where  $\mathbf{x}^{(t)} \in \mathbb{C}^{M_r \times 1}$  and its  $j^{th}$  element is the only non-zero entry. Thus the normal symbols are differentially encoded as

$$\mathbf{s}_n = s_r \mathbf{x}^{(t)}. \quad (2)$$

The received signal  $\mathbf{y}^{(t)} \in \mathbb{C}^{M_r \times 1}$  is represented as

$$\mathbf{y}^{(t)} = \mathbf{H}^{(t)} \mathbf{s}_n + \mathbf{n}_t \quad (3)$$

where  $\mathbf{H}^{(t)} \in \mathbb{C}^{M_r \times M_t}$  is the channel fading matrix during the transmission of normal symbols and  $\mathbf{n}_t \in \mathbb{C}^{M_r \times 1}$  is the noise vector, whose elements follow i.i.d complex Gaussian distribution  $\mathcal{CN}(0, \sigma_n^2)$ . The received normal symbol of (3) can also be reformulated as

$$\mathbf{y}^{(t)} = \mathbf{h}_j^{(t)} x + \mathbf{n}_t \quad (4)$$

where  $\mathbf{h}_j^{(t)} \in \mathbb{C}^{M_r \times 1}$  is the activated transmit antenna which is also the  $j^{th}$  column vector of  $\mathbf{H}^{(t)}$ . Using the assumption that the channel is quasi-static during an entire frame consisting of  $K$  symbols we have  $\mathbf{H}_r \approx \mathbf{H}^{(t)}$ , and thus the received normal symbol is rewritten using (2) and (4) as

$$\mathbf{y}^{(t)} = \mathbf{y}_{rj} x + \mathbf{n}_t - \mathbf{n}_{rj} x^{(t)} \quad (5)$$

The ML detector can be derived from (5) as

$$[\hat{j}, \hat{x}] = \arg \min_{\forall \hat{x} \in \mathcal{G}, \hat{j} \in [1, M_t]} \|\mathbf{y}^{(t)} - \mathbf{y}_{rj} \hat{x}\|_F^2 \quad (6)$$

The reference signal  $\mathbf{y}_{rj}$  gives the estimate of the channel matrix and is used to recover the information from  $\mathbf{y}^{(t)}$ . The combination of reference signals and the modulation symbol which minimizes the metric in (6) is the best estimate of the activated transmit antenna and the modulated symbol.

### A. Optimal power allocation

The authors have also suggested a higher transmit power for the reference symbols than the normal symbols [12]. The objective of power allocation is to maximize the average output signal-to-noise ratio (SNR). The  $K$  symbols of each frame is divided into blocks of size  $P = \frac{K}{M_t}$ , where the first block always consists of  $M_t$  reference symbols from all the transmit antennas and the rest of the  $P - 1$  blocks consists of normal symbols. If  $\bar{\gamma}$  is the average transmit power, the power allocation between the reference and normal data blocks is given by

$$\bar{\gamma}_r = \frac{P\bar{\gamma}}{1 + \sqrt{P-1}} \quad (7)$$

$$\bar{\gamma}_n = \frac{P\bar{\gamma}}{P-1 + \sqrt{P-1}} \quad (8)$$

where  $\bar{\gamma}_r$ ,  $\bar{\gamma}_n$  are the average transmit power of the reference and normal blocks. Thus the power in a frame is divided as per (7), (8) and is made available to  $s_r$  and  $\mathbf{x}^{(t)}$  respectively.

### B. Proposed detector

We assume that the channel coefficients during successive time slots remains the same during a single frame consisting of reference and normal symbols. This assumption of quasi-static fading channel, is  $\mathbf{H}_r \approx \mathbf{H}^{(t)} = \mathbf{H}$ , all throughout a frame of symbols is utilized to find the transmit antenna from the received symbol.  $\mathbf{Y}_r \in \mathbb{C}^{M_r \times M_t}$  represents the received reference symbol and it is written as,

$$\mathbf{Y}_r = \mathbf{H}_r s_r + \mathbf{N}_r. \quad (9)$$

In order to reduce the impact of noise, the received symbols during a time instant are normalized and are represented as  $\bar{\mathbf{Y}}_r$ ,  $\bar{\mathbf{y}}^{(t)}$ . The inner product between the

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**Algorithm 1** Low complexity detector for GD-SM
 

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- 1: **Input:**  $\mathbf{y}^{(t)}$ ,  $\mathbf{Y}_r$ : the normal and reference symbols,  $\mathbf{x}^{(t)}$ : symbol to be detected and  $M_t$ : transmit antenna number.
  - 2:  $\bar{\mathbf{y}}^{(t)}$  and  $\bar{\mathbf{Y}}_r$  are the column normalized matrices of  $\mathbf{y}^{(t)}$  and  $\mathbf{Y}_r$ .
  - 3: Inner product operation:  $\mathbf{Yabs} = |\bar{\mathbf{Y}}_r^H * \bar{\mathbf{y}}^{(t)}|$
  - 4: Store the location:  $[a_j, j] = \max(\mathbf{Yabs})$
  - 5: Find the MPSK symbol:  $\hat{x} = (\bar{\mathbf{y}}_{r,j})^\dagger * (\bar{\mathbf{y}}^{(t)})$
  - 6: **Output GD-SM symbol:** MPSK symbol:  $\hat{x}$ . Transmit antenna :  $j$
- 

received symbols (3) and (9) yields the actual transmitted symbol as,

$$\begin{aligned}
 \mathbf{Y}_r^H \mathbf{y}^{(t)} &= (\mathbf{H}_r \mathbf{s}_r + \mathbf{N}_r)^H (\mathbf{H}^{(t)} \mathbf{s}_n + \mathbf{n}_t) \\
 &\approx \mathbf{s}_r^H \mathbf{H}^H \mathbf{h}_j s_r \mathbf{x} + \mathbf{N}' \\
 &= s_r^H \mathbb{H} s_r \mathbf{x} + \mathbf{N}' \\
 &= \mathbb{Z} x + \mathbf{N}'
 \end{aligned} \tag{10}$$

where  $\mathbf{N}' = s_r^H \mathbf{H}_r \mathbf{n}_t + \mathbf{N}_r^H \mathbf{H}^{(t)} \mathbf{s}_n + \mathbf{N}_r^H \mathbf{n}_r$  is the noise term and  $s_r^* s_r = 1$ . Assuming that  $\mathbf{H}_r$  is a full rank channel fading matrix and as the columns are normalized before being used in the system model, the elements of the column matrix  $\mathbb{H}$  are  $\mathbb{H}_j \approx 1$  and  $\mathbb{H}_i \ll 1, \forall i \neq j$ . Similarly the elements of the column matrix  $\mathbb{Z}$  are  $\mathbb{Z}_j \approx 1$  and  $\mathbb{Z}_i \ll 1, \forall i \neq j$ . Thus the basic structure of the GD-SM symbol  $x^{(t)}$  is preserved. Thus the inner product between  $\mathbf{H}$  and  $\mathbf{h}_j$  gives a  $M_t \times 1$  column matrix named  $\mathbb{H}$ . The quasi-static assumption helps in preserving high correlation or an inner product value close to one for one of the columns of  $\mathbf{H}$  corresponding to the transmit antenna coefficient  $\mathbf{h}_j$ . By applying  $\ell_\infty$ -norm to the inner product operation of (10) we can localize the active transmit antenna in time through,

$$[a_j, j] = \left\| \left( |\bar{\mathbf{Y}}_r^H * \bar{\mathbf{y}}^{(t)}| \right) \right\|_{\infty} \quad \text{where, } i \in (1, M_t) \tag{11}$$

which returns the largest element, and its location in the column matrix points to the active transmit antenna during that time slot. Once the active antenna is found as  $j$  the corresponding reference symbol  $\mathbf{y}_{r,j}$  is selected from  $\mathbf{Y}_r$  so as to decode the transmitted MPSK symbol as shown below,

$$\hat{x}^{(t)} = \bar{\mathbf{y}}_{r,j}^\dagger * \bar{\mathbf{y}}^{(t)} \quad \text{where, } j \in (1, M_t). \tag{12}$$

The steps required to perform the two-stage low complexity detection process is detailed in the Algorithm 1.

### III. COMPLEXITY ANALYSIS

The complexity is defined in terms of the number of real valued multiplications required to detect  $L$  normal GD-SM symbols occurring in one frame. For the proposed detector described in Algorithm 1, the real multiplications involved for the different steps and operations are: (i) Step 2, for normalization of  $\mathbf{Y}_r$  and  $\mathbf{y}^{(t)}$ ,  $(4M_r + 1)M_t + (4M_r + 1)L$  (ii) Step 3, for the inner product and absolute value operation,  $(4M_r M_t + 2M_r)L$ , (iii) Step 5, for decoding the MPSK symbol,  $(4M_r + 1)M_t + 4M_r L$ . When we compared the

complexity of the proposed detector to that of the ML-based detector [12], the former has a significantly lower number of computations given by,

$$C_{ml} = 4M_r M_t M + 2M_r M_t M L \tag{13}$$

$$C_{proposed} = 8M_r M_t + 2M_t + (10M_r + 1 + 4M_r M_t) L \tag{14}$$

where  $M$  is the MPSK modulation order. We can see from Figure 1 that the proposed detector requires only a very low number of computations to detect the symbol. Thus

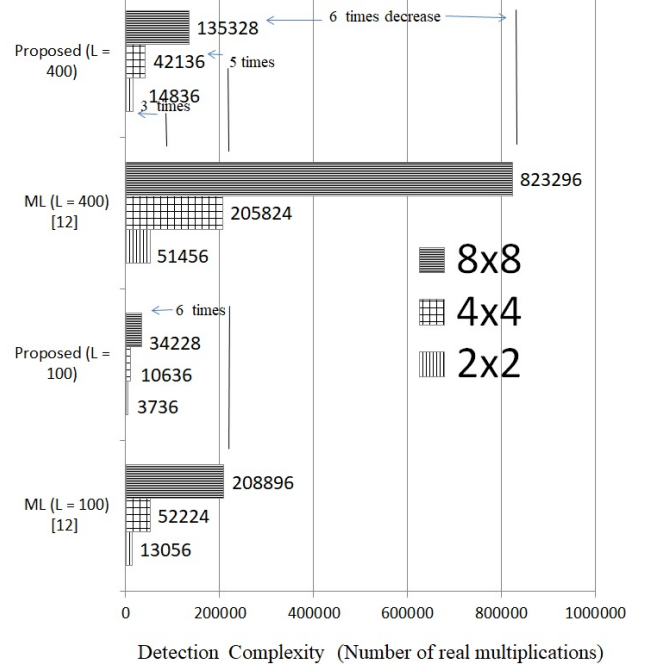


Fig. (1) Complexity of detectors for GD-SM schemes for different  $M_t \times M_r$  configurations

the computations are considered for decoding all the  $L$  normal symbols present in a frame of  $K = M_t + L$  symbols. As the complexity of the proposed detector is independent of the modulation order  $M$ , the number of computations to decode the symbols doesn't grow exponentially as in the ML detector. The reduction in complexity is calculated as

$$Reduction \text{ in complexity} = \frac{C_{ml} - C_{proposed}}{C_{ml}} \times 100\%. \tag{15}$$

The proposed detector achieves significant complexity reduction of 70% to 83% for  $M_r = M_t = 2, 8$  and for  $L = 100, 400$  symbols, compared to ML and the same is detailed in Table I and is also plotted in Figure 1 by fixing the MPSK modulation order as,  $M = 16$ .

TABLE (I) Numerical value of complexity in terms of the number of real multiplications

Methods	No. of transmit antennas		
	2	4	8
$C_{ml} (L = 100)$ [12]	$1.3 \times 10^4$	$5.2 \times 10^4$	$2 \times 10^5$
$C_{proposed} (L = 100)$	3736	10636	$3.4 \times 10^4$
$C_{ml} (L = 400)$ [12]	$5.1 \times 10^4$	$2 \times 10^5$	$8.2 \times 10^5$
$C_{proposed} (L = 400)$	$1.5 \times 10^4$	$4.2 \times 10^4$	$1.3 \times 10^5$

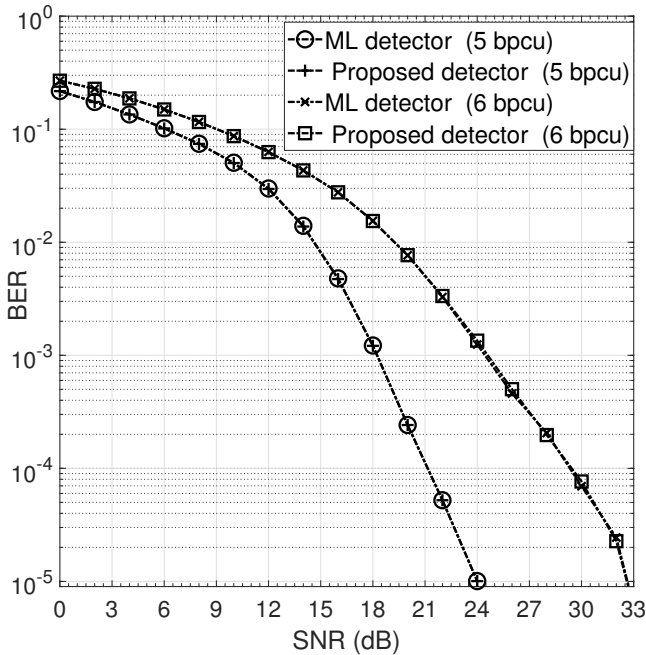


Fig. (2) BER of the proposed and ML detectors for a  $2 \times 2$  GD-SM scheme for different throughput

#### IV. SIMULATION STUDIES AND DISCUSSION

In this section, we compare the error performance of the ML-based detector and the proposed detector under Rayleigh fading channel for identical throughput and there is only a negligible penalty in bit error rate (BER). The BER for higher order modulation is evaluated for  $2 \times 2$  and  $4 \times 4$  antenna configurations working with a frame size of  $L = 100$  normal symbols and is observed to have nearly the same performance as the ML detector. The number of bits per channel use (bpcu) is the metric used to describe throughput and it is defined as the total number of bits that the transmitter can send at a given instant when only one of the transmit antenna is active during the normal block. The error performance for higher throughput is also found in Figure 2, 3 and it is observed that, irrespective of the throughput the proposed detector attains almost the same performance as that of the optimal detector in [12]. The proposed detector has low complexity due to the selection of the highly correlated reference symbol from the first block of the same frame and by using it to decode the MPSK symbol as well. This two-stage detection process reduces the complexity when compared to the joint minimization technique performed in an ML detector.

#### V. CONCLUSION

In this work, we proposed a detector for GD-SM as an alternative to the computationally prohibitive ML-based detector. This novel detector attains such a complexity reduction of up to 83% without compromising on the error performance. As this differential scheme employs a power allocation strategy for the reference symbols, it could attain a BER performance close to the coherent SM. Thus GD-SM schemes are more relevant in the context of energy efficiency as it avoids CE at the receiver, and

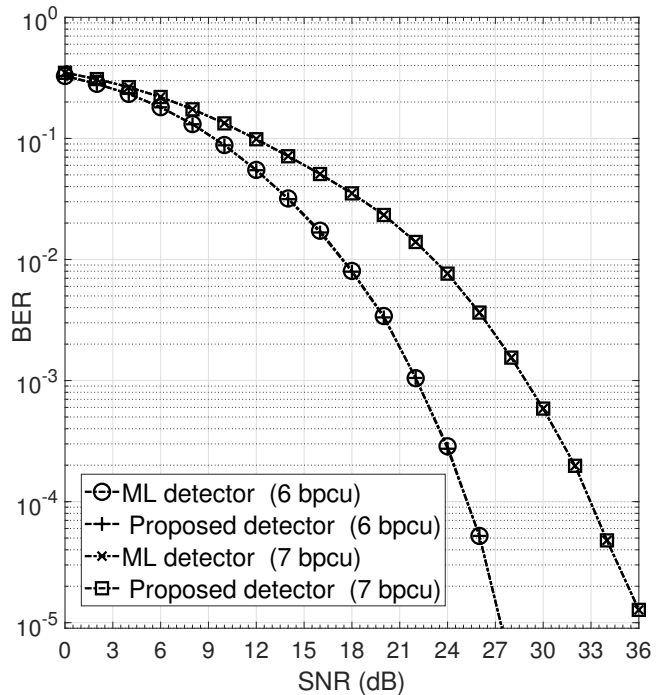


Fig. (3) BER of the proposed and ML detectors for a  $4 \times 4$  GD-SM scheme for different throughput

to save the energy and processing capability further, a computationally efficient detector such as the one proposed in this paper helps in the overall requirement of making the communication systems greener.

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