Physical Layer Secrecy Performance Analysis of Imperfect Feedback Based 4×1 MISO System

Sunil Kumar†, Ankit Garg*, and Manav R. Bhatnagar†
†Department of Electrical Engineering, Indian Institute of Technology-Delhi, New Delhi, India 110016
*Department of Electronics and Communication Engineering, Jaypee Institute of Information Technology, Noida, India 201309
Emails:{kumar85sunil,ankit88garg}@gmail.com, manav@ee.iitd.ac.in

***Abstract*—In this paper, we analyze the physical layer secrecy performance of two-bit feedback based 4×1 multiple-input-single-output communication system in the presence of a single antenna equipped passive eavesdropper over Rayleigh faded channel. To achieve full-rate and spatial diversity real orthogonal space-time block code is employed in the considered wiretap communication system. In a practical wireless communication system, it is not always possible to acquire full channel knowledge. Thereby, in this paper, we exploit the partial channel state information at the transmitter with the help of two-bit quantized feedback based diagonal precoder. Further, exact closed-form expressions of the secrecy outage probabilities are derived over perfect and imperfect feedback bits by using order statistics. It is shown in the numerical results that an optimized power allocation scheme successfully improves secrecy performance gain under erroneous feedback in comparison to other schemes.**

I. Introduction

Due to unguided transmission medium, wireless communication systems are susceptible to malicious eavesdropping. A within range passive eavesdropper intercepts the transmit information without detected by the transmitter and receiver. To secure data and avoid a higher-layer cryptographic and encryption techniques, physical layer security has recently gained interest in wiretap communication systems [1], [2]. Multiple-input multiple-output (MIMO) technique has been exploited to enhance the secrecy performance of wireless communication [3]. It is well known that the beamforming technique achieves perfect secrecy gain (i.e., non-zero secrecy capacity) even if the quality of the eavesdropper's channel is higher than that of the main channel [4]. However, in a practical wireless communication system, it is not always possible to acquire full channel state information (CSI) of legitimate and eavesdropping channels (especially for purely passive eavesdroppers), to perform beamforming. Thus, transmit antenna selection (TAS) schemes are proposed in [5]–[10] that uses few feedback bits to convey the partial CSI at the transmitter (CSIT). In [11], two TAS based MIMO wiretap system is studied employing Alamouti space-time code (STBC). In that, it is shown that the secrecy performance of the Alamouti based STBC is superior to that of best TAS (BTAS) scheme. All the aforementioned works consider perfect feedback based TAS. However, due to the fading nature of the wireless feedback link, it is not always possible for the transmitter (Alice) to receive error free feedback bits that are sent from the receiver (Bob). This may severely degrade the secrecy performance of the system depending on the amount of feedback error. Therefore, it is imperative to study the effect of feedback error on the secrecy performance of MIMO wireless systems that uses TAS. Recently, in [12] investigated the secrecy performance of a one-bit (*erroneous*) feedback based diagonal precoding scheme over a 2×1 in the presence of a single antenna equipped eavesdropper (Eve). An optimized transmit power allocation scheme was proposed to attain the diversity order in the presence of imperfect feedback information. However, in higher order MISO systems (*more than two transmit antennas*), it is quite challenging to find optimum power allocation (OPA) corresponding to the erroneous feedback. Besides that, the main challenge is to deal with the system involving a significant mathematical complexity, and allocating transmit powers using involved order statistics. Motivated by the aforementioned discussion, in this paper, we study the secrecy performance of a 4×1 multiple-input single-output (MISO) system with imperfect feedback in the presence of a single antenna equipped eavesdropper (Eve). To achieve full-rate and spatial diversity real orthogonal space-time block code (ROSTBC) is employed in the considered wiretap communication system. An OPA scheme is provided with the help of a diagonal precoder to optimize the secrecy performance of the considered system in the presence of feedback errors.

In this paper, an OPA scheme is employed to retain the full diversity in the case of imperfect feedback bits. The exact closed-form expression of secrecy outage probabilities (SOPs) of the OPA scheme are derived under perfect and imperfect feedback involving order statistics. The derived expression is used to evaluate the SOPs of BTAS scheme and uniform power allocation (UPA) for the considered wiretap system. Numerical results show that imperfect feedback based BTAS scheme looses diversity gain. Whereas, the OPA scheme provides a full diversity and also significant secrecy outage coding gain over the UPA scheme under perfect and imperfect feedback.

II. Preliminaries

A. System Model

We consider a ROSTBC precoded MISO wiretap communication system that consists of four transmit antennas at the Alice and a single antenna at legitimate and unintended receiver (i.e., Bob and Eve, respectively) as shown in Fig. 1. We assume that the main channel and the eavesdropper's channel are subject to quasi-static block Rayleigh fading,

978-1-7281-8895-9/20/$31.00

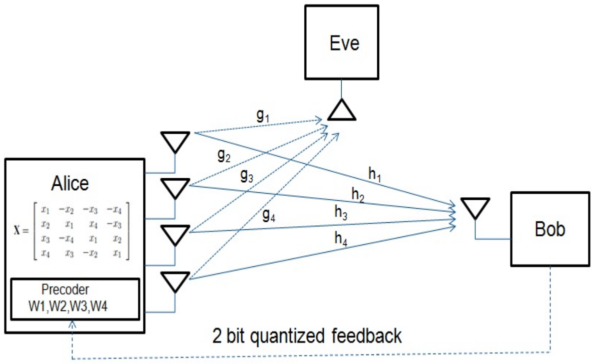

Fig. 1. Block diagram of a 4×1 MISO system with precoder, quantized feedback, and ROSTBC.

where each block covers four symbol intervals, and Alice, Bob, and Eve are static. In quasi-static environments, the channel remains constant during transmission of an encoded ROSTBC block. Therefore, the transmitter can acquire the CSI using a feedback link and update it in every frame. Further, all the three nodes operate over a Time Division Multiple Access (TDMA) system in half duplex mode.

Let x_1, x_2, x_3, and x_4 to be the four symbols from a real signal constellation with respective Power per Symbol. $E\left\{|x_1|^2\right\} = E\left\{|x_2|^2\right\} = E\left\{|x_3|^2\right\} = E\left\{|x_4|^2\right\} = P_s$, where $E\left\{\cdot\right\}$ stands for the expectation operator. The code structure of the ROSTBC for four transmit antennas [13] is

$$\mathbf{X} = \begin{bmatrix} x_1 & -x_2 & -x_3 & -x_4 \\ x_2 & x_1 & x_4 & -x_3 \\ x_3 & -x_4 & x_1 & x_2 \\ x_4 & x_3 & -x_2 & x_1 \end{bmatrix}. \quad (1)$$

The ROSTBC signal matrix is proportional to unitary matrix, satisfies the condition $\mathbb{E}[\mathbf{X}\mathbf{X}^T] = P_t\mathbf{I}_4$, where P_t is the total transmit power at Alice, $(\cdot)^T$ denotes transpose operator, $\mathbf{I}_4$ is the 4×4 identity matrix, and $\mathbb{E}[\cdot]$ is the expectation operator. The ROSTBC matrix is of full rate which implies that four symbols are transmitted on four symbol intervals. The legitimate user estimate the channel with the help of training sequence or by pilot symbols and feedback the decision in form of quantized two bits, to Alice with no delay for selecting the appropriate Diagonal Precoder matrix for appropriate power allocation to the corresponding transmit antennas. The Precoder matrices $\{\mathbf{W}_1, \mathbf{W}_2, \mathbf{W}_3, \mathbf{W}_4\}$ are the diagonal precoding matrices defined as: $\mathbf{W} = \text{diag}\left\{w_1, w_2, w_3, w_4\right\}$, where $\text{diag}\left\{\cdot,\cdot,\cdot,\cdot\right\}$ denotes a 4×4 diagonal matrix, and $w_1^2 + w_2^2 + w_3^2 + w_4^2 = 1$; w_1^2, w_2^2, w_3^2, and w_4^2 represent the transmit power allocation to the respective transmit antenna. Due to the broadcast nature, wireless feedback link is inherently insecure. The Eve within the range of a wireless transmission obtains information about the precoder selected. However, due to independence of the legitimate channel and the eavesdropper channel, the exclusive precoder selected is optimum only for the Bob. Hence, the proposed feedback based precoding scheme improves the secrecy of the wiretap channel. Assuming Eve's distance from Alice is known and the path loss exponent is known and all the assumptions about average SNR of the Eve is same as in [11], which can be justified in several practical scenarios. The received signal vector at Bob and Eve are, respectively, given by:

$$\mathbf{y}_B = \mathbf{h}^T\mathbf{W}_k\mathbf{X} + \mathbf{e}, \quad \mathbf{y}_E = \mathbf{g}^T\mathbf{W}_k\mathbf{X} + \mathbf{n}, \quad (2)$$

where $k = 1, 2, 3, 4$, $\mathbf{h} = [h_1, h_2, h_3, h_4]^T$, and $\mathbf{g} = [g_1, g_2, g_3, g_4]^T$. Then $\mathbf{e}$ and $\mathbf{n}$ are 1×4 noise vectors whose elements are circularly symmetric complex Gaussian random variables with zero-mean and N_0 variance, respectively.

III. Precoder Design

The single parameter based diagonal precoder is used with ROSTBC signal transmission to improve the performance metric of the the system. This diagonal precoder matrices contains only the diagonal weighting components which are used for proportional power allocation to the respective antennas. In this section, we design the optimal diagonal precoding scheme [14] for 4×1 MISO system for the transmission of real signal constellation based ROSTBC.

Receiver after estimating the channel, transmits two bits of quantized feedback to the transmitter for the selection of appropriate diagonal precoder matrix out of the following diagonal precoding matrices:

$$\mathbf{W}_1 = \text{diag}\left\{a_1, a_2, a_2, a_2\right\} \text{ or } \mathbf{W}_2 = \text{diag}\left\{a_2, a_1, a_2, a_2\right\} \text{ or}$$
$$\mathbf{W}_3 = \text{diag}\left\{a_2, a_2, a_1, a_2\right\} \text{ or } \mathbf{W}_4 = \text{diag}\left\{a_2, a_2, a_2, a_1\right\}. (3)$$

Here a_1^2 is an Power Allocation parameter defined as $a_1 = a$, $a_2 = \left(\frac{1-a^2}{3}\right)^{1/2}$ where a_1^2 satisfies $0.25 \leq a_1^2 \leq 1$. For $a_1^2 = 0.25$, system works as an unweighted UPA transmission MISO system and $a_1^2 = 1$, system works as BTAS transmission MISO system. The precoder is selected according to the feedback bits received by the transmitter as per the selection rule stated in reference [14]. In practice the feedback signal also degrade with fading and consequently transmitter selects wrong precoder. Let $X_i = \frac{P_t|h_i|^2}{N_o}$ $\forall\ i \in 1, 2, 3, 4$ are the exponential distributed random variables (RVs) with $\bar{\gamma}_B$ as parameter, where $\bar{\gamma}_B = P_t\Omega_h/N_o$ is the average SNR of the legitimate channel. These random variables are used to define order statistics as follows.

$$\begin{aligned} X_{(1)} &= \min(X_1, X_2, X_3, ..., X_N) \\ X_{(2)} &= 2^{nd} \text{ smallest of } X_1, X_2, ..., X_N \\ &\vdots \\ X_{(N)} &= \max(X_1, X_2, X_3, ..., X_N) \end{aligned}$$

where N is the numbers of RVs. The joint PDF of these ordered random variables (RVs) is defined as follows

$$f_{X_{(1)},X_{(2)},...,X_{(N)}}(x_1, x_2, ..., x_N) = N! f(x_1) f(x_2) \cdots f(x_N). \quad (4)$$

Let $K, L, M, N \in \{X_1, X_2, X_3, X_4\}$, and $K = X_{(4)} \geq L = X_{(3)} \geq M = X_{(2)} \geq N = X_{(1)}$. The joint PDF of these ordered random variables K, L, M, N are defined as in equation (5)

$$f_{K,L,M.N}(k, l, m, n) = \frac{24}{\bar{\gamma}_B^4} \exp\left(-\frac{k + l + m + n}{\bar{\gamma}_B}\right). \quad (5)$$

If feedback is received error-free, then the proposed precoding scheme multiplies a_1^2 to the random variable K and a_2^2 to other remaining RVs. However, there are three cases of wrong transmit antenna selection under erroneous feedback, depicted as $wr1$, $wr2$, and $wr3$ in the below equations. The received instantaneous SNR for the correct and wrong choices, can be expressed by using (3) [14] as shown in (6)

$$\begin{aligned}\Gamma_{B_c}^{OPA} &= a_1^2K + a_2^2L + a_2^2M + a_2^2N\\ \Gamma_{B_{wr1}}^{OPA} &= a_2^2K + a_1^2L + a_2^2M + a_2^2N\\ \Gamma_{B_{wr2}}^{OPA} &= a_2^2K + a_2^2L + a_1^2M + a_2^2N\\ \Gamma_{B_{wr3}}^{OPA} &= a_2^2K + a_2^2L + a_2^2M + a_1^2N. \end{aligned} \tag{6}$$

The SNR at Eve is independent of the feedback sent by Bob. This can be expressed as:

$$\Gamma_E^{OPA} = a_1^2Y_1 + a_2^2Y_2 + a_2^2Y_3 + a_2^2Y_4 \tag{7}$$

where $Y_i \in \left\{\frac{P_t}{N_0}|g_1|^2, \frac{P_t}{N_0}|g_2|^2, \frac{P_t}{N_0}|g_3|^2, \frac{P_t}{N_0}|g_4|^2\right\}$. Each Y_i's is exponential distributed with parameter $\bar{\gamma}_E$, where $\bar{\gamma}_E = P_t\Omega_g/N_o$ is the average SNR of eavesdropper channel.

IV. SOP Analysis

Secrecy Outage Probability (SOP) is defined as the probability that the Secrecy Capacity (C_s) is less than the target Secrecy rate (R_s). Secrecy Capacity is defined as the difference between capacities of the legitimate user and eavesdropper. SOP can be obtained by CDF and PDF expressions for Bob's SNR and Eve's SNR respectively. The CDF expression of SNR at Bob depend upon reliability of feedback. So, there are four possible cases depending upon the quality of feedback.

A. CDF & PDF Analysis at the Receivers

The CDF of SNR at Bob under correct feedback (6) is defined as

$$F_{\Gamma_{B_c}}^{OPA}(\gamma) = \Pr\left(a_1^2K + a_2^2L + a_2^2M + a_2^2N \le \gamma\right)$$

$\Pr[\cdot]$ denotes the probability operator. As $K \ge L \ge M \ge N \ge 0$ the volume bounded by the planes $K = L, L = M, M = N, N = 0$ and $a_1^2K + a_2^2L + a_2^2M + a_2^2N = \gamma$ can be specified by the following integration.

$$F_{\Gamma_{B_c}}^{OPA}(\gamma) = \int_{n=0}^{\gamma}\int_{m=n}^{\frac{\gamma-a_2^2n}{1-a_2^2}}\int_{l=m}^{\frac{\gamma-a_2^2m-a_2^2n}{a_1^2+a_2^2}}\int_{k=l}^{\frac{\gamma-a_2^2l-a_2^2m-a_2^2n}{a_1^2}} \times f_{KLMN}(k,l,m,n)\; dk\, dl\, dm\, dn, \tag{8}$$

making use of (5) in (8) and solving the integral gives the following expression.

$$\begin{aligned}F_{\Gamma_{B_c}}^{OPA}(\gamma) &= 1 - e^{-\frac{4\gamma}{\bar{\gamma}_B}} - \frac{4}{A_4}e^{-\frac{3\gamma}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right)\\ &+\frac{6}{{A_2}^3}e^{-\frac{2\gamma}{(a_1^2+a_2^2)\bar{\gamma}_B}}\left(1-e^{-\frac{2\gamma A_2}{\bar{\gamma}_B}}\right) - \frac{4}{{A_3}^3}e^{-\frac{\gamma}{a_1^2\bar{\gamma}_B}}\left(1-e^{-\frac{3\gamma A_3}{\bar{\gamma}_B}}\right)\\ &-\frac{12}{A_1{A_2}^3}e^{-\frac{2\gamma}{(a_1^2+a_2^2)\bar{\gamma}_B}}e^{-\frac{\gamma A_2}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right)\\ &+\frac{12}{A_4{A_3}^2}e^{-\frac{\gamma}{a_1^2\bar{\gamma}_B}}e^{-\frac{2\gamma A_3}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right). \end{aligned} \tag{9}$$

Now, considering the situation when feedback received incorrectly due to fading nature of the channel. The expression of CDF are calculated by following the same guideline as in the case of correct feedback. The expressions for the three cases under erroneous feedback are given as below.

Case 1.

$$F_{\Gamma_{B_{wr1}}}^{OPA}(\gamma) = \Pr\left(a_2^2K + a_1^2L + a_2^2M + a_2^2N \le \gamma\right)$$

$$\begin{aligned}F_{\Gamma_{B_{wr1}}}^{OPA}(\gamma) &= 1 - e^{-\frac{4\gamma}{\bar{\gamma}_B}} - \frac{4}{A_4}e^{-\frac{3\gamma}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right)\\ &-\frac{6}{{A_2}^3}e^{-\frac{2\gamma}{(a_1^2+a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{2\gamma A_2}{\bar{\gamma}_B}}\right) + \frac{24}{{A_5}^3}e^{\frac{-\gamma}{a_2^2\bar{\gamma}_B}}\left(1 - e^{\frac{\gamma A_5}{\bar{\gamma}_B}}\right)\\ &+\frac{12}{A_1{A_2}^3}e^{-\frac{2\gamma}{(a_1^2+a_2^2)\bar{\gamma}_B}}e^{-\frac{\gamma A_2}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right)\\ &+\frac{24}{A_4{A_5}^2}e^{\frac{-\gamma}{a_2^2\bar{\gamma}_B}}e^{\frac{\gamma A_5}{(1-a_2^2)\bar{\gamma}_B}}\left(1 - e^{-\frac{\gamma A_4}{\bar{\gamma}_B}}\right). \end{aligned} \tag{10}$$

Case 2.

$$F_{\Gamma_{B_{wr2}}}^{OPA}(\gamma) = \Pr\left(a_2^2K + a_2^2L + a_1^2M + a_2^2N \le \gamma\right)$$

$$\begin{aligned}F_{\Gamma_{B_{wr2}}}^{OPA}(\gamma) = 1 &- \underbrace{\left(1 - \frac{4}{A_4} - \frac{12B}{A_5} - \frac{12}{a_2^2{A_5}^3}\right)}_{\alpha_1} e^{-\beta_1\gamma}\\ &-\underbrace{\frac{4\left(1 + 3a_2^2A_6\right)}{A_4}}_{\alpha_2} e^{-\beta_2\gamma} - \underbrace{\frac{12\left(2 + a_1^2 - 2a_2^2\right)}{a_2^2{A_5}^3}}_{\alpha_3} e^{-\beta_3\gamma}\\ &-\underbrace{\frac{12}{a_2^2{A_5}^2\bar{\gamma}_B}}_{\alpha_4}\gamma e^{-\beta_3\gamma}, \end{aligned} \tag{11}$$

where β_1, β_2 and β_3 are defined as below.

$$\beta_1 = \frac{4}{\bar{\gamma}_B} \qquad \beta_2 = \frac{3}{(1-a_2^2)\bar{\gamma}_B} \qquad \beta_3 = \frac{1}{a_2^2\bar{\gamma}_B}$$

Case 3.

$$F_{\Gamma_{B_{wr3}}}^{OPA}(\gamma) = \Pr\left(a_2^2K + a_2^2L + a_2^2M + a_1^2N \le \gamma\right)$$

$$\begin{aligned}F_{\Gamma_{B_{wr3}}}^{OPA}(\gamma) &= 1 - \left(1 + 4\left(\frac{1}{A_5} + \frac{1}{A_7} + \frac{1}{A_8}\right)\right)e^{-\frac{4\gamma}{\bar{\gamma}_B}} + 4\left\{\frac{1}{A_5}\right.\\ &\left.\times\left(1 + \frac{\gamma}{a_2^2\bar{\gamma}_B} + \frac{\gamma^2}{2a_2^4\bar{\gamma}_B^2}\right) + \frac{1}{A_7}\left(1 + \frac{\gamma}{a_2^2\bar{\gamma}_B}\right) + \frac{1}{A_8}\right\}e^{-\frac{\gamma}{a_2^2\bar{\gamma}_B}}, \end{aligned} \tag{12}$$

where $A_1, A_2, A_3, A_4, A_5, A_6, A_7$ and A_8 used in the above equations (9), (10), (11) and (12) are given as:

$$A_1 = \frac{1-2a_2^2}{1-a_2^2} \quad A_3 = \frac{a_1^2 - a_2^2}{a_1^2} \quad A_5 = \frac{a_1^2 - a_2^2}{a_2^2}$$

$$A_2 = \frac{a_1^2 - a_2^2}{a_1^2 + a_2^2} \quad A_4 = \frac{a_1^2 - a_2^2}{1 - a_2^2} \quad A_6 = \frac{1 + a_1^2 - 2a_2^2}{\left(a_1^2 - a_2^2\right)^2}$$

$$A_7 = \frac{\left(a_1^2 - a_2^2\right)^2}{a_2^2} \qquad A_8 = \frac{\left(a_1^2 - a_2^2\right)^3}{a_2^2}.$$

The SNR at Eve is independent of the feedback decision received at Alice. So, CDF expression of SNR at Eve can be written as follows.

$$F_{\Gamma_E}^{OPA}(\gamma) = \Pr\left(a_1^2 Y_1 + a_2^2 Y_2 + a_2^2 Y_3 + a_2^2 Y_4 \leq \gamma\right)$$

$$F_{\Gamma_E}^{OPA}(\gamma) = \int_{u=0}^{\frac{\gamma}{a_2^2}} \int_{t=u}^{\frac{\gamma - a_2^2 y}{a_1^2}} f_{Y_1 Y}(y_1, y) dy_1 \, dy, \quad (13)$$

where $f_{Y_1Y}(y_1, y) = f_{Y_1}(y_1) f_Y(y)$ because Y_1 and Y are statistically independent.

$$Y = Y_2 + Y_3 + Y_4.$$

The PDF of Y is $\chi^2(6)$ distributed.

$$f_Y(\gamma) = \frac{\gamma^2}{2\bar{\gamma}_E^3} e^{-\frac{\gamma}{\bar{\gamma}_E}} \quad \forall \quad \gamma \geq 0 \quad (14)$$

making use of (14) in (13) and solving the integral gives the CDF expression as below.

$$F_{\Gamma_E}^{OPA}(\gamma) = \frac{1}{2\bar{\gamma}_E^3}\left[2\bar{\gamma}_E^3 - \bar{\gamma}_E\left(\frac{\gamma^2}{a_2^4} + 2\frac{\gamma\bar{\gamma}_E}{a_2^2} + 2\bar{\gamma}_E^2\right) e^{-\frac{\gamma}{a_2^2\bar{\gamma}_E}} - e^{-\frac{\gamma}{a_1^2\bar{\gamma}_E}}\left\{2\bar{\gamma}_E'^3 - \bar{\gamma}_E'\left(\frac{\gamma^2}{a_2^4} + 2\frac{\gamma\bar{\gamma}_E'}{a_2^2} + 2\bar{\gamma}_E'^2\right) e^{-\frac{\gamma}{a_2^2\bar{\gamma}_E'}}\right\}\right]. \quad (15)$$

The PDF of SNR at Eve can be obtained after differentiation of the respective CDF expression.

$$f_{\Gamma_E}^{OPA}(\gamma) = \underbrace{\frac{1}{2a_2^6\bar{\gamma}_E^3}}_{\zeta_1}\gamma^2 e^{-\tau_1\gamma} - \underbrace{\frac{\bar{\gamma}_E'^2}{a_1^2 a_2^2 \bar{\gamma}_E^4}}_{\zeta_2}\gamma e^{-\tau_2\gamma} - \underbrace{\frac{\bar{\gamma}_E'^3}{a_1^2\bar{\gamma}_E^4}}_{\zeta_3} e^{-\tau_2\gamma} \underbrace{- \frac{1}{2a_2^4\bar{\gamma}_E^3}\left(\frac{1}{a_2^2} + \frac{\bar{\gamma}_E'}{a_1^2\bar{\gamma}_E}\right)}_{\zeta_4}\gamma^2 e^{-\tau_2\gamma} + \underbrace{\frac{\bar{\gamma}_E'^3}{a_1^2\bar{\gamma}_E^4}}_{\zeta_5} e^{-\tau_3\gamma}, \quad (16)$$

where τ_1, τ_2 and τ_3 are defined as:

$$\tau_1 = \frac{1}{a_2^2\bar{\gamma}_E} \qquad \tau_2 = \frac{1}{a_2^2\bar{\gamma}_E'} + \frac{1}{a_1^2\bar{\gamma}_E} \qquad \tau_3 = \frac{1}{a_1^2\bar{\gamma}_E}.$$

$\bar{\gamma}_E' = a_1^2\bar{\gamma}_E/(a_1^2 - a_2^2)$. The CDF expressions for Best Transmit Antenna Selection (BTAS) and Uniform Power Allocation (UPA) can be obtained with $a_1^2 = 1$ and $a_1^2 = 0.25$ respectively.

B. SOP Calculation

SOP of the system model (Fig. 1) is expressed as below.

$$P_{Sout_x}^{OPA}(R_s) = \Pr\left[\log_2\left(\frac{1 + \Gamma_{B_x}^{OPA}}{1 + \Gamma_E^{OPA}}\right) < R_s\right] \quad (17)$$

$$= \int_0^\infty F_{\Gamma_{B_x}}^{OPA}(2^{R_s}(1+y) - 1) f_{\Gamma_E}^{OPA}(y) \, dy, \quad (18)$$

where x indicates either correct or wrong feedback case, R_s is the Secrecy rate and y is dummy variable. The SOP expression for 2^{nd} case of wrong feedback is evaluated as follows.

The CDF expression (11) of SNR at Bob for 2^{nd} case of wrong feedback and PDF of SNR at Eve (16) can be written in compact form as shown below.

$$F_{\Gamma_{B_{wr2}}}^{OPA}(\gamma) = 1 + \sum_{i=1}^{3} \alpha_i e^{-\beta_i\gamma} + \alpha_4\gamma e^{-\beta_3\gamma}. \quad (19)$$

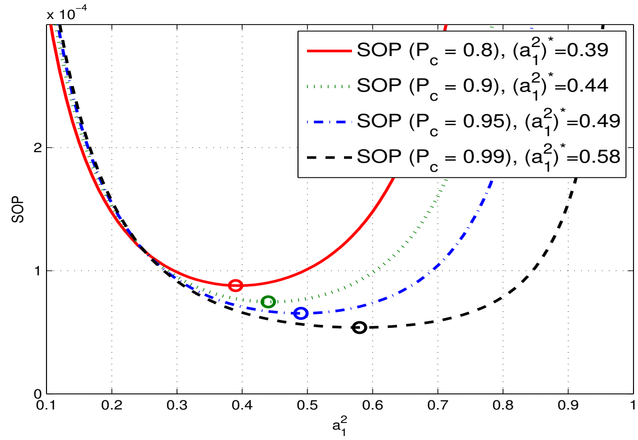

Fig. 2. SOP of the 4×1 MISO system versus a_1^2 with $\bar{\gamma}_E = 5$ dB

$$f_{\Gamma_E}^{OPA}(\gamma) = \zeta_1\gamma^2 e^{-\tau_1\gamma} + \zeta_2\gamma e^{-\tau_2\gamma} + \zeta_3 e^{-\tau_2\gamma} + \zeta_4\gamma^2 e^{-\tau_2\gamma} + \zeta_5 e^{-\tau_3\gamma}. \quad (20)$$

Then the SOP for the above mentioned case can be calculated as follows.

$$P_{Sout_{wr2}}^{OPA}(R_s) = 1 + 2\zeta_1\sum_{i=1}^{3}\frac{\alpha_i e^{-\beta_i(2^{R_s}-1)}}{(\beta_i 2^{R_s} + \tau_1)^3} + \zeta_2\sum_{i=1}^{3}\frac{\alpha_i e^{-\beta_i(2^{R_s}-1)}}{(\beta_i 2^{R_s} + \tau_2)^2} + \zeta_3\sum_{i=1}^{3}\frac{\alpha_i e^{-\beta_i(2^{R_s}-1)}}{(\beta_i 2^{R_s} + \tau_2)} + 2\zeta_4\sum_{i=1}^{3}\frac{\alpha_i e^{-\beta_i(2^{R_s}-1)}}{(\beta_i 2^{R_s} + \tau_2)^3} + \zeta_5\sum_{i=1}^{3}\frac{\alpha_i e^{-\beta_i(2^{R_s}-1)}}{(\beta_i 2^{R_s} + \tau_3)} + \alpha_4 e^{-\beta_3(2^{R_s}-1)}\left(\frac{6\zeta_1}{(\beta_3 2^{R_s} + \tau_1)^4} + \frac{2\zeta_2}{(\beta_3 2^{R_s} + \tau_2)^3} + \frac{\zeta_3}{(\beta_3 2^{R_s} + \tau_3)^2} + \frac{6\zeta_4}{(\beta_3 2^{R_s} + \tau_4)^4} + \frac{\zeta_5}{(\beta_3 2^{R_s} + \tau_5)^2}\right). \quad (21)$$

By following the same procedure as for the 2^{nd} case of wrong feedback, the Secrecy Outage Probability can be found for correct and other wrong feedback cases. The Secrecy Outage Probability can be expressed as follows.

$$P_{Sout}^{OPA}(R_s) = P_c P_{Sout_c}^{OPA}(R_s) + \frac{(1 - P_c)}{3}\sum_{i=1}^{3} P_{Sout_{wri}}^{OPA}(R_s). \quad (22)$$

where P_c is the probability of correct feedback signal and $i \in 1, 2, 3$. The Secrecy Outage Probability of BTAS and UPA scheme are the special cases of OPA scheme which can be evaluated by putting $a_1^2 = 1$ and $a_1^2 = 0.25$ in the above equation (22).

V. Numerical Results and Conclusions

In this section, we provide a detailed discussion upon the proposed optimized transmit power allocation with diagonal precoding scheme for 4×1 system under imperfect quantized two bit feedback and comparison with other scheme such as BTAS and UPA. The results are obtained at average SNR at Eve $(\bar{\gamma}_E) = 5$ dB and Secrecy rate $R_s = 2$ bps/Hz. The analytical plots are obtained by using the derived theoretical results as mentioned in previous sections; the simulation

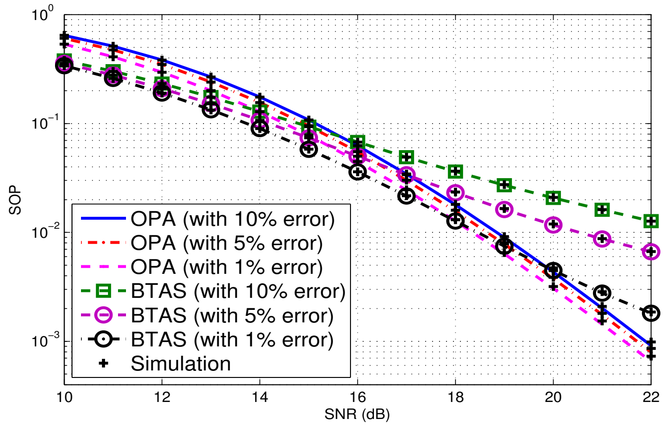

Fig. 3. Analytical and Simulated SOPs of OPA and BTAS Scheme at $\bar{\gamma}_E = 5$ dB for 4×1 MISO system.

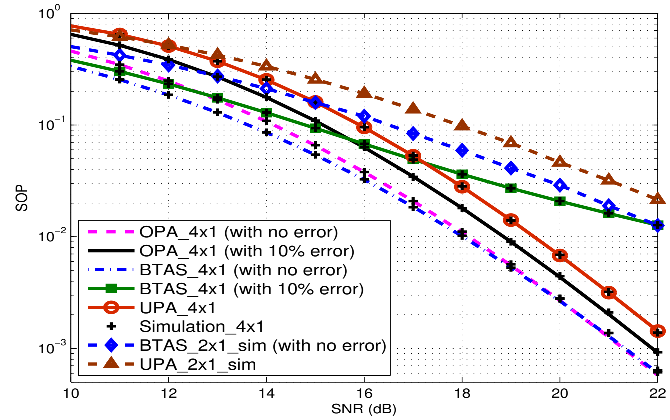

Fig. 4. Analytical & Simulated SOP of 4×1 MISO and Simulated SOP of 2×1 MISO system at $\bar{\gamma}_E = 5$ dB.

results are obtained by simulating the SOP with ROSTBC transmit matrix over uncorrelated Rayleigh fading channels through MATLAB.

The SOP has been plotted as a function of a_1^2 for different values of $P_c = 0.80, 0.90, 0.95, 0.99$ at average SNR at Bob $(\bar{\gamma}_B)$ = 25 dB as shown in Fig. 2. The optimum value of $(a_1^2)^*$ parameter for OPA scheme, which minimizes the SOP are $0.39, 0.44, 0.49$ and 0.58 respectively for 20% $(P_c = 0.80)$, 10% $(P_c = 0.90)$, 5% $(P_c = 0.95)$ and 1% $(P_c = 0.99)$ of feedback error. On the other hand, a_1^2 approaches to unity with increasing value of P_c. The OPA scheme is relatively insensitive to feedback errors and performs better than UPA and BTAS scheme under imperfect feedback as shown in Fig. 3 of BTAS and OPA scheme comparison. The plot has been made for different value of P_c indicates degradation of BTAS scheme with small amount of imperfection in feedback. The simulation and analytical plots of SOP versus SNR (dB) of 4×1 system is shown in Fig. 4 for various transmission schemes along with the simulated results of 2×1 system. We can draw the following inferences regarding 4×1 system: First, The Secrecy Outage Coding (SOC) gain improvement of the BTAS scheme with perfect feedback based precoding scheme is ≥ 1 dB better than the UPA scheme at SOP $\approx 10^{-3}$. However, the performance degrades with the feedback error and becomes poorer than UPA at high SNR. Second, The SOC gain improvement of approximately 1 dB is obtained by the proposed Optimal Power Allocation scheme (no feedback error) compared to the UPA and remains insensitive to the error in the feedback. For example, at SNR $(\bar{\gamma}_B)$= 22 dB, the SOP is 6.1×10^{-4} for BTAS scheme under perfect feedback and it increases to 1.27×10^{-2} under $P_c = 0.9$, where as for the OPA scheme SOP at the same value of average SNR is 6.0×10^{-4} and 9.12×10^{-4} without/with feedback error respectively. The analytical and simulated results are matched which indicate the SOP expressions are accurate. SOP of BTAS scheme with perfect feedback and OPA scheme with/without feedback error gains full SOD of 4 while 2×1 MISO system gains full SOD of 2 in BTAS (with no error) and UPA schemes as shown in Fig. 4. However, BTAS scheme for 4×1 MISO system looses its SOD to unity in case of small percentage of feedback bit error.

References

[1] M. Bloch, J. Barros, M. R. D. Rodrigues, and S. W. McLaughlin, "Wireless information-theoretic security," *IEEE Trans. Inf. Theory*, vol. 54, no. 6, pp. 2515–2534, Jun. 2008.

[2] Y. Shiu, S. Y. Chang, H. Wu, S. C. Huang, and H. Chen, "Physical layer security in wireless networks: a tutorial," *IEEE Commun. Mag.*, vol. 18, no. 5, pp. 66–74, Apr. 2011.

[3] F. Oggier and B. Hassibi, "The secrecy capacity of the mimo wiretap channel," *IEEE Trans. Inf. Theory*, vol. 57, no. 8, pp. 4961–4972, Aug. 2011.

[4] A. Mukherjee and A. L. Swindlehurst, "Robust beamforming for security in MIMO wiretap channels with imperfect CSI," *IEEE Trans. Signal Process.*, vol. 59, no. 1, pp. 351–361, Jan. 2011.

[5] M. Z. I. Sarkar and T. Ratnarajah, "Enhancing security in correlated channel with maximal ratio combining diversity," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6745–6751, Dec. 2012.

[6] N. Yang, P. L. Yeoh, M. Elkashlan, R. Schober, and J. Yuan, "MIMO wiretap channels: Secure transmission using transmit antenna selection and receive generalized selection combining," *IEEE Commun. Lett.*, vol. 17, no. 9, pp. 1754–1757, Sept. 2013.

[7] N. Yang, P. L. Yeoh, M. Elkashlan, R. Schober, and I. B. Collings, "Transmit antenna selection for security enhancement in MIMO wiretap channels," *IEEE Trans. Commun.*, vol. 61, no. 1, pp. 144–154, Jan. 2013.

[8] L. Wang, M. Elkashlan, J. Huang, R. Schober, and R. K. Mallik, "Secure transmission with antenna selection in MIMO Nakagami-m fading channels," *IEEE Trans. Wireless Commun.*, vol. 13, no. 11, pp. 6054–6067, Nov. 2014.

[9] H. Alves, R. D. Souza, M. Debbah, and M. Bennis, "Performance of transmit antenna selection physical layer security schemes," *IEEE Signal Process. Lett.*, vol. 19, no. 6, pp. 372–375, Jun. 2012.

[10] J. Zhu, Y. Zou, G. Wang, Y. D. Yao, and G. K. Karagiannidis, "On secrecy performance of antenna-selection-aided MIMO systems against eavesdropping," *IEEE Trans. Veh. Technol.*, vol. 65, no. 1, pp. 214–225, Jan. 2016.

[11] S. Yan, N. Yang, R. Malaney, and J. Yuan, "Transmit antenna selection with alamouti coding and power allocation in MIMO wiretap channels," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1656–1667, Mar. 2014.

[12] A. Garg, H. K. Boddapati, and M. R. Bhatnagar, "Secrecy performance of imperfect one bit feedback based alamouti MISO system," *IEEE Commun. Lett.*, vol. 21, no. 12, pp. 2690–2693, 2017.

[13] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," vol. 45, no. 5, pp. 1456–1467, Jul. 1999.

[14] A. Garg, M. R. Bhatnagar, O. Berder, and B. Vrigneau, "Order statistics based diagonal precoding for 4×1 MISO system with real orthogonal space time block codes," *2016 International Conference on Signal Processing and Communications (SPCOM)* , pp. 1–5, Jun. 2016.