Impact of User and Relay Hardware Impairments on Spectral Efficiency of HD Massive MIMO Relay

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***Abstract*—Multi-pair two-way massive multiple-input multiple-output (mMIMO) relaying is being widely investigated. Most of the spectral efficiency (SE) investigations in mMIMO relaying assume ideal hardware. We consider a multi-pair two-way mMIMO half-duplex (HD) relay with user and relay hardware impairments. We derive a novel closed-form SE expression with maximum ratio relay processing and show that the SE, *primarily due to the user hardware impairments*, asymptotically saturates to a finite value despite the number of relay antennas N going to infinity. We also scale the HD relay hardware impairments as N^z with $z \geq 0$, and analyze the asymptotic SE limits for four different power scaling schemes. We use them to investigate the rate of increase in relay hardware impairments with increase in N that can be tolerated without compromising the SE.**

***Index Terms*—Hardware impairment, hardware scaling, relay.**

I. Introduction

Multi-pair two-way relaying (TWR) enables information exchange between multiple user-pairs in two channel uses via a shared half-duplex (HD) relay [1], [2]. Massive multiple-input multiple-output (mMIMO) technology used in [1], [2] for multi-pair TWR improves its spectral efficiency (SE) by mitigating co-channel interference using simple linear transmit/receive processing. References [1] and [2] derived the power scaling laws and closed-form SE expressions for two-way multi-pair mMIMO amplify-and-forward (AF) relaying. The aforementioned references assume that each antenna is connected to a *high-quality* radio frequency (RF) transceiver, which can prohibitively increase the system cost. mMIMO systems, to reduce the cost, are being planned to be built with low-cost hardware components, which will be prone to impairments like non-linearities and phase noise [3]–[5].

Reference [6] derives bounds on the uplink and downlink SE for a single hop mMIMO system, considering hardware impairments, both at the base station and the users. Reference [3] analyzed the impact of relay hardware impairments on a multi-pair mMIMO amplify and forward (AF) HD relaying, with maximum ratio relay processing. This work derived hardware scaling laws by varying the hardware impairments with number of relay antennas N. This work however did not study power scaling schemes for the hardware-impaired systems. Reference [4] derived power scaling laws for one-way relaying by considering relay hardware impairments. This work does not scale the relay hardware impairments with N and therefore, ignores the fact that the relay can employ low-cost hardware, without compromising its SE. Further, both these works neglect hardware impairments at the users. The **main contributions** of the current work bridge these gaps, and are listed as following:

1) We derive a closed-form SE expression for a multi-pair two-way AF HD mMIMO relaying system wherein both relay and user have low-quality hardware. Reference [3] derived an SE expression by considering the relay hardware impairments alone. We note that the user hardware impairments, particularly transmitter, complicates the SE analysis as it also affects both relay transmit and receive processing. This closed-form SE expression, which to the best of our knowledge is completely novel, *shows that the SE asymptotically saturates to a finite value as $N \to \infty$, primarily due to the user hardware impairments.* The impact of relay hardware impairments however asymptotically vanishes.

2) We derive asymptotic SE limits for four different power scaling schemes by varying the relay hardware impairments as N^z, where $z \geq 0$ is the relay hardware scaling exponent. This analysis is practically crucial as it helps us in determining desirable z values, for these power scaling cases, with which the hardware impairments can be scaled with N, without significantly degrading the SE. We show that the asymptotic SE saturates to finite value for $0 \leq z < 1$, but goes to zero for $z > 1$.

II. System Model

We consider a HD multi-pair TWR system, where K single-antenna HD user-pairs (U_{2m-1}, U_{2m}), $(m = 1, \cdots, K)$ exchange information via a shared N-antenna mMIMO AF HD relay operating in time division duplex (TDD) mode. We assume that the direct link between user-pairs is absent because of heavy shadowing and large path loss [2]. We also assume that the mMIMO relay and all the $2K$ users, are equipped with low quality transmit/receive hardware which are prone to impairments. The transmit and receive hardware impairments at the relay and at the users can be modeled as additive independent distortion terms, which are proportional to the transmit and receive signal power, respectively [5].

The data exchange in HD TWR occurs in two phases - the multiple access (MA) phase and the broadcast (BC) phase. These two phases have reciprocal channels due to the TDD relay [2]. In the MA phase, all the users simultaneously transmit their respective signals $\sqrt{P_S}x_k$ to the relay, where P_S is the transmit power of user U_k with $\mathbb{E}[|x_k|^2] = 1$. However, due to the transmit hardware impairments of the users, the transmit signal of the kth user gets effected by an additive distortion term $\eta_{tu_k} \sim \mathcal{CN}(0, \kappa_{tu}^2 P_S)$. The scalar constant κ_{tu} characterizes the level of user transmit hardware impairment, and can be interpreted as a metric which indirectly characterizes user transmit error vector magnitude (EVM) [5]. The effective transmit signal of the kth user, therefore, is $\bar{x}_k = \sqrt{P_S}x_k + \eta_{tu_k}$.

The received signal $\mathbf{y} \in \mathbb{C}^{N\times 1}$ at the relay is written as

$$\mathbf{y} = \sum_{k=1}^{2K} \mathbf{g}_k(\sqrt{P_S}x_k + \eta_{tu_k}) + \boldsymbol{\eta}_{rr} + \mathbf{n}_r = \mathbf{G}(\sqrt{P_S}\mathbf{x} + \boldsymbol{\eta}_{tu}) + \boldsymbol{\eta}_{rr} + \mathbf{n}_r, \quad (1)$$

where the vector $\mathbf{x} = [x_1, \cdots, x_{2K}]^T$, and the vector $\mathbf{g}_k \in \mathbb{C}^{N\times 1}$ for $k = 1, \cdots, 2K$ denotes the channel between the kth user and the relay, which has independent and identically distributed (i.i.d) elements with pdf $\mathcal{CN}(0, \sigma_k^2)$. Here σ_k^2 is the large-scale fading coefficient. The concatenated channel matrix $\mathbf{G} = [\mathbf{g}_1, \cdots, \mathbf{g}_{2K}] \in \mathbb{C}^{N\times 2K}$, which accounts for both small-scale fading and large-scale fading can be expressed as $\mathbf{G} = \mathbf{H}\mathbf{D}^{1/2}$. Here $\mathbf{H} \sim \mathcal{CN}(0, 1)$ is the small scale fading, and the diagonal matrix $\mathbf{D}$ is the large scale fading with σ_k^2 as the kth diagonal element. The vector $\mathbf{n}_r$ is the additive white Gaussian noise (AWGN) at the relay, and is distributed as $\mathcal{CN}(\mathbf{0}, \sigma_{nr}^2\mathbf{I}_N)$, where the symbol $\mathbf{I}_N$ represent an $N \times N$ identity matrix. The vector $\boldsymbol{\eta}_{tu} = [\eta_{tu_1}, \cdots, \eta_{tu_{2K}}]^T \sim \mathcal{CN}(\mathbf{0}, \kappa_{tu}^2 P_S \mathbf{I}_{2K})$ represents the transmit hardware impairments of all the $2K$ users. The term $\boldsymbol{\eta}_{rr} \in \mathbb{C}^{N\times 1}$ represents the relay receiver hardware impairments and is modeled as

$$\boldsymbol{\eta}_{rr} \sim \mathcal{CN}(\mathbf{0}, (1 + \kappa_{tu}^2)\kappa_{rr}^2 P_S \mathbf{W}_D). \quad (2)$$

Here $\mathbf{W}_D = \text{diag}\{W_{kk}\}_{k=1,\cdots,2K}$ is a diagonal matrix with W_{kk} being the kth diagonal element of $\mathbf{W} = \sum_{i=1}^{2K} \mathbf{g}_i\mathbf{g}_i^H$ [3]. The constant κ_{rr} can be interpreted as the relay receive EVM [5].

In the BC phase, the AF relay generates its transmit signal $\mathbf{x}_R = \alpha\mathbf{F}\mathbf{y}$. Here the precoder $\mathbf{F} = \mathbf{G}^*\mathbf{P}\mathbf{G}^H = \sum_{k=1}^{2K} \mathbf{g}_k^*\mathbf{g}_{k'}^H$ is designed based on MRC/MRT principles, where (k, k') represents a user pair. The term α is designed to satisfy the relay transmit power constraint and the permutation matrix $\mathbf{P} = \text{diag}(\mathbf{P}_1, \cdots, \mathbf{P}_K)$, with $\mathbf{P}_k = [0, 1; 1, 0]$ for $k = 1, \cdots, K$ enables information exchange between user-pairs. The relay then broadcasts $\mathbf{x}_R$ to all the users.

Due to the relay transmit hardware impairments, its transmit signal gets affected by an additive distortion term $\boldsymbol{\eta}_{tr} \sim \mathcal{CN}(0, \kappa_{tr}^2 \frac{P_R}{N}\mathbf{I}_N)$. The proportionality constant κ_{tr} can be interpreted as the relay transmit EVM [5]. The effective relay transmit signal is therefore given as $\tilde{\mathbf{x}}_R = \alpha\mathbf{F}\big(\mathbf{G}(\sqrt{P_S}\mathbf{x} + \boldsymbol{\eta}_{tu}) + \boldsymbol{\eta}_{rr} + \mathbf{n}_r\big) + \boldsymbol{\eta}_{tr}$. The signal received by the user $U_{k'}$ gets affected by an additional distortion term $\eta_{ru_{k'}}$ due to its receive hardware impairments and is given as

$$y_{k'} = \mathbf{g}_{k'}^T\tilde{\mathbf{x}}_R + \eta_{ru_{k'}} + n_{k'}$$
$$= \underbrace{\sqrt{P_S}\alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k x_k}_{\text{desired signal}} + \underbrace{\sqrt{P_S}\alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_{k'}x_{k'}}_{\text{self- interference}} + \underbrace{\sqrt{P_S}\alpha\sum_{i\neq k,k'}\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i x_i}_{\text{inter-user interference}}$$
$$+ \underbrace{\alpha\mathbf{g}_{k'}^T\mathbf{F}\boldsymbol{\eta}_{rr} + \mathbf{g}_{k'}^T\boldsymbol{\eta}_{tr}}_{\text{relay hardware impairments}} + \underbrace{\alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\boldsymbol{\eta}_{tu} + \eta_{ru_{k'}}}_{\text{user hardware impairments}} + \underbrace{\alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{n}_r + n_{k'}}_{\text{compound noise}}. \quad (3)$$

Here $n_{k'}$ is the AWGN at the k'th user, which is distributed as $\mathcal{CN}(0, \sigma_{nu}^2)$. The term $\eta_{ru_{k'}} \sim \mathcal{CN}(\mathbf{0}, \kappa_{ru}^2 w)$ represents the receive hardware impairment at the k'th user, with w being its receive power, given by $w = \mathbb{E}_{|\mathbf{G}}\{|\mathbf{g}_{k'}^T\tilde{\mathbf{x}}_R|^2\} = \alpha^2 P_S(1 + \kappa_{tu}^2)(\|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\|^2 + \kappa_{rr}^2\mathbf{g}_{k'}^T\mathbf{F}\mathbf{W}_D\mathbf{F}^H\mathbf{g}_{k'}^*) + \alpha^2\sigma_{nr}^2\|\mathbf{g}_{k'}^T\mathbf{F}\|^2 + \frac{\kappa_{tr}^2 P_R}{N}\|\mathbf{g}_{k'}\|^2$. Since the user $U_{k'}$ knows its transmit signal $x_{k'}$, it can cancel the self-interference term from its receive signal. The received signal $\bar{y}_{k'}$ after self-interference cancellation is

$$\bar{y}_{k'} = \sqrt{P_S}\alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k x_k + \sqrt{P_S}\alpha\sum_{i\neq k,k'}\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i x_i + \alpha\mathbf{g}_{k'}^T\mathbf{F}\boldsymbol{\eta}_{rr} + \mathbf{g}_{k'}^T\boldsymbol{\eta}_{tr} + \alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\boldsymbol{\eta}_{tu} + \eta_{ru_{k'}} + \alpha\mathbf{g}_{k'}^T\mathbf{F}\mathbf{n}_r + n_{k'}. \quad (4)$$

We assume, similar to [2], that the relay and the users have perfect channel knowledge that is obtained using pilots.

III. Spectral Efficiency Analysis

We now derive a closed form SE expression for the $U_k - U_{k'}$ link, which we then use to derive the asymptotic SE limits for four different power scaling cases. We also investigate the level of hardware imperfections that can be tolerated with increase in the number of relay antennas, while maintaining a high SE.

Using (4), the ergodic SE of the transmission link $U_k \to U_{k'}$ can be written as

$$R_{k'} = \mathbb{E}\left\{C\left(\frac{\alpha^2 P_S|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}{\alpha^2 P_S\sum_{i\neq k,k'}|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i|^2 + \rho + \omega + \zeta}\right)\right\}, \quad (5)$$

where $\rho = \alpha^2(1 + \kappa_{tu}^2)\kappa_{rr}^2 P_S\mathbf{g}_{k'}^T\mathbf{F}\mathbf{W}_D\mathbf{F}^H\mathbf{g}_{k'}^* + \frac{\kappa_{tr}^2 P_R}{N}\|\mathbf{g}_{k'}\|^2$, $\omega = \alpha^2\kappa_{tu}^2 P_S\|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\|^2 + \kappa_{ru}^2 w$ $\zeta = \alpha^2\sigma_{nr}^2\|\mathbf{g}_{k'}^T\mathbf{F}\|^2 + \sigma_{nu}^2$ and the function $C(x) \triangleq 0.5\log_2(1 + x)$.

It is extremely challenging to further simplify (5) [1]. We therefore, similar to [1], approximate it using Jensen's inequality which becomes tight as $N \to \infty$:

$$R_{k'} \geq \bar{R}_{k'} = C\left(\frac{1}{\text{IP}_{k'} + \text{RI}_{k'} + \text{UI}_{k'} + \text{RN}_{k'} + \text{UN}_{k'}}\right). \quad (6)$$

Here IP, RN and UN denote the inter-pair interference, amplified noise from relay and noise at the user respectively. We define RI = RRI + TRI and UI = URI + UTI where RRI/TRI and URI/UTI represents the receive/transmit hardware impairments at the relay and at the users respectively. These terms are given as,

$$\text{IP}_{k'} = \sum_{i\neq k,k'}\mathbb{E}\left\{\frac{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}, \text{TRI}_{k'} = \frac{\kappa_{tr}^2 P_R}{N\alpha^2 P_S}\mathbb{E}\left\{\frac{\|\mathbf{g}_{k'}\|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\},$$

$$\text{URI}_{k'} = \frac{\kappa_{ru}^2}{\alpha^2 P_S}\mathbb{E}\left\{\frac{w}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}, \text{UTI}_{k'} = \kappa_{tu}^2\mathbb{E}\left\{\frac{\|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\},$$

$$\text{RN}_{k'} = \frac{\sigma_{nr}^2}{P_S}\mathbb{E}\left\{\frac{\|\mathbf{g}_{k'}^T\mathbf{F}\|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}, \text{UN}_{k'} = \frac{\sigma_{nu}^2}{\alpha^2 P_S}\mathbb{E}\left\{\frac{1}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\},$$

$$\text{and } \text{RRI}_{k'} = (1 + \kappa_{tu}^2)\kappa_{rr}^2\mathbb{E}\left\{\frac{\mathbf{g}_{k'}^T\mathbf{F}\mathbf{W}_D\mathbf{F}^H\mathbf{g}_{k'}^*}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}. \quad (7)$$

We now simplify α, designed to satisfy the total relay power constraint at the relay as $P_R = \mathbb{E}[\|\alpha\mathbf{F}\mathbf{y}\|^2]$. Therefore

$$\alpha = \sqrt{\frac{P_R}{\text{Tr}\{P_S(1+\kappa_{tu}^2)(\mathbf{F}\mathbf{G}\mathbf{G}^H\mathbf{F}^H + \kappa_{rr}^2\mathbf{F}\mathbf{W}_D\mathbf{F}^H) + \sigma_{nr}^2\mathbf{F}\mathbf{F}^H\}}}$$
$$\stackrel{(a)}{=} \sqrt{\frac{P_R}{P_S(1+\kappa_{tu}^2)(N^3\Delta_1 + N^2\kappa_{rr}^2\Delta_2) + N^2\sigma_{nr}^2\Delta_3}}. \quad (8)$$

Here $\Delta_1 = \text{Tr}\left\{\left(\mathbf{G}^H\mathbf{G}/N\right)^T\mathbf{P}\left(\mathbf{G}^H\mathbf{G}/N\right)^2\mathbf{P}\right\}$, $\Delta_2 = \text{Tr}\left\{\left(\mathbf{G}^H\mathbf{G}/N\right)^T\mathbf{P}\left(\mathbf{G}^H\mathbf{W}_D\mathbf{G}/N\right)\mathbf{P}\right\}$,

$$\bar{R}_{k'}=C\left(\frac{NP_SP_R\sigma_k^4\sigma_{k'}^4}{P_R\sigma_k^2((1+\kappa_{ru}^2)(P_S(1+\kappa_{tu}^2)(\Delta_4+\sigma_k^2\Delta_0\kappa_{rr}^2)+\sigma_{k'}^2\sigma_k^2\sigma_{nr}^2+\kappa_{tr}^2\Lambda)+NP_S\sigma_{k'}^2\sigma_k^4(\Omega-\frac{2\sigma_{k'}^2}{N\sigma_k^2}))+\sigma_{nu}^2\Lambda}\right). \quad (14)$$

$$\bar{R}_{k'}^{(22)}=C\Big(\frac{E_S\sigma_k^4\sigma_{k'}^2}{\sigma_k^2\sigma_{k'}^2(E_S\sigma_k^2\Omega+(1+\kappa_{ru}^2)(\sigma_{nr}^2+E_S\Delta_0(1+\kappa_{tu}^2)\kappa_{rr0}^2))+\kappa_{tr0}^2(1+\kappa_{ru}^2)(E_S(1+\kappa_{tu}^2)(\Delta_1+\kappa_{rr0}^2\Delta_2)+\sigma_{nr}^2\Delta_3)}\Big). \quad (20)$$

$\Delta_3 = \text{Tr}\left\{\left(\mathbf{G}^H\mathbf{G}/N\right)^T\mathbf{P}\left(\mathbf{G}^H\mathbf{G}/N\right)\mathbf{P}\right\}$. Equality in (a) is obtained by substituting $\mathbf{F}=\mathbf{G}^*\mathbf{P}\mathbf{G}^H$, and by using the fact: $\text{Tr}(\mathbf{AB})=\text{Tr}(\mathbf{BA})$. Based on the Lemma 1 in [2] and using (2), we have $\left(\frac{\mathbf{G}^H\mathbf{G}}{N}\right)\xrightarrow[N\to\infty]{a.s.}\mathbf{D}$ and $\left(\frac{\mathbf{G}^H\mathbf{W}_D\mathbf{G}}{N}\right)\xrightarrow[N\to\infty]{a.s.}\bar{\mathbf{D}}$, where $\bar{\mathbf{D}}$ is a diagonal matrix whose kth diagonal element is given as $\sigma_k^2\left(\sum_{j=1}^{2K}\sigma_j^2+\sigma_k^2\right)$ for $k=1,\cdots,2K$. Hence, we can re-write Δ_1, Δ_2 and Δ_3 as

$$\Delta_1=\sum_{i=1}^{K}\sigma_i^2\sigma_{i'}^2(\sigma_i^2+\sigma_{i'}^2),\ \Delta_3=\sum_{i=1}^{K}2\sigma_i^2\sigma_{i'}^2,$$
$$\Delta_2=\sum_{i=1}^{K}\sigma_i^2\sigma_{i'}^2(2\Delta_0+\sigma_i^2+\sigma_{i'}^2),\ \text{where}\ \Delta_0=\sum_{j=1}^{2K}\sigma_j^2. \quad (9)$$

The amplification factor using (8) and (9) can be given by

$$\alpha^2-\frac{P_R}{N^3\Lambda}\xrightarrow[N\to\infty]{a.s.}0, \quad (10)$$

where $\Lambda=(1+\kappa_{tu}^2)P_S(\Delta_1+\frac{\kappa_{rr}^2}{N}\Delta_2)+\frac{\sigma_{nr}^2}{N}\Delta_3$.

We next evaluate the terms in (7). We start with the user transmit impairment term $\text{UTI}_{k'}$:

$$\text{UTI}_{k'}=\kappa_{tu}^2\mathbb{E}\left\{\frac{\|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{G}\|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}=\kappa_{tu}^2\left(\sum_{i\neq k}\mathbb{E}\left\{\frac{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i|^2}{|\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k|^2}\right\}+1\right), \quad (11)$$

Based on Lemma 1 in [7] and substituting $\mathbf{F}=\mathbf{G}^*\mathbf{P}\mathbf{G}^H=\sum_{j=1}^{2K}\mathbf{g}_j^*\mathbf{g}_{j'}^H$, we have $\frac{\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_k}{N^2}\xrightarrow[N\to\infty]{a.s.}\sigma_k^2\sigma_{k'}^2$ and $\mathbb{E}\left\{\left|\frac{\mathbf{g}_{k'}^T\mathbf{F}\mathbf{g}_i}{N\sqrt{N}}\right|^2\right\}=\sum_{j=1}^{2K}\mathbb{E}\left\{\left|\frac{\mathbf{g}_{k'}^T\mathbf{g}_j^*\mathbf{g}_{j'}^H\mathbf{g}_i}{N\sqrt{N}}\right|^2\right\}\xrightarrow[N\to\infty]{a.s.}\sigma_i^2\sigma_{k'}^2(\sigma_{k'}^2\sigma_k^2+\sigma_i^2\sigma_{i'}^2)$. Therefore, we can write

$$\text{UTI}_{k'}-\kappa_{tu}^2\left(\frac{1}{N}\sum_{i\neq k}\left(\frac{\sigma_i^2}{\sigma_k^2}+\frac{\sigma_i^4\sigma_{i'}^2}{\sigma_k^4\sigma_{k'}^2}\right)+1\right)\xrightarrow[N\to\infty]{a.s.}0. \quad (12)$$

On similar lines the other terms in (7) can be expressed as

$$\text{IP}_{k'}-\sum_{i\neq k,k'}\frac{1}{N}\left\{\frac{\sigma_i^2}{\sigma_k^2}+\frac{\sigma_i^4\sigma_{i'}^2}{\sigma_{k'}^2\sigma_k^4}\right\}\xrightarrow[N\to\infty]{a.s.}0,\ \text{RN}_{k'}-\frac{\sigma_{nr}^2}{NP_S\sigma_k^2}\xrightarrow[N\to\infty]{a.s.}0,$$

$$\text{RRI}_{k'}-\frac{(1+\kappa_{tu}^2)\kappa_{rr}^2\Delta_0}{N\sigma_k^2}\xrightarrow[N\to\infty]{a.s.}0,\ \text{TRI}_{k'}-\frac{\kappa_{tr}^2\Lambda}{NP_S\sigma_{k'}^2\sigma_k^4}\xrightarrow[N\to\infty]{a.s.}0,$$

$$\text{UN}_{k'}-\frac{\sigma_{nu}^2\Lambda}{NP_SP_R\sigma_k^4\sigma_{k'}^4}\xrightarrow[N\to\infty]{a.s.}0\ \text{and}$$

$$\text{URI}_{k'}-\frac{\kappa_{ru}^2}{N}\Big((1+\kappa_{tu}^2)\Big(\sum_{i\neq k}\Big(\frac{\sigma_i^2}{\sigma_k^2}+\frac{\sigma_i^4\sigma_{i'}^2}{\sigma_k^4\sigma_{k'}^2}\Big)+1\Big)+\frac{(1+\kappa_{tu}^2)\kappa_{rr}^2\Delta_0}{\sigma_k^2}+\frac{\sigma_{nr}^2}{P_S\sigma_k^2}+\frac{\kappa_{tr}^2\Lambda}{P_S\sigma_{k'}^2\sigma_k^4}\Big)\xrightarrow[N\to\infty]{a.s.}0. \quad (13)$$

Substituting the expressions from (12) and (13) in (6) gives the closed form SE expression for the k'th user given in (14) (at the top of this page) where $\Omega=\kappa_{tu}^2+\kappa_{ru}^2(1+\kappa_{tu}^2)$ and $\Delta_4=\sum_{i\neq k}\sigma_i^2(\sigma_{k'}^2\sigma_k^2+\sigma_i^2\sigma_{i'}^2)$.

Remark 1. SE insights: We observe from (14) that unlike the relay hardware impairment terms $(\kappa_{tr},\kappa_{rr})$, the user hardware impairment terms $(\kappa_{tu},\kappa_{ru})$ in the denominator gets multiplied by N. Therefore the detrimental impact of user hardware impairments on the SE is much higher than that of the relay hardware impairments. We also notice that the user hardware impairments terms $(\kappa_{tu},\kappa_{ru})$ gets multiplied with the relay hardware impairment terms $(\kappa_{tr},\kappa_{rr})$, relay noise variance (σ_{nr}^2) and user noise variance (σ_{nu}^2) which further degrades the SE. We note that references [3] and [4] did not consider the user impairments and therefore could not provide these crucial insights.

IV. Scaling analysis with hardware imperfections

We next derive the asymptotic SE limit for the link $U_k\to U_{k'}$ for four power scaling schemes (for fixed E_R and E_S)

- Case1: $P_S=E_S$ and $P_R=E_R$.
- Case2: $P_S=E_S/N$ and $P_R=E_R$.
- Case3: $P_S=E_S$ and $P_R=E_R/N$.
- Case4: $P_S=E_S/N$ and $P_R=E_R/N$.

We then investigate the level of relay hardware imperfections that can be tolerated with increase in the number of relay antennas, while maintaining high SE. To this end, similar to [5], we express the relay hardware imperfection parameters as

$$\kappa_{rr}^2=\kappa_{rr0}^2N^z\ \text{and}\ \kappa_{tr}^2=\kappa_{tr0}^2N^z, \quad (15)$$

for some given hardware scaling exponent $z\geq 0$, and some initial value $\kappa\geq 0$. Here z indicates the rate of increase of hardware impairments.

Case1: $P_S=E_S$ and $P_R=E_R$: Substituting $P_S=E_S$, $P_R=E_R$ in (14) we have

$$\bar{R}_{k'}=C\left(\frac{NE_SE_R\sigma_k^4\sigma_{k'}^4}{E_R\sigma_k^2(\Theta_{k'}+NE_S\sigma_{k'}^2\sigma_k^4(\Omega-\frac{2\sigma_{k'}^2}{N\sigma_k^2}))+\sigma_{nu}^2\Lambda_0}\right), \quad (16)$$

where, $\Theta_{k'}=(1+\kappa_{ru}^2)(E_S(1+\kappa_{tu}^2)(\Delta_4+\sigma_i^2\sigma_{i'}^2)+\sigma_k^2\Delta_0\kappa_{rr0}^2N^z)+\sigma_{k'}^2\sigma_k^2\sigma_{nr}^2+\kappa_{tr0}^2N^z\Lambda_0)$ and $\Lambda_0=(1+\kappa_{tu}^2)E_S\Delta_1+(1+\kappa_{tu}^2)\kappa_{rr0}^2N^{z-1}E_S\Delta_2+\frac{\sigma_{nr}^2}{N}\Delta_3$. We next derive the asymptotic SE limit as $N\to\infty$ for different values of relay hardware impairment levels z.

1) $0\leq z<1$: Using (16) as $N\to\infty$ the asymptotic limit on $\bar{R}_{k'}$ is

$$\bar{R}_{k'}^{(11)}=C\Big(\frac{1}{\kappa_{tu}^2+\kappa_{ru}^2(1+\kappa_{tu}^2)}\Big). \quad (17)$$

We observe from (17) that as $N\to\infty$ the SE saturates to $\bar{R}_{k'}^{(11)}$ which depends only on the user hardware impairment terms $(\kappa_{tu},\kappa_{ru})$. This is because both the user hardware impairment terms $(\kappa_{tu},\kappa_{ru})$ in the denominator and the desired signal power in the numerator of (16) gets multiplied by N and hence grows at the same rate as $N\to\infty$. This is unlike the results derived in [3] and [4], where the SE $\to\infty$ as $N\to\infty$. Note that [3] and [4] ignore the user hardware impairments.

$$\bar{R}_{k'}^{(32)}=C\Big(\frac{E_R\sigma_k^4\sigma_{k'}^4}{(E_R\sigma_{k'}^2\kappa_{tr0}^2(1+\kappa_{ru}^2)+\sigma_{nu}^2)(1+\kappa_{tu}^2)(\Delta_1+\kappa_{rr0}^2\Delta_2)+E_R\sigma_k^2\sigma_{k'}^4(\sigma_k^2\Omega+(1+\kappa_{tu}^2)\kappa_{rr0}^2(1+\kappa_{ru}^2)\Delta_0)}\Big). \quad (22)$$

$$\bar{R}_{k'}^{(41)}=C\Big(\frac{E_R\sigma_k^4\sigma_{k'}^4}{E_R\sigma_k^2\sigma_{k'}^4(E_S\sigma_k^2\Omega+(1+\kappa_{ru}^2)\sigma_{nr}^2)+\sigma_{nu}^2(E_S(1+\kappa_{tu}^2)\Delta_1+\sigma_{nr}^2\Delta_3)}\Big). \quad (23)$$

$$\bar{R}_{k'}^{(42)}=C\Big(\frac{E_R\sigma_k^4\sigma_{k'}^4}{E_R\sigma_k^2\sigma_{k'}^4(\sigma_k^2\Omega+(1+\kappa_{ru}^2)(\frac{\sigma_{nr}^2}{E_S}+(1+\kappa_{tu}^2)\kappa_{rr0}^2\Delta_0))+(E_R\sigma_{k'}^2\kappa_{tr0}^2(1+\kappa_{ru}^2)+\sigma_{nu}^2)((1+\kappa_{tu}^2)(\Delta_1+\kappa_{rr0}^2\Delta_2)+\frac{\sigma_{nr}^2}{E_S}\Delta_3)}\Big). \quad (24)$$

2) $z=1$: With $N\rightarrow\infty$ the asymptotic limit on $\bar{R}_{k'}$ is

$$\bar{R}_{k'}^{(12)}=C\Big(\frac{\sigma_k^4\sigma_{k'}^2}{\sigma_k^2\sigma_{k'}^4\Omega+(1+\kappa_{ru}^2)(1+\kappa_{tu}^2)\Psi}\Big), \quad (18)$$

where, $\Psi=\kappa_{tr0}^2(\Delta_1+\kappa_{rr0}^2\Delta_2)+\kappa_{rr0}^2\sigma_k^2\sigma_{k'}^2\Delta_0$. We note that $\bar{R}_{k'}^{(12)}$ now additionally depends on the relay hardware impairment terms $(\kappa_{rr0},\kappa_{tr0})$ which gets multiplied with the large scale fading coefficients of all the $2K$ users $(\Delta_0,\Delta_1,$ and $\Delta_2)$. This further degrades the SE.

3) $z>1$: With $N\rightarrow\infty$ the terms $\mathrm{RRI}_{k'}$, $\mathrm{TRI}_{k'}$ and $\mathrm{URI}_{k'}$ in (13) goes to infinity. Therefore, for $z>1$, $\bar{R}_{k'}\xrightarrow[N\rightarrow\infty]{a.s.}0$. This implies that for $z>1$ the relay hardware impairments dominate the array gain provided by large N.

Case2: $P_S=E_S/N$ and $P_R=E_R$ By substituting $P_S=E_S/N$, $P_R=E_R$ in (14) we next derive the asymptotic SE limit as $N\rightarrow\infty$ for different values of relay hardware impairment levels z.

1) $0\leq z<1$: As $N\rightarrow\infty$ the asymptotic limit on $\bar{R}_{k'}$ is

$$\bar{R}_{k'}^{(21)}=C\Big(\frac{E_S\sigma_k^2}{E_S\sigma_k^2\Omega+(1+\kappa_{ru}^2)\sigma_{nr}^2}\Big). \quad (19)$$

We observe from (19) that the asymptotic SE expression depends on the user hardware impairment terms $(\kappa_{tu},\kappa_{ru})$ and the relay noise variance (σ_{nr}^2) while the relay hardware impairment term $(\kappa_{rr0},\kappa_{tr0})$ along with the user noise variance (σ_{nu}^2) disappears.

2) $z=1$: As $N\rightarrow\infty$ the asymptotic limit on $\bar{R}_{k'}$ $(\bar{R}_{k'}^{(22)})$ is given in (20) (at the top of the previous page). We see that $\bar{R}_{k'}^{(22)}$ now additionally contains relay hardware impairment terms $(\kappa_{rr0},\kappa_{tr0})$ along with the relay noise variance (σ_{nr}^2) and user impairment terms $(\kappa_{ru},\kappa_{tu})$, which degrades the SE when compared with the case $0\leq z<1$.

3) $z>1$: For this case, as $N\rightarrow\infty$ the terms $\mathrm{RHI}_{k'}$, $\mathrm{THI}_{k'}$, $\mathrm{UN}_{k'}$ and $\mathrm{UTI}_{k'}$ in (13) asymptotically become infinity. Hence for $z>1$, we have $\bar{R}_{k'}\xrightarrow[N\rightarrow\infty]{a.s.}0$. This shows that if we degrade the hardware quality by choosing hardware scaling exponent $z>1$, the hardware impairments become severe and the SE approaches zero as $N\rightarrow\infty$.

Case3: $P_S=E_S$ and $P_R=E_R/N$: Substituting $P_S=E_S$ and $P_R=E_R/N$ in (14) we now derive the asymptotic SE when $N\rightarrow\infty$ for different z values as follows:

1) $0\leq z<1$: With $N\rightarrow\infty$ the asymptotic limit on $\bar{R}_{k'}$ is

$$\bar{R}_{k'}^{(31)}=C\Big(\frac{E_R\sigma_k^4\sigma_{k'}^4}{E_R\sigma_k^4\sigma_{k'}^4\Omega+(1+\kappa_{tu}^2)\sigma_{nu}^2\Delta_1}\Big). \quad (20)$$

We observe from (20) that as $N\rightarrow\infty$ the relay hardware impairment terms $(\kappa_{rr0},\kappa_{tr0})$ vanish along with the relay noise variance (σ_{nr}^2) from $\bar{R}_{k'}^{(31)}$. The user impairment terms $(\kappa_{tu},\kappa_{ru})$ and the user noise variance (σ_{nu}^2) nevertheless remain in the asymptotic SE expression. Moreover, the user noise variance (σ_{nu}^2) gets multiplied with the large scale fading coefficients of all $2K$ users (i.e. Δ_1) and the transmit user impairment term (κ_{tu}) in the asymptotic SE expression, which leads to lower SE as compared to Case 1.

2) $z=1$: With $N\rightarrow\infty$ the asymptotic SE expression for this case is given in (22) (at the top of this page). We notice that for this case the relay hardware impairment terms $(\kappa_{rr0},\kappa_{tr0})$ appears in addition to the user noise variance (σ_{nu}^2) and user impairment terms $(\kappa_{ru},\kappa_{tu})$. This further degrades the SE.

3) $z>1$: The terms $\mathrm{RRI}_{k'}$, $\mathrm{TRI}_{k'}$, $\mathrm{UN}_{k'}$ and $\mathrm{URI}_{k'}$ in (13) asymptotically becomes infinity. Hence for $z>1$, we have $\bar{R}_{k'}\xrightarrow[N\rightarrow\infty]{a.s.}0$.

Case4: $P_S=E_S/N$ and $P_R=E_R/N$: By substituting $P_S=E_S/N$ and $P_R=E_R/N$ in (14) the asymptotic SE when $N\rightarrow\infty$ for $0\leq z<1$ and $z=1$ are respectively given by (23) and (24) respectively (at the top of this page). We see from (23) and (24) that both the relay noise variance (σ_{nr}^2) and the user noise variance (σ_{nu}^2) remains and gets multiplied with the relay power (E_R) and user power (E_S) respectively in the asymptotic SE expression. Moreover the impact of relay and user noise is further amplified by the large scale fading coefficients σ_k^2 of all $2K$ users (i.e. Δ_1 and Δ_3) and the user hardware impairment terms $(\kappa_{ru},\kappa_{tu})$. This leads to lowest SE among all the cases. We also notice from (24) that with $N\rightarrow\infty$, the relay hardware impairment terms $(\kappa_{rr0},\kappa_{tr0})$ additionally appears in the asymptotic SE expression. For $z>1$ the SE asymptotically becomes zero.

V. Simulation results

We now numerically investigate the sum SE (in bps/Hz) defined as $\bar{R}=\sum_{k=1}^{2K}\bar{R}_k$ for different values of relay and user impairments, and verify the tightness of the derived closed form expression. We also investigate the sum-SE for different power scaling schemes for different z values. We consider $K=10$ and set i) $\sigma_{nr}^2=\sigma_{nu}^2=1$; ii) $E_S=10$ dB, $E_R=16$ dB with respect to σ_{nr}^2 [8]; iii) $\sigma_k^2=1$ for $k=1,\cdots,2K$ [8].

We plot in Fig. 1 the sum-SE for different relay and user hardware impairment levels and see that the derived closed-form expressions overlap with the ergodic expressions. We observe that sum-SE monotonously increases with N and goes to infinity as $N\rightarrow\infty$, when no user impairments is considered. This is similar to the results in [3] and [4] which ignored the user impairments. We also observe that the sum-SE falls with the increase in the relay impairments. This is because of the relay hardware impairment terms $(\kappa_{tr},\kappa_{rr})$ which appears in

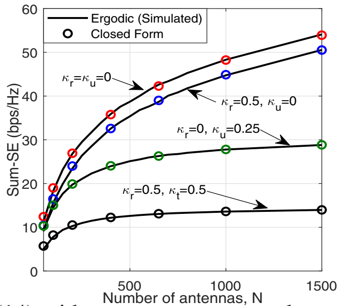
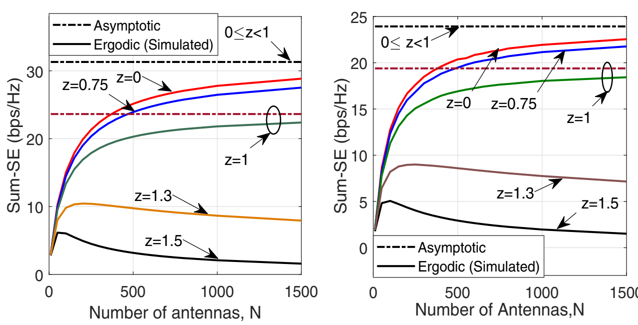


Fig. 1: Validation of (14) with $\kappa_{tr} = \kappa_{rr} = \kappa_r$ and $\kappa_{tu} = \kappa_{ru} = \kappa_u$

the denominator of the SE expression in (14). We however, notice that as soon as we introduce the user impairments, the sum-SE severely degrades. The degradation in the sum-SE with the increase in user impairments is much larger than with the increase in the relay hardware impairments. This is because of the fact that, the user impairment terms $(\kappa_{tu}, \kappa_{ru})$ gets multiplied with N unlike the relay impairment terms $(\kappa_{rr}, \kappa_{tr})$ in the denominator of the SE-expression in (14).

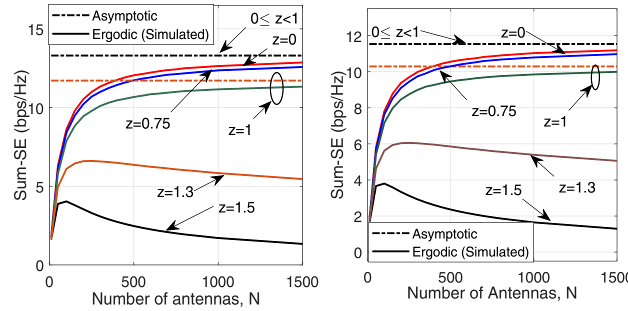
We now plot in Fig. 2a to Fig. 3b the sum SE for the four power scaling cases viz Case1, Case2 Case3 and Case4 respectively, for $0 \leq z < 1$, $z = 1$ and $z > 1$. For this numerical study we set $\kappa_{ru} = \kappa_{tu} = 0.25$ and $\kappa_{tr0} = \kappa_{rr0} = 0.05$. We notice from Fig. 2a, where we plot the SE versus N for the Case1, that for $0 \leq z < 1$ the SE initially increases with N, and then saturates to the derived asymptotic limit as $N \to \infty$. This is because of the fact that the user impairments terms $(\kappa_{tu}, \kappa_{ru})$ gets multiplied by N in the denominator and hence grows at the same rate as the signal power in the numerator. This is unlike [3] and [4], which ignored the user impairments, where the SE goes to infinity as $N \to \infty$. The SE for $z = 1$ also saturates to a value lower than $0 \leq z < 1$ case. This is because, as observed from (18), the relay hardware impairment terms additionally appears in $\bar{R}_{k'}$ for $z = 1$. We also notice that for $z > 1$, the SE asymptotically approaches zero.



(a) **(b)**

Fig. 2: SE vs N: a) Case1; and b) Case2.

We next plot sum-SE vs N for power scaling the Case2 in Fig. 2b and observe that for $0 \leq z \leq 1$ the sum-SE initially increases with N and then saturates to the derived asymptotic limits as $N \to \infty$. We, however, notice that the asymptotic limit of the sum-SE for $z = 1$ is around 18.97% lower than that for $0 \leq z < 1$. This is because as observed from (19) and (20), the relay hardware impairment terms vanish for $0 \leq z < 1$ from the SE expression as $N \to \infty$ but does not vanish for $z = 1$. We also see that for $z > 1$ the sum-SE asymptotically approaches zero.



(a) **(b)**

Fig. 3: SE vs N: a) Case3; and b) Case4.

We plot the SE for the Case3 in Fig. 3a and observe that the SE asymptotic limit for both $0 \leq z < 1$ and $z = 1$ is almost half as that of the Case1. We also notice that for $N > 250$, the SE for $0 \leq z < 1$ is only 11.95% higher than $z = 1$. This is because, the $\bar{R}_{k'}$ in (20) depends additionally on the large scale fading coefficients σ_k^2 of all $2K$ users as compared to (19), which dominates the relay hardware impairment terms. We plot the SE for the Case4 in Fig. 3b and observe that Case4 has the lowest SE among all power scaling cases. This is because, the sum-SE depends on both the relay and user noise variance $(\sigma_{nr}^2, \sigma_{nu}^2)$ which further gets amplified by the relay power (E_R), user power (E_S) and the large scale fading coefficients σ_k^2 of all $2K$ users $(\Delta_1$ and $\Delta_3)$.

VI. Conclusion

We derived a closed-form SE expression for two-way AF mMIMO relaying, incorporating both relay and user hardware impairments. We show that the SE asymptotically saturates to a finite value as number of relay antennas goes to infinity, primarily due to the user hardware impairments. We also derived asymptotic SE expressions for four power scaling schemes with different hardware impairment levels. We verified that as $N \to \infty$ the ergodic SE approaches the derived asymptotic SE. We also examined how fast hardware impairments can be increased while maintaining a high SE.

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