Algorithms for Coded Random Access and Inference in Large Dimensional Spaces

Jean-Francois Chamberland and Krishna R. Narayanan
Vamsi K. Amalladinne, Asit K. Pradhan, et al.

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This material is based upon work supported, in part, by NSF under Grant No. 1619085

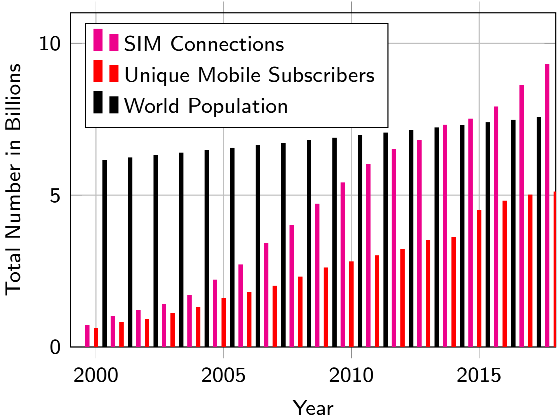
This material is also based upon work support, in part, by Qualcomm Technologies, Inc., through their University Relations Program

Part I

An Evolving Digital Landscape

Mobile Device Market Penetration

There are now more subscribed wireless devices than humans on Earth



Sources: United Nations, GSMA

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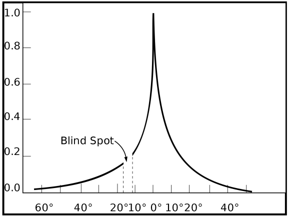
Clarity of Vision – Reaching the Limit



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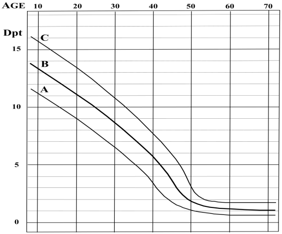

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Visual Resolution

Peak visual resolution of 20/20 human is

$$\frac{1}{\text{Visual Acuity}} = \frac{1}{20/20} \text{ min. of arc}$$

$$\approx 0.0167 \text{ degrees}$$

Sharp drops limit viewing angle to ± 20 degrees

Amplitude of Accommodation

Diopters capture eye adaptability in reciprocal of focal length, Crystalline limits minimum range

Visual Acuity and Display Technology

Apple Super Retina HD

Screen Distance

The distance at which the super retina HD display matches this resolution is

$$\text{Distance} = \frac{1}{2} \cdot \frac{1}{458} \cdot \cot \frac{1}{120}$$
$$= 1.876 \text{ in.}$$

Mobile VR Headsets

©Oculus Rift

Content-Rich Applications

Video and Mobile Statistics

- 63% of all US online traffic comes from smartphones and tablets – Stone Temple
- More than 70% of YouTube viewing happens on mobile devices – Comscore
- 65% of all digital media time is spent on mobile devices – Business2Community

©Real

Options to Stay the Course

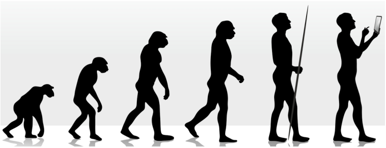
Spend More Time on Mobile Devices

Average time spent on mobile phone in US is 3h45m per day

– eMarketer

©Dreamworks

Wait for Eye Evolution



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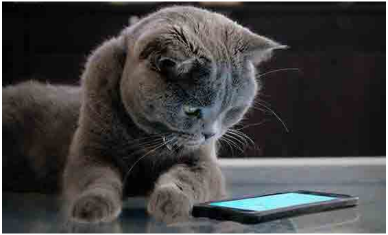
Diversify User Population



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Summary of Quality of Experience

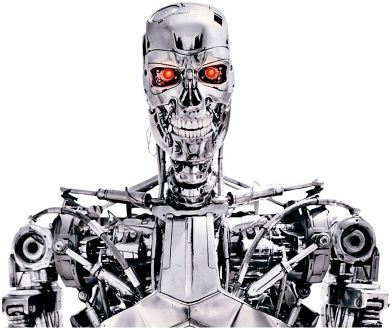
Current Wireless Landscape

- **Growth and Market Penetration**: Near saturation
  - Number of connected wireless devices exceeds world population
  - Almost every human who wants mobile phone has one (or more)
- **Screen Quality**: At limit of eye acuity
  - Screens are near boundary of visual resolution
  - Viewing distance is constrained by amplitude of accommodation
- **Content-Rich Apps**: Video watching & gaming are prevalent
  - On average, a person spends 4 hours on mobile device per day
  - More videos are watch on phones than elsewhere

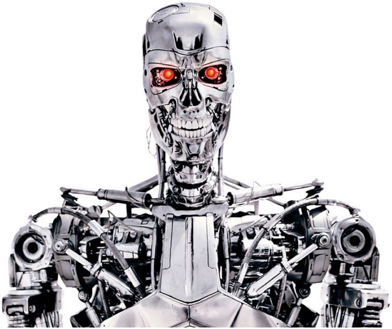
Wireless Research and the Future

What's Next?

The Rise of the Machine



The Rise of the Machine



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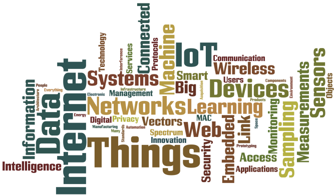
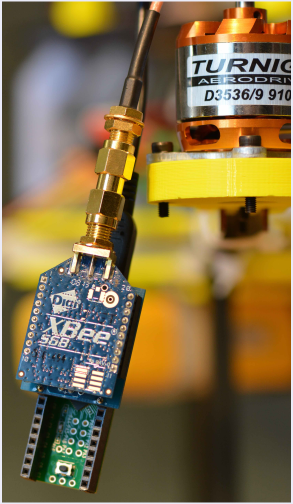


Internet of Things



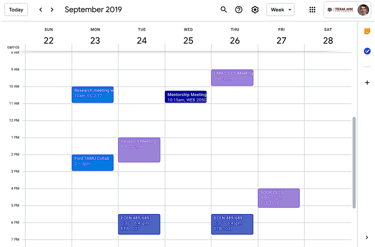
Contrasting Machines and Human Behaviors

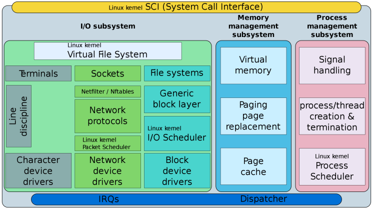
Typical Human Calendar

- YouTube video earns 1 view when watched for $\geq$ 30 sec
- 47% of visitors expect website to load in $\leq$ 2 sec
- Callers notice roundtrip voice delays of $\geq$ 250 ms

Machine Scheduler

- OS timeslice $\approx$ 10 ms
- LTE schedule $\approx$ 1 ms (transmission time interval)
- Microcontroller interrupt latency is $\leq$ 10 μs





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Information and Inference

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 51, NO. 2, FEBRUARY 2003 407

Decentralized Detection in Sensor Networks

Jean-François Chamberland, *Student Member, IEEE,* and Venugopal V. Veeravalli, *Senior Member, IEEE*

Abstract—**In this paper, we investigate a binary decentralized detection problem in which a network of wireless sensors provides relevant information about the state of nature to a fusion center. Each sensor transmits its data over a multiple access channel. Upon reception of the information, the fusion center attempts to accurately reconstruct the state of nature. We consider the scenario where the sensor network is constrained by the capacity of the wireless channel over which the sensors are transmitting, and we study the structure of an optimal sensor configuration. For the problem of detecting deterministic signals in additive Gaussian noise, we show that having a set of identical binary sensors is asymptotically optimal, as the number of observations per sensor goes to infinity. Thus, the gain offered by having more sensors exceeds the benefits of getting detailed information from each sensor. A thorough analysis of the Gaussian case is presented along with some extensions to other observation distributions.**

Index Terms—**Bayesian estimation, decentralized detection, sensor network, wireless sensors.**

problem have been studied in the past. Notably, the class of decentralized detection problems where each sensor must select one of D possible messages has received much attention. In this setting, which was originally introduced by Tenney and Sandell [1], the goal is to find what message should be sent by which sensor and when. See Tsitsiklis [2] and the references contained therein for an elaborate treatment of the decentralized detection problem. More recently, the problem of decentralized detection with correlated observations has also been addressed (see, e.g., [3] and [4]).

In essence, having each sensor select one of D possible messages upper bounds the amount of information available at the fusion center. Indeed, the quantity of information relayed to the fusion center by a network of L sensors, each sending one of D possible messages, does not exceed $L\lceil \log_2 D \rceil$ bits per unit time. In the standard decentralized problem formulation, the number of sensors L and the number of distinct messages D are

Payload Design Guideline

- Most of information for inference is contained in first few bits!

Information and Inference

A Telemetering System by Code Modulation —Δ-Σ Modulation*

H. INOSE†, MEMBER, IRE, Y. YASUDA†, AND J. MURAKAMI‡

Summary—A communication system by code modulation is described which incorporates an integration process in the original delta modulation system and is named delta-sigma modulation after its modulation mechanism. It has an advantage over delta modulation in dc level transmission and stability of performance, although both require essentially an equal bandwidth and complexity of circuitry. An experimental telemetering system employing delta-sigma modulation is also described.

the input signal before it enters the modulator so as to generate output pulses carrying the information corresponding to the amplitude of the input signal. The delta-sigma modulation (Δ-ΣM) system is a realization of this principle.

THE PRINCIPLE OF THE Δ-ΣM SYSTEM

Payload Design Guideline

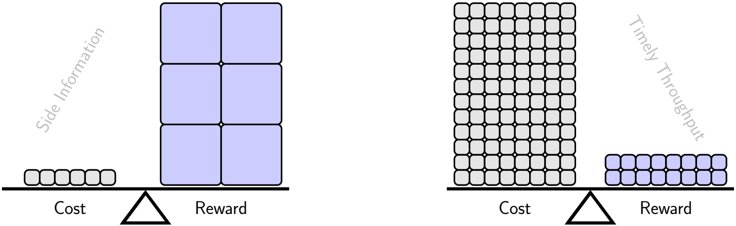
- Signals are tracked well using small, yet frequent updates
- Δ-Σ modulation

Losing the Connection

Emerging M2M Traffic Characteristics

- Device density – Massive versus small
- Connectivity profile – Sporadic versus sustained
- Packet payloads – Minuscule versus moderate-to-long

Anticipated traffic characteristics invalidate the acquisition-estimation-scheduling paradigm!



Revival of Uncoordinated Access

A New Reality

- Must address sporadic nature of machine-driven communications
- Transfer of small payloads without ability to amortize cost of acquiring channel and buffer states over long connections
- Preclude use of opportunistic scheduling
- Evinced by departure from scheduling-based solutions

Communication and Identity

When number of devices is massive, with only subset of them active, problem of allocating resources (e.g., codebook, subcarriers, signature sequences) to every user as to manage interference becomes very complex

Uncoordinated, Unsourced MAC

Coding and Compressed Sensing with Approximate Message Passing Part II - Problem Formulation and Benchmarks

J.-F. Chamberland and Krishna Narayanan

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Electrical and Computer Engineering
Texas A&M University

IISc, India
2020

This material is based upon work supported, in part, by NSF under Grant No. 1619085

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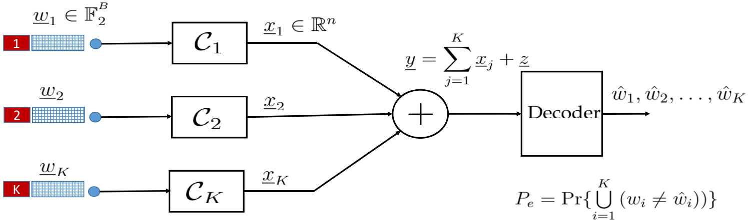
Outline of Part II

- Problem formulation for uncoordinated multiple access
- What information theoretic benchmarks are relevant?
- Very brief review of coding for the Gaussian multiple access channel

Traditional Gaussian multiple access channel (GMAC)

- K users, each user has a B-bit message
- n channel uses
- Classical information theory - fix K and let $n, B \to \infty$



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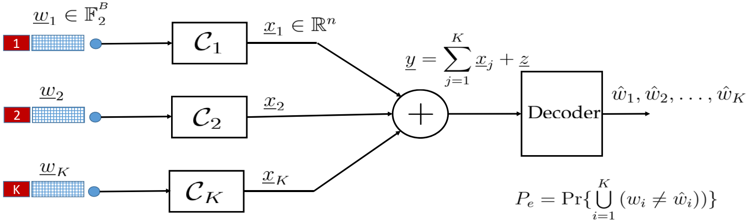
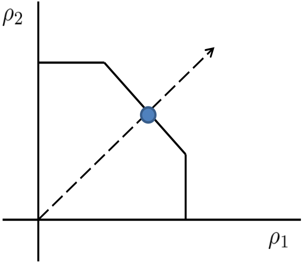


Assumptions

- User identity is conveyed separately
- Resources are allocated based on identity
- Codebooks are different but assumed to be known at the decoder

Capacity of the Traditional GMAC



Achievable rates

$$
\begin{aligned}
\rho_i &< \frac{1}{2}\log\left(1+\frac{P}{\sigma^2}\right), \quad i = 1,\ldots,K \\
\sum_{i=1}^{K}\rho_i &< \frac{1}{2}\log\left(1+\frac{KP}{\sigma^2}\right)
\end{aligned}
$$

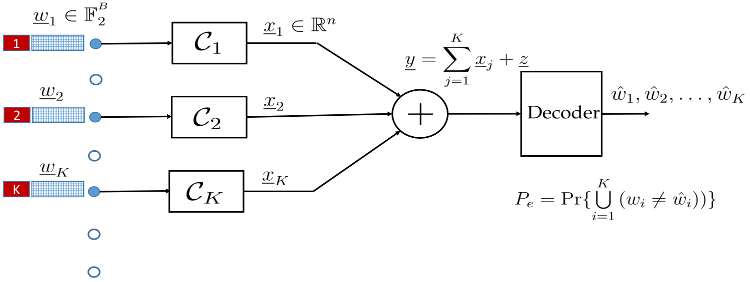
Achieving points on the GMAC region

- K is fixed and $n \to \infty$
- Relies on joint typicality of $X_1, X_2, \ldots, X_k, Y$
- For any subset of RVs, A.E.P holds
- Number of subsets is exponential in K

Many-Access Channels with Random User Activity

Model

- Total # users ℓ_n grows with block length n
- Each user is active independently with probability $\alpha_n \in [0,1]$
- Average # active users $K_n = \alpha_n \ell_n$ is unbounded and grows as $\mathcal{O}(n)$
- ℓ_n does not grow exponentially with n
- Need to identify the set of active users and decode their messages



Chen, Chen, Guo. *Capacity of Gaussian many-access channels*. Transactions on information theory, vol. 63, No. 6, June 2017

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- ℓ_n does not grow exponentially with n

Main Result

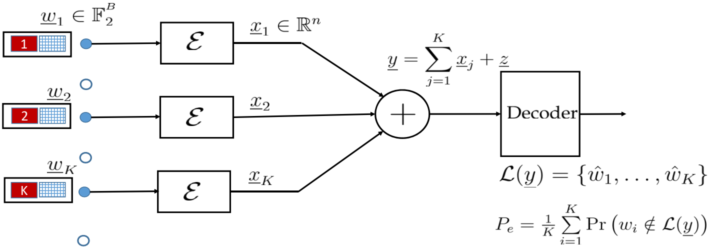
- Symmetric message length capacity

$$C(n) = \max\left(\underbrace{\frac{n}{2K_n}\log\left(1+\frac{K_n P}{\sigma^2}\right)}_{(1)} - \underbrace{\frac{\ell_n H_2(\alpha_n)}{K_n}}_{(2)}, 0\right) \text{ bits/user}$$

- (1): Symmetric capacity of K_n-user MAC with known user activities
- (2): Capacity penalty due to uncertainty in resolving user activities

Chen, Chen, Guo. *Capacity of Gaussian many-access channels*. Transactions on information theory, vol. 63, No. 6, June 2017

Uncoordinated Multiple Access Channel (MAC)



LoRa-Inspired Parameters

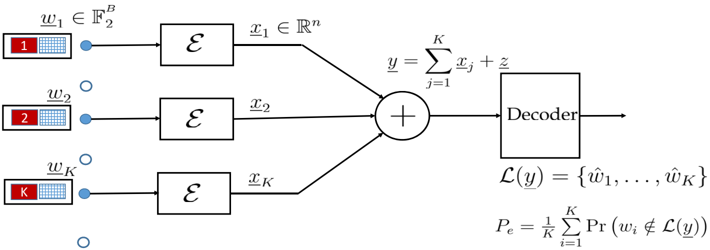
- K active users out of K_{tot} total users, $K \in [25 : 300]$
- Each user has B-bit message, B is small ≈ 100
- N channel uses available, $N \approx 30,000$

M. Berioli, G. Cocco, G. Liva and A. Munari, *Modern Random Access Protocols*. Foundations and Trends in Networking, 2016

F. Clazzer, A. Munari, G. Liva, F. Lazaro, C. Stefanovic, P. Popovski, *From 5G to 6G: Has the Time for Modern Random Access Come?*, arXiv 2019

Uncoordinated Unsourced MAC [Polyanskiy' 17]

- K active users out of K_{tot} total users $K \in [25 : 300]$,
- K_{tot} is very large
- Each user has a B-bit message. B is small ≈ 100
- n channel uses available $n \approx 30,000$

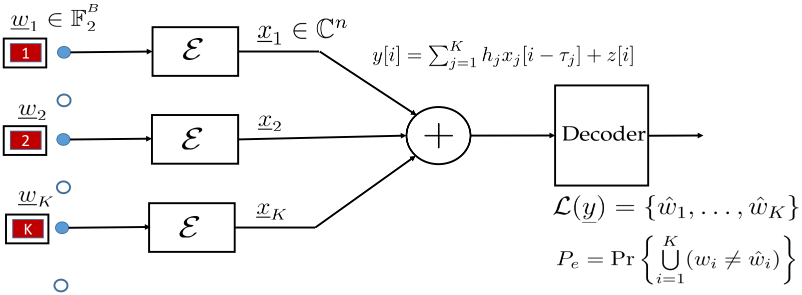


- Uncoordinated/Unsourced: All devices employ same encoder
- Decoding done upto permutation of messages
- Finite block length regime
- Error metric : Per-user error probability (PUPE)

Neighbor discovery

Problem statement

- Identify network interface addresses (NIAs) of nodes within one hop
- Address space is $\{0, 1, \cdots, 2^{48} - 1\}$ ($K_{tot} = 2^{48}$ potentially)



Gaussian Random Codes & Performance Bounds

A perspective on massive random-access

Yury Polyanskiy

Abstract—**This paper discusses the contemporary problem of providing multiple-access (MAC) to a massive number of uncoordinated users. First, we define a random-access code for K_a-user Gaussian MAC to be a collection of norm-constrained vectors such that the noisy sum of any K_a of them can be decoded with a given (suitably defined) probability of error. An achievability bound for such codes is proposed and compared against popular practical solutions: ALOHA, coded slotted ALOHA, CDMA, and treating interference as noise. It is found out that as the number of users increases existing solutions become vastly energy-inefficient.**

MAC [11], [12]). Already 30 years ago R. Gallager [13] called for "a coding technology that is applicable for a large set of transmitters of which a small, but variable, subset simultaneously use the channel." It appears (to this author) that this call has not been completely answered still. One reason for this could be that the models in each of three categories are different and thus solutions are not directly comparable. Our first goal, thus, is to define a notion of random-access code that would appeal to all three communities. This we do next.

Theorem: Fix $P' < P$. There exists an (M, n, ϵ) random-access code for the K-user GMAC satisfying power-constraint P and

$$\epsilon \leq \sum_{t=1}^{K} \frac{t}{K} \min(p_t, q_t) + p_0.$$

Constants p_0, p_t, and q_t are given in Polyanskiy's paper

Y. Polyanskiy. *A Perspective on Massive Random-Access*. ISIT, 2017

Random Coding (RC) Achievability Bound

Encoder

- Generate $M = 2^B$ codewords $\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_{2^B} \overset{i.i.d.}{\sim} \mathcal{N}(0, P'), P' < P$
- Power constraint: If $\|\mathbf{s}_{\mathbf{w}_j}\|_2^2 > nP$, user j transmits $\mathbf{0}$

Random Coding (RC) Achievability Bound

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- Power constraint: If $\|\mathbf{s}_{\mathbf{w}_j}\|_2^2 > nP$, user j transmits $\mathbf{0}$

Decoder

- Decoder outputs set $\hat{S}$ of size K minimizing $\|\mathbf{y} - \sum_{j \in \hat{S}} \mathbf{s}_j\|_2^2$
- Must search through $\binom{2^B}{K}$ possibilities for $\hat{S}$ assuming no collisions

Random Coding (RC) Achievability Bound

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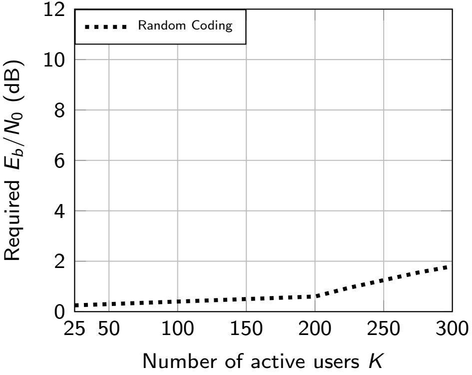
Achievability benchmark

- Per-user probability of error (PUPE) of this scheme is an achievability bound for UMAC
- RC achievability obtained by optimizing over P'

Finite Block Length Bound; $B = 100, n = 30000$

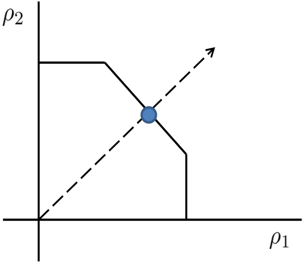


Key insight from the bound

For fixed n,

- Small K - bound is dominated by FBL penalty due to finite B
- Large K - bound is dominated by multi-access interference

Coding for the Traditional GMAC



Achievable sum rate

$$\sum_{i=1}^{K} \rho_i < \frac{1}{2} \log \left(1 + \frac{KP}{\sigma^2}\right)$$

Achieving points on the GMAC region

- Corner points can be achieved using successive interference cancellation
- Any point can be achieved through rate-splitting
- These require coordination among users
- Equal rate point is harder to achieve without coordination

Coding Schemes for the Equal Rate Point

- Time/Frequency/Code Division Multiple Access (T/F/CDMA)
- Ping *et al.* - Interleave division multiple access (IDMA)
- Yedla, Pfister, N. ' 11 - Spatially coupled LDPC
- Truhachev, Schlegel - Spatially coupled MA
- Sasoglu *et al.*'13 - Polar codes for MAC

All these schemes require coordination between users to pick parameters

TDMA/FDMA/CDMA

- TDMA/FDMA
  - Requires coordinated allocation of time/frequency slots
  - Without coordination, there will be collisions

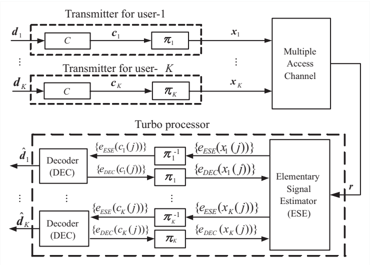
TDMA/FDMA/CDMA

- TDMA/FDMA
  - Requires coordinated allocation of time/frequency slots
  - Without coordination, there will be collisions

- Orthogonal CDMA
  - Users need to be ‘assigned’ spreading sequences
  - $K_{tot} \gg K$ - spreading sequence length will be too large
  - $K_{tot} \approx 10000$, $n = 30000$ and $B = 100$
  - Not enough dimensions for coding

Interleave Division Multiple Access - Ping *et al.*' 06

- Each user encodes with the same code & picks a different interleaver
- Message passing decoding and demodulation
- Close to capacity performance for small number of users



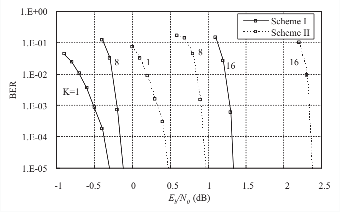
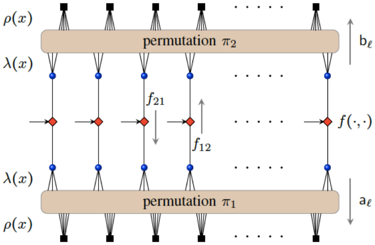


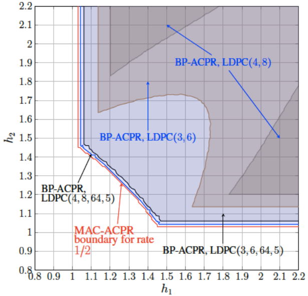
Fig. 7. Performance of IDMA systems based on the turbo-Hadamard code [31] and turbo code over AWGN channels. $N_r = 1$, $It = 30$, $N_{\text{info}} = 4095$ for Scheme I and $N_{\text{info}} = 4096$ for Scheme II.

- The interleavers have to be different and known to the receiver
- Performance is not very good for large number of users

SC-LDPC for GMAC - Yedla, Pfister, N '11

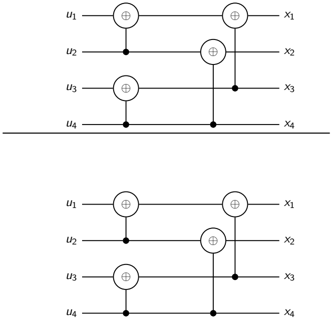
- Spatially coupled LDPC codes with different interleavers
- Empirically shown to be universal for MAC





- Interleavers need to be chosen in a coordinated manner
- Interleavers need to be known at the receiver
- Not a good solution for short block lengths

Polar Codes for MAC - Sasoglu'13



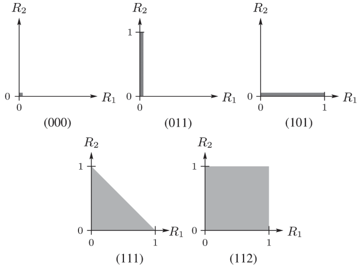


Fig. 1. Capacity regions of the five extremal MACs.

- Polar codes can be optimized for MAC
- Frozen bits have to be chosen in a coordinated fashion

Takeaways

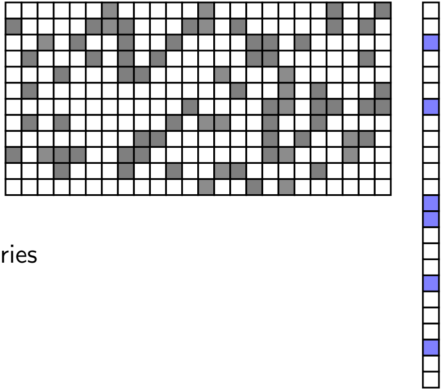
Main points from this part

- Traditional GMAC channel model is not suitable for modeling IoT
- Many access channel models number of users growing with n
- Finite Block Length achievability bounds serve as a good benchmark
- Existing coding schemes for GMAC need to be modified

Part I

A Quest for Low-Complexity: Coded Compressed Sensing

Abstract CS Challenge



Problem setting

- Noisy compressed sensing

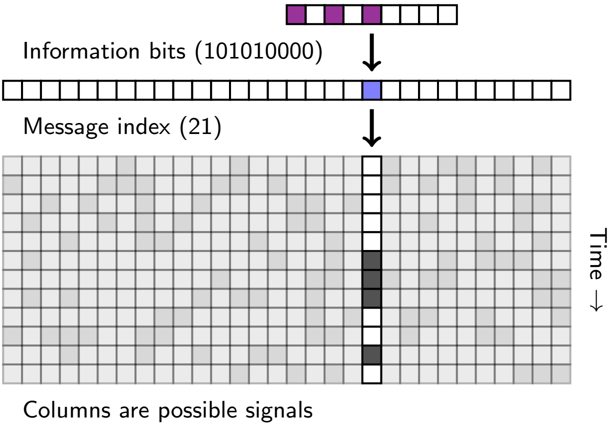
$$y = As + z$$

  where s is K sparse
- s has non-negative integer entries
- $\mathrm{A}.\texttt{shape} \approx 30,000 \times 2^{128}$
- z is additive Gaussian noise

Performance evaluation

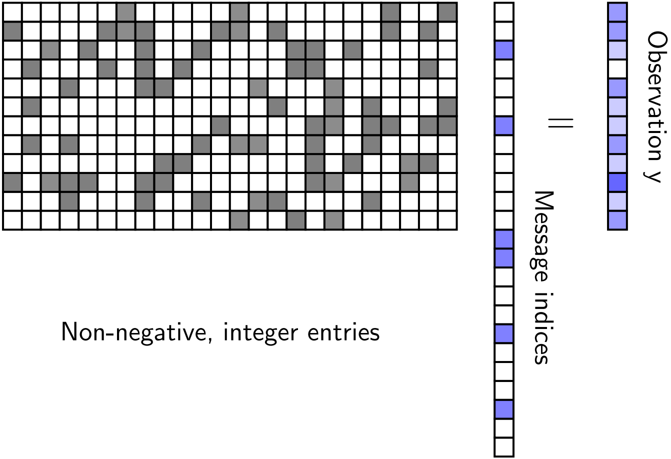
- Number of mistakes in support recovery normalized by K
- Related to the per user probability of error in MAC setting
- Performance target on order of 1–5%

Multiple Perspectives for Individual Messages



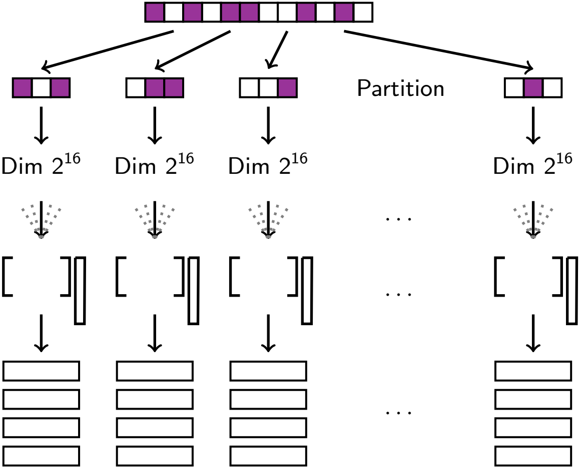
- Binary message or information content
- Integer value of message
- Index representation in vector form

Unified CS/UMAC perspective



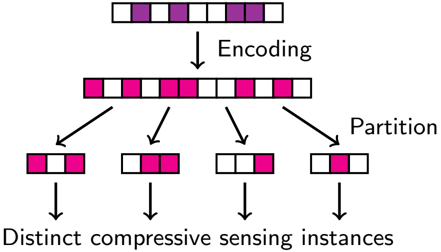
Non-negative, integer entries

Idea: Compressive Sensing Applied to Fragments



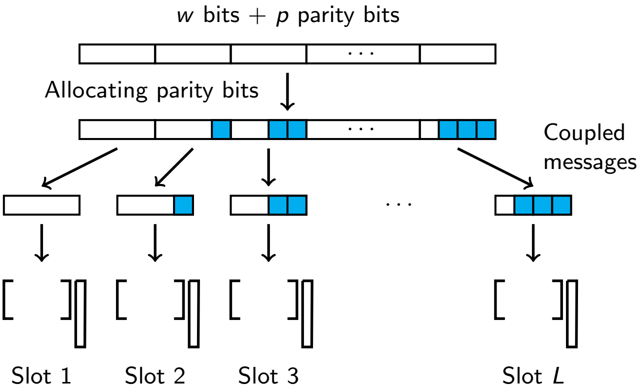
- Issue: unordered lists of fragments!

Idea: Divide and Conquer Information Bits



- Split problem into sub-components suitable for CS framework
- Get lists of sub-packets, one list for every slot
- Stitch pieces of one packet together using error correction

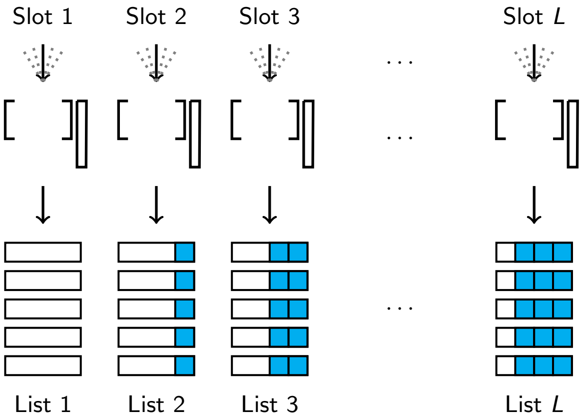
Coded Compressive Sensing – Device Perspective



- Collection of L CS matrices and 1-sparse vectors
- Each CS generated signal is sent in specific time slot

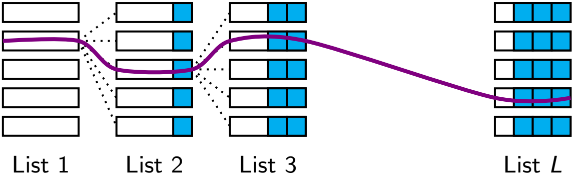
V. Amalladinne, A. Vem, D. Soma, K. R. Narayanan, JFC. *Coupled Compressive Sensing Scheme for Unsourced Multiple Access*. ICASSP 2018

Coded Compressive Sensing – Multiple Access



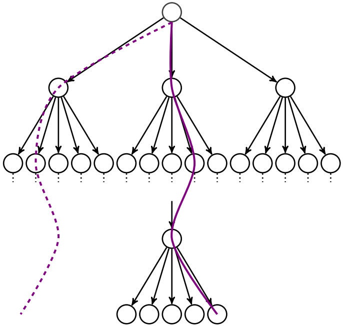
- L instances of CS problem, each solved with non-negative LS
- Produces L lists of K decoded sub-packets (with parity)
- Must piece sub-packets together using tree decoder

Coded Compressive Sensing – Stitching Process

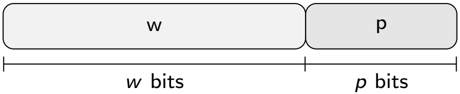


Tree decoding principles

- Every parity is linear combination of bits in preceding blocks
- Late parity bits offer better performance
- Early parity bits decrease decoding complexity
- Correct fragment is on list



Coded Compressive Sensing – Understanding Parity Bits



- Consider binary information vector w of length k
- Systematically encoded using generator matrix G, with $\mathrm{p} = \mathrm{w}\mathrm{G}$
- Suppose alternate vector w_{r} is selected at random from $\{0, 1\}^k$

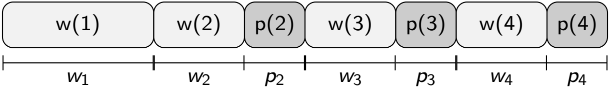
Lemma

Probability that randomly selected information vector w_{r} produces same parity sub-component is given by

$$\Pr(\mathrm{p} = \mathrm{p}_{\mathrm{r}}) = 2^{-\operatorname{rank}(\mathrm{G})}$$

Proof: $\{\mathrm{p} = \mathrm{p}_{\mathrm{r}}\} = \{\mathrm{w}\mathrm{G} = \mathrm{w}_{\mathrm{r}}\mathrm{G}\} = \{\mathrm{w} + \mathrm{w}_{\mathrm{r}} \in \operatorname{nullspace}(\mathrm{G})\}$

Coded Compressive Sensing – General Parity Bits



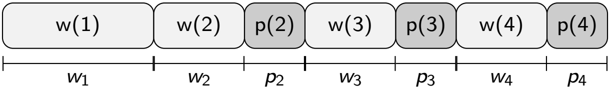
- True vector $(\mathrm{w}_{i_1}(1), \mathrm{w}_{i_1}(2), \mathrm{w}_{i_1}(3), \mathrm{w}_{i_1}(4))$
- Consider alternate vector with information sub-block $(\mathrm{w}_{i_1}(1), \mathrm{w}_{i_2}(2), \mathrm{w}_{i_3}(3), \mathrm{w}_{i_4}(4))$ pieced from lists
- To survive stage 4, candidate vector must fulfill parity equations

$$(\mathrm{w}_{i_1}(1) - \mathrm{w}_{i_2}(1)) \begin{bmatrix} G_{1,2} \end{bmatrix} = 0$$

$$(\mathrm{w}_{i_1}(1) - \mathrm{w}_{i_3}(1), \mathrm{w}_{i_2}(2) - \mathrm{w}_{i_3}(2)) \begin{bmatrix} G_{1,3} \\ G_{2,3} \end{bmatrix} = 0$$

$$(\mathrm{w}_{i_1}(1) - \mathrm{w}_{i_4}(1), \mathrm{w}_{i_2}(2) - \mathrm{w}_{i_4}(2), \mathrm{w}_{i_3}(3) - \mathrm{w}_{i_4}(3)) \begin{bmatrix} G_{1,4} \\ G_{2,4} \\ G_{3,4} \end{bmatrix} = 0$$

Coded Compressive Sensing – General Parity Bits



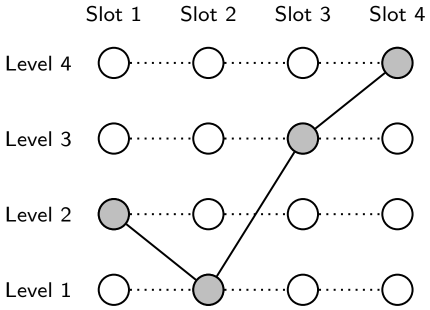
- When indices are not repeated in $(\mathrm{w}_{i_1}(1), \mathrm{w}_{i_2}(2), \mathrm{w}_{i_3}(3), \mathrm{w}_{i_4}(4))$, probability is governed by

$$\mathrm{rank}\left(\begin{bmatrix} G_{1,2} & G_{1,3} & G_{1,4} \\ 0 & G_{2,3} & G_{2,4} \\ 0 & 0 & G_{3,4} \end{bmatrix}\right)$$

- But, when indices are repeated, sub-blocks may disappear

$$\mathrm{rank}\left(\begin{bmatrix} G_{1,2}1_{\{i_2 \neq i_1\}} & G_{1,3}1_{\{i_3 \neq i_1\}} & G_{1,4}1_{\{i_4 \neq i_1\}} \\ 0 & G_{2,3}1_{\{i_3 \neq i_2\}} & G_{2,4}1_{\{i_4 \neq i_2\}} \\ 0 & 0 & G_{3,4}1_{\{i_4 \neq i_3\}} \end{bmatrix}\right)$$

Candidate Paths and Bell Numbers

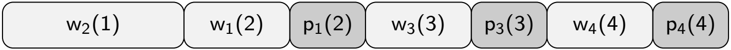


Probability that wrong path is consistent with parities is

$$\Pr(p = p_r) = 2^{-\,\mathsf{rank}(G)}$$

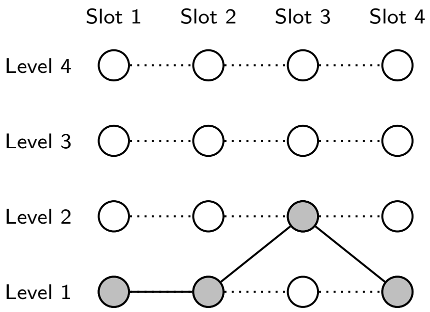
where

$$G = \begin{bmatrix} G_{1,2} & G_{1,3} & G_{1,4} \\ 0 & G_{2,3} & G_{2,4} \\ 0 & 0 & G_{3,4} \end{bmatrix}$$



When Levels Do NOT Repeat

Candidate Paths and Bell Numbers

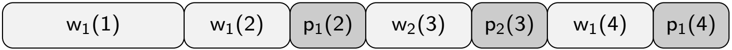


Probability that wrong path is consistent with parities is

$$\Pr(p = p_{\mathrm{r}}) = 2^{-\operatorname{rank}(G)}$$

where

$$G = \begin{bmatrix} 0 & G_{1,3} & 0 \\ 0 & G_{2,3} & 0 \\ 0 & 0 & G_{3,4} \end{bmatrix}$$

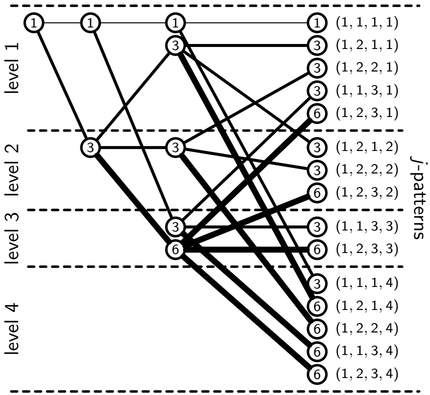


When Levels Repeat

Bell Numbers and j-patterns

Integer Sequences

- K^L paths
- Reduce complexity through equivalence
- Online Encyclopedia of Integer Sequences (OEIS) A000110
- Bell numbers grow rapidly
- Hard to compute expected number of surviving paths



Need Approximation

Allocating Parity Bits (approximation)

- p_ℓ: # parity bits in sub-block $\ell \in 2, \ldots, L$,
- P_ℓ: # erroneous paths that survive stage $\ell \in 2, \ldots, L$,
- Complexity C_{tree}: # nodes on which parity check constraints verified

Expressions for $\mathbb{E}[P_\ell]$ and C_{tree}

- $P_\ell | P_{\ell-1} \sim B((P_{\ell-1}+1)K - 1, \rho_\ell)$, $\rho_\ell = 2^{-p_\ell}$, $q_\ell = 1 - \rho_\ell$

$$\begin{aligned}
\mathbb{E}[P_\ell] &= \mathbb{E}[\mathbb{E}[P_\ell | P_{\ell-1}]] \\
&= \mathbb{E}[((P_{\ell-1}+1)K - 1)\rho_\ell] \\
&= \rho_\ell K \mathbb{E}[P_{\ell-1}] + \rho_\ell (K-1) \\
&= \sum_{r=1}^{\ell} K^{\ell-r}(K-1) \prod_{j=r}^{\ell} \rho_j
\end{aligned}$$

- $C_{\text{tree}} = K + \sum_{\ell=2}^{L-1} [(P_\ell + 1)K]$
- $\mathbb{E}[C_{\text{tree}}]$ can be computed using the expression for $\mathbb{E}[P_\ell]$

Optimization of Parity Lengths

- p_ℓ: # parity bits in sub-block $\ell \in 2, \ldots, L$,
- P_ℓ: # erroneous paths that survive stage $\ell \in 2, \ldots, L$,

Relaxed geometric programming optimization

$$
\begin{aligned}
\underset{(p_2,\ldots,p_L)}{\text{minimize}} \quad & \mathbb{E}[C_{\text{tree}}] \\
\text{subject to} \quad & \Pr(P_L \geq 1) \leq \varepsilon_{\text{tree}} && \text{Erroneous paths} \\
& \sum_{\ell=2}^{L} p_\ell = M - B && \text{Total \# parity bits} \\
& p_\ell \in \{0, \ldots, N/L\} \quad \forall\, \ell \in 2, \ldots, L && \text{Integer constraints}
\end{aligned}
$$

- Solved using standard convex solver, e.g., CVX

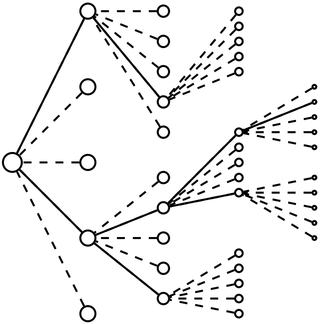
Choice of Parity Lengths

- $K = 200$, $L = 11$, $N/L = 15$

$\varepsilon_{\text{tree}}$	$\mathbb{E}[C_{\text{tree}}]$	Parity Lengths $p_2, \ldots, p_L$
0.006	Infeasible	Infeasible
0.0061930	3.2357×10^{11}	$0, 0, 0, 0, 15, 15, 15, 15, 15, 15$
0.0061931	3357300	$0, 3, 8, 8, 8, 8, 10, 15, 15, 15$
0.0061932	1737000	$0, 4, 8, 8, 8, 8, 9, 15, 15, 15$
0.0061933	926990	$0, 5, 8, 8, 8, 8, 8, 15, 15, 15$
0.0061935	467060	$1, 8, 8, 8, 8, 8, 8, 11, 15, 15$
0.0062	79634	$1, 8, 8, 8, 8, 8, 8, 11, 15, 15$
0.007	7357.8	$6, 8, 8, 8, 8, 8, 8, 8, 13, 15$
0.008	6152.7	$7, 8, 8, 8, 8, 8, 8, 8, 12, 15$
0.02	5022.9	$6, 8, 8, 9, 9, 9, 9, 9, 9, 14$
0.04	4158	$7, 8, 8, 9, 9, 9, 9, 9, 9, 13$
0.6378	3066.3	$9, 9, 9, 9, 9, 9, 9, 9, 9, 9$

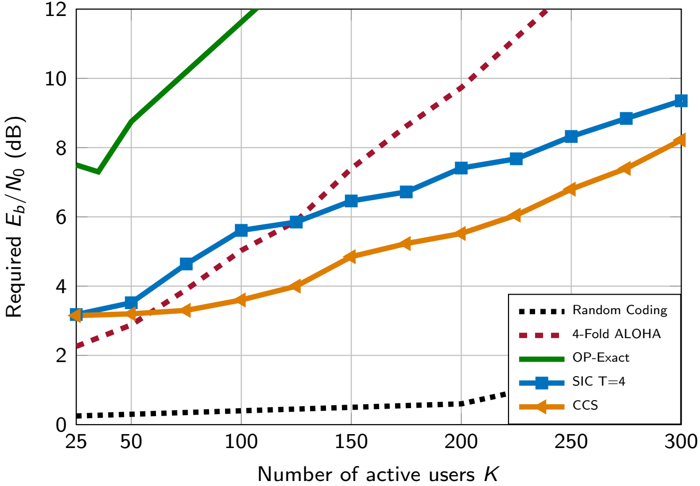
Choice of Parity Lengths

- $K = 200$, $L = 11$, $N/L = 15$



Parity Lengths $p_2, \ldots, p_L$
$0, 0, 0, 0, 15, 15, 15, 15, 15, 15$
$0, 3, 8, 8, 8, 8, 10, 15, 15, 15$
$0, 4, 8, 8, 8, 8, 9, 15, 15, 15$
$0, 5, 8, 8, 8, 8, 8, 15, 15, 15$
$1, 8, 8, 8, 8, 8, 8, 11, 15, 15$
$1, 8, 8, 8, 8, 8, 8, 11, 15, 15$
$6, 8, 8, 8, 8, 8, 8, 8, 13, 15$
$7, 8, 8, 8, 8, 8, 8, 8, 12, 15$
$6, 8, 8, 9, 9, 9, 9, 9, 9, 14$
$7, 8, 8, 9, 9, 9, 9, 9, 9, 13$
$9, 9, 9, 9, 9, 9, 9, 9, 9, 9$

Performance of CCS and Previous Schemes



Leveraging CCS Framework

CHIRRUP: a practical algorithm for unsourced multiple access

Robert Calderbank, Andrew Thompson

(Submitted on 2 Nov 2018)

Unsourced multiple access abstracts grantless simultaneous communication of a large number of devices (messages) each of which transmits (is transmitted) infrequently. It provides a model for machine-to-machine communication in the Internet of Things (IoT), including the special case of radio-frequency identification (RFID), as well as neighbor discovery in ad hoc wireless networks. This paper presents a fast algorithm for unsourced multiple access that scales to 2^{100} devices (arbitrary 100 bit messages). The primary building block is multiuser detection of binary chirps which are simply codewords in the second order Reed Muller code. The chirp detection algorithm originally presented by Howard et al. is enhanced and integrated into a peeling decoder designed for a patching and slotting framework. In terms of both energy per bit and number of transmitted messages, the proposed algorithm is within a factor of 2 of state of the art approaches. A significant advantage of our algorithm is its computational efficiency. We prove that the worst-case complexity of the basic chirp reconstruction algorithm is $\mathcal{O}[nK(\log_2 n + K)]$, where n is the codeword length and K is the number of active users, and we report computing times for our algorithm. Our performance and computing time results represent a benchmark against which other practical algorithms can be measured.

Subjects: **Signal Processing (eess.SP)**
Cite as: **arXiv:1811.00879 [eess.SP]**
(or **arXiv:1811.00879v1 [eess.SP]** for this version)

Submission history
From: Andrew Thompson [view email]
[v1] Fri, 2 Nov 2018 14:25:46 UTC (470 KB)

Which authors of this paper are endorsers? | *Disable MathJax* (*What is MathJax?*)

- Hadamard matrix based compressing scheme + CSS
- Ultra-low complexity decoding algorithm

S. D. Howard, A. R. Calderbank, S. J. Searle. *A Fast Reconstruction Algorithm for Deterministic Compressive Sensing using Second Order Reed-Muller Codes.* CISS 2008

Example: CHIRRUP

- Sensing matrix based on 2nd-order Reed-Muller functions,

$$\phi_{R,b}(a) = \frac{(-1)^{\mathrm{wt}(b)}}{\sqrt{2^m}} i^{(2b+Ra)^T a}$$

  R is binary symmetric matrix with zeros on diagonal, wt represent weight, and $i = \sqrt{-1}$

- Every column of form

$$\begin{matrix} | \\ x_{R,b} \\ | \end{matrix} = \begin{bmatrix} \phi_{R,b}([0]_2) \\ \phi_{R,b}([1]_2) \\ \vdots \\ \phi_{R,b}([2^m - 1]_2) \end{bmatrix}$$

  $[\cdot]_2$ is integer expressed in radix of 2

- Information encoded into R and b
- **Fast recovery:** Inner-products, Hardmard project onto Walsh basis, get R row column at a time, dechirp, Hadamard project to b

Leveraging CCS Framework

Non-Bayesian Activity Detection, Large-Scale Fading Coefficient Estimation, and Unsourced Random Access with a Massive MIMO Receiver

Alexander Fengler, Saeid Haghighatshoar, Peter Jung, Giuseppe Caire

In this paper, we study the problem of user activity detection and large-scale fading coefficient estimation in a random access wireless uplink with a massive MIMO base station with a large number M of antennas and a large number of wireless single-antenna devices (users). We consider a block fading channel model where the M-dimensional channel vector of each user remains constant over a coherence block containing L signal dimensions in time-frequency. In the considered setting, the number of potential users K_{tot} is much larger than L but at each time slot only $K_a << K_{\text{tot}}$ of them are active. Previous results, based on compressed sensing, require that $K_a \leq L$, which is a bottleneck in massive deployment scenarios such as Internet-of-Things and unsourced random access. In this work we show that such limitation can be overcome when the number of base station antennas M is sufficiently large. We also provide two algorithms. One is based on Non-Negative Least-Squares, for which the above scaling result can be rigorously proved. The other consists of a low-complexity iterative componentwise minimization of the likelihood function of the underlying problem. Finally, we use the proposed approximated ML algorithm as the decoder for the inner code in a concatenated coding scheme for unsourced random access, where all users make use of the same codebook, and the massive MIMO base station must come up with the list of transmitted messages irrespectively of the identity of the transmitters. We show that reliable communication is possible at any E_b/N_0 provided that a sufficiently large number of base station antennas is used, and that a sum spectral efficiency in the order of $\mathcal{O}(L\log(L))$ is achievable.

Comments: 50 pages, 8 figures, submitted to IEEE Trans. Inf. Theory
Subjects: **Information Theory (cs.IT)**
Cite as: arXiv:1910.11266 **[cs.IT]**
(or arXiv:1910.11266v1 **[cs.IT]** for this version)

Bibliographic data

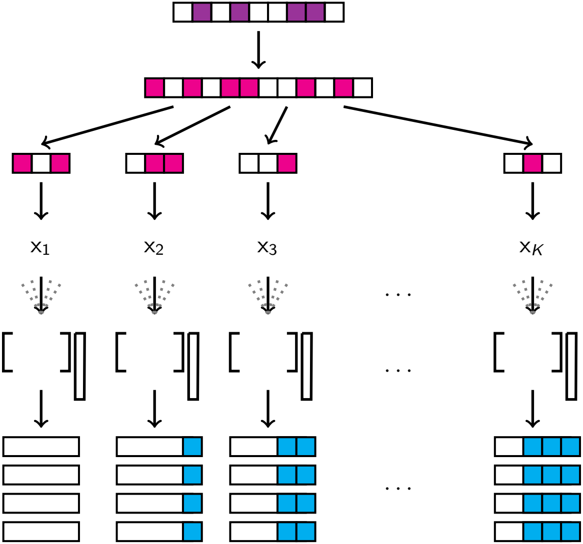
[Enable Bibex (What is Bibex?)]

Submission history

From: Alexander Fengler [view email]
[v1] Thu, 24 Oct 2019 16:32:30 UTC (661 KB)

- Activity detection in random access
- Massive MIMO Receiver

Coded Compressed Sensing – Summary



Pertinent References

- V. Amalladinne, J.-F. Chamberland, and K. R. Narayanan. A coded compressed sensing scheme for uncoordinated multiple access. *Accepted for publication in the IEEE Transactions on Information Theory*, 2020.
- V. K. Amalladinne, A. Vem, D. K. Soma, K. R. Narayanan, and J.-F. Chamberland. A coupled compressive sensing scheme for unsourced multiple access. In *International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, Calgary, Canada, April 2018. IEEE.
- R. Calderbank and A. Thompson. CHIRRUP: A practical algorithm for unsourced multiple access. *Information and Inference: A Journal of the IMA*, 2018.
- A. Fengler, S. Haghighatshoar, P. Jung, and G. Caire. Non-Bayesian activity detection, large-scale fading coefficient estimation, and unsourced random access with a massive MIMO receiver. *arXiv preprint arXiv:1910.11266*, 2019.

Part II

Coded Compressed Sensing: Some Recent Advances

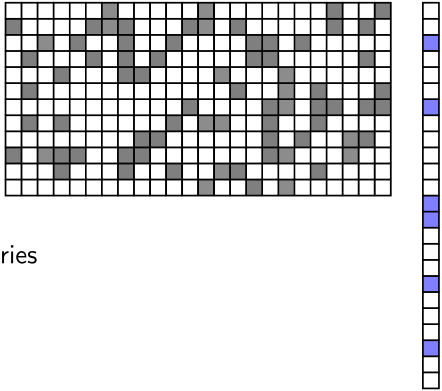
Abstract CS Challenge – Recap

Problem setting

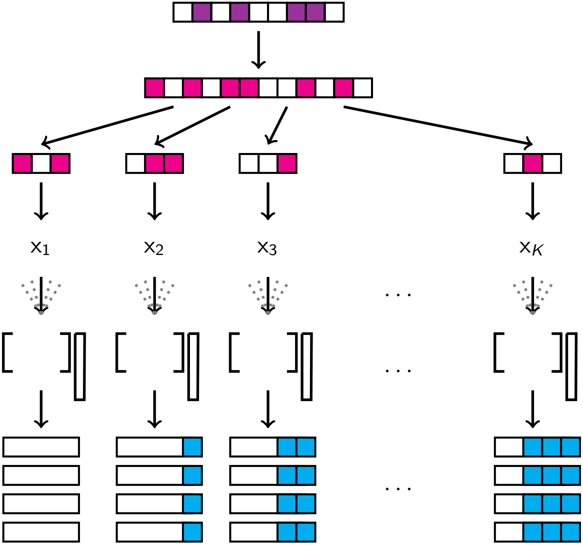
- Noisy compressed sensing

$$y = As + z$$

  where s is K sparse
- s has non-negative integer entries
- $A.\mathtt{shape} \approx 30,000 \times 2^{128}$
- z is additive Gaussian noise



Coded Compressed Sensing – Recap



Enhanced Coded Compressed Sensing

An enhanced decoding algorithm for coded compressed sensing

Vamsi K. Amalladinne, Jean-Francois Chamberland, Krishna R. Narayanan

Coded compressed sensing is an algorithmic framework tailored to sparse recovery in very large dimensional spaces. This framework is originally envisioned for the unsourced multiple access channel, a wireless paradigm attuned to machine-type communications. Coded compressed sensing uses a divide-and-conquer approach to break the sparse recovery task into sub-components whose dimensions are amenable to conventional compressed sensing solvers. The recovered fragments are then stitched together using a low complexity decoder. This article introduces an enhanced decoding algorithm for coded compressed sensing where fragment recovery and the stitching process are executed in tandem, passing information between them. This novel scheme leads to gains in performance and a significant reduction in computational complexity. This algorithmic opportunity stems from the realization that the parity structure inherent to coded compressed sensing can be used to dynamically restrict the search space of the subsequent recovery algorithm.

Comments: Submitted to ICASSP2020
Subjects: **Information Theory (cs.IT)**; Signal Processing (eess.SP)
Cite as: arXiv:1910.09704 **[cs.IT]**
(or arXiv:1910.09704v1 **[cs.IT]** for this version)

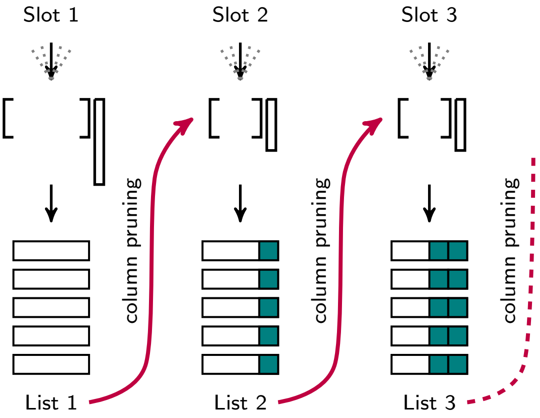
Bibliographic data
[Enable Bibex (What is Bibex?)]

Submission history
From: Vamsi Amalladinne [view email]
[v1] Tue, 22 Oct 2019 00:17:37 UTC (65 KB)

Leverage algorithmic opportunity

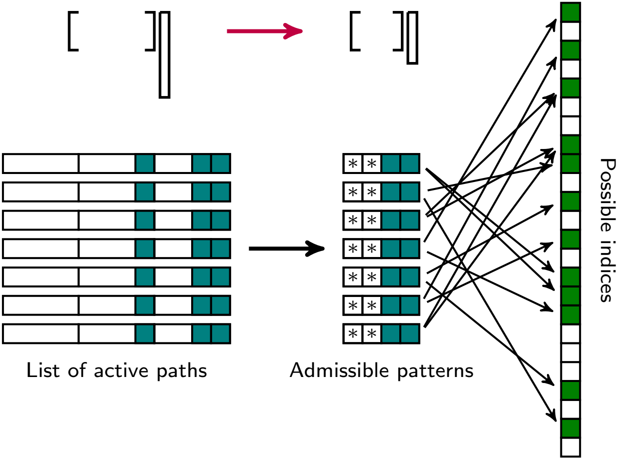
- Extending CCS framework by integrating tree code
- Decisions at early stages inform later parts
- Algorithmic performance improvements

Coded Compressive Sensing with Column Pruning



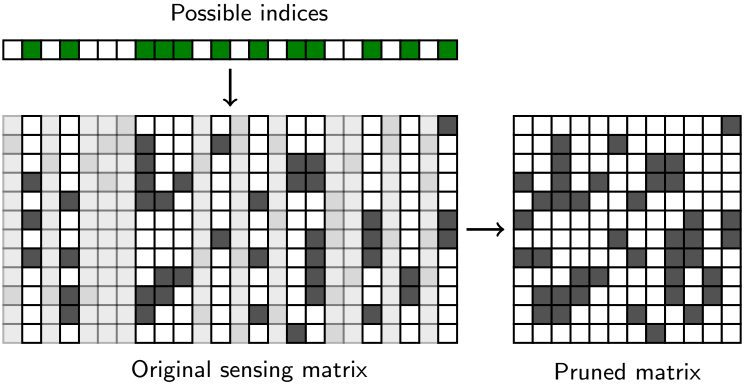
- Active partial paths determine possible parity patterns
- Admissible indices for next slot determined by possible parities
- Inadmissible columns can be pruned before CS algorithm

Coded Compressive Sensing – Dimensionality Reduction



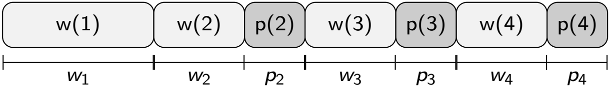
- Every surviving path produces parity pattern
- Only fragments with these pattern can appear in subsequent slot
- On average, there are $K(1 + \mathrm{E}[P_\ell])$ possibilities parity patterns

Coded Compressive Sensing with Column Pruning



- For K small, width of sensing matrix is greatly reduced
- Actual sensing matrix is determined dynamically at run time
- Complexity of CS algorithm becomes much smaller

Dynamic Dimensions of Sensing Matrices



- $P_\ell^{(i)}$ is # erroneous paths that survive stage ℓ for root i
- Total number of active paths at stage ℓ

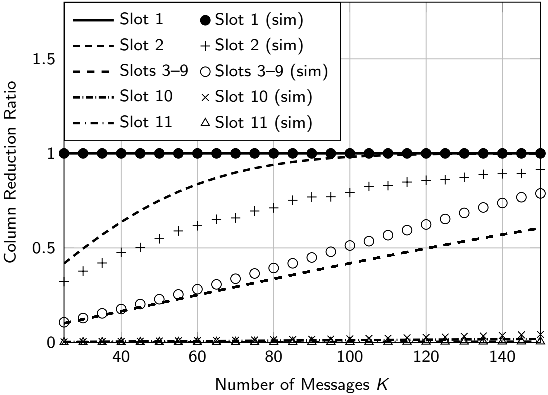
$$\text{Count}(\ell) = \sum_{i=1}^{K} \left(1 + P_\ell^{(i)}\right) \approx K + K\mathbb{E}[P_\ell]$$

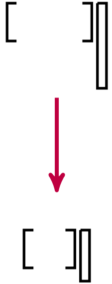
- Assuming path counts have concentrated, the number of active parity patterns become

$$|\mathcal{P}_\ell| \approx \underbrace{2^{p_\ell}}_{\#\text{ of patterns}} \times \underbrace{\left(1 - \left(1 - 2^{-p_\ell}\right)^{\text{Count}(\ell)}\right)}_{\text{survival probability}}$$

- Expected column reduction ratio is $\left(1 - \left(1 - 2^{-p_\ell}\right)^{\text{Count}(\ell)}\right) \ll 1$ for typical $(p_1, \ldots, p_L)$ and small K

Expected Column Reduction Ratio





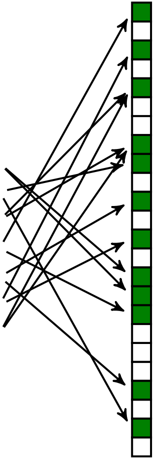
- Parity allocation parameters, with $w_\ell + p_\ell = 15$,

$$(p_1, p_2, \ldots, p_{10}) = (6, 8, 8, 8, 8, 8, 8, 8, 13, 15)$$

- Pruning is more pronounced at later stages
- Effective width of sensing matrix is greatly reduced

Additional Implications of Dynamic Pruning

1. When previous stages list actual sub-blocks, sensing matrix for next stage is trimmed down correctly
   - Reduces search space for CS solver and **improves performance**
2. If erroneous partial path survives, then pruned sensing matrix retains all the columns with parity patterns that match erroneous path, but discards other columns
   - Steers CS solver towards list that includes sub-blocks consistent with erroneous path
   - Increases propensity for **error propagation**
3. If valid sub-block is absent from CS list, then corresponding parity pattern may disappear
   - When this occurs, received vector for subsequent slot is no longer of the form $y(\ell) = As(\ell) + z(\ell)$ because of missing columns
   - Results in **noise amplification** for other messages



Consequences of Dynamic Pruning

Overall Performance Improvements

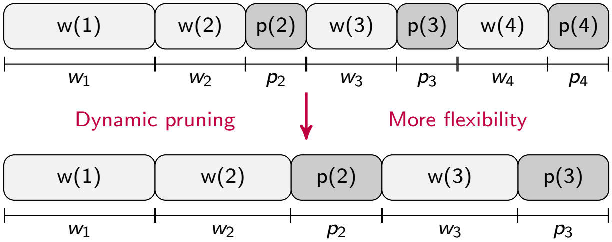
Immediate considerations – reparametrization

- Can one increase number of bits per slot?
- How should we allocate information and parity bits within slots?
- What about channel uses?

New compressed sensing challenge

- How can one design good deterministic sensing matrices tailored to stochastic pruning?

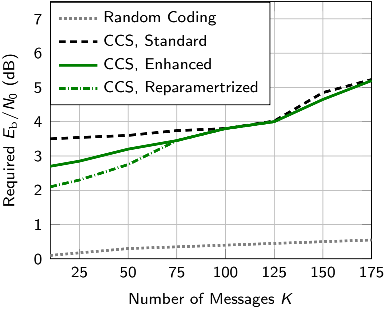
Dynamic Pruning Leads to Reparametrization

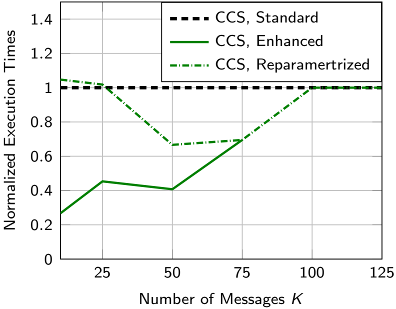


- Originally, width of matrices constrained by CS decoder complexity
- Recovery now takes place over dynamically pruned matrices
- Opportunity to **assign information and parity bits differently**
- Perhaps **reduce slot count** and circumvent FBL limitations
- **Reallocate channel uses** based on sampling complexity

$$K \log \left(\frac{N}{K} \right) \quad \text{where } N \text{ is (random) matrix width}$$

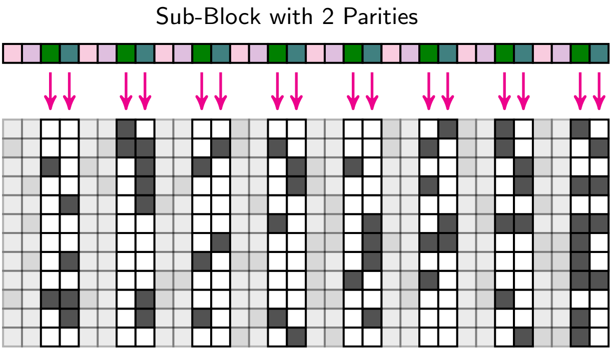
Preliminary Performance Enhanced CCS





- Performrmance improves significantly with enhanced CCS decoding, especially for smaller K values
- Computational complexity is reduced drastically
- Reparametrization offer additional gains in performance, even for preliminary exploration

Sensing Matrix Design for Stochastic Pruning



- For 2 parities, there are $\binom{4}{2} = 6$ possible pruned matrices
- Columns within parity group well separated
- Columns from distinct groups less likely to appear together

What Are Good Designs?

Asynchronous Coded Compressed Sensing

Asynchronous Neighbor Discovery Using Coupled Compressive Sensing

Vamsi K. Amalladinne, Krishna R. Narayanan, Jean-Francois Chamberland, Dongning Guo

(Submitted on 2 Nov 2018)

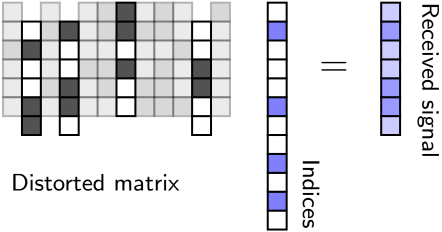
The neighbor discovery paradigm finds wide application in Internet of Things networks, where the number of active devices is orders of magnitude smaller than the total device population. Designing low-complexity schemes for asynchronous neighbor discovery has recently gained significant attention from the research community. Concurrently, a divide-and-conquer framework, referred to as coupled compressive sensing, has been introduced for the synchronous massive random access channel. This work adapts this novel algorithm to the problem of asynchronous neighbor discovery with unknown transmission delays. Simulation results suggest that the proposed scheme requires much fewer transmissions to achieve a performance level akin to that of state-of-the-art techniques.

Subjects: **Signal Processing (eess.SP)**; Information Theory (cs.IT)
Cite as: **arXiv:1811.00687 [eess.SP]**
(or **arXiv:1811.00687v1 [eess.SP]** for this version)

Building robust sensing matrices

- Extending CCS framework with low sample complexity
- Addressing issues pertaining to asynchrony
- Context of neighbor discovery

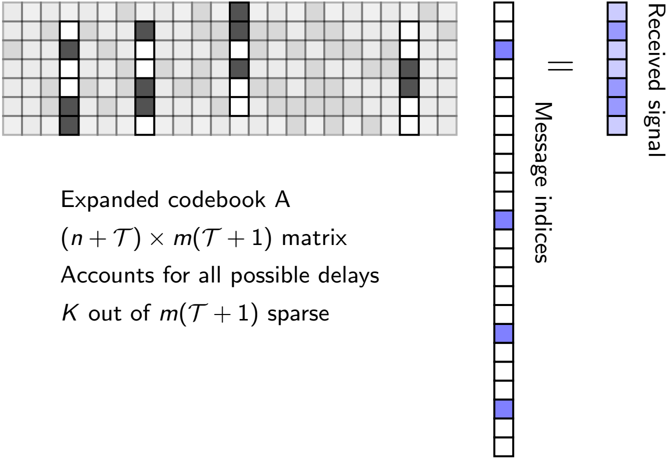
Dealing with Jitter and Asynchrony



Asynchronous signals

- $\mathsf{y} = \tilde{\mathsf{A}}\tilde{\mathsf{s}} + \mathsf{z}$ with $\|\mathsf{s}\|_0 = K$
- $\tilde{A}.\mathtt{shape}$ is $(n + \mathcal{T}) \times m$ unknown due to random delays
- Max delay $\mathcal{T}$ known to the decoder

Expanded Codebook through Sensing Matrix



Expanded codebook A

$(n + \mathcal{T}) \times m(\mathcal{T} + 1)$ matrix

Accounts for all possible delays

K out of $m(\mathcal{T} + 1)$ sparse

- Increases computational load of CS solvers

Extending CCS Framework

SPARCs for Unsourced Random Access

Alexander Fengler, Peter Jung, Giuseppe Caire

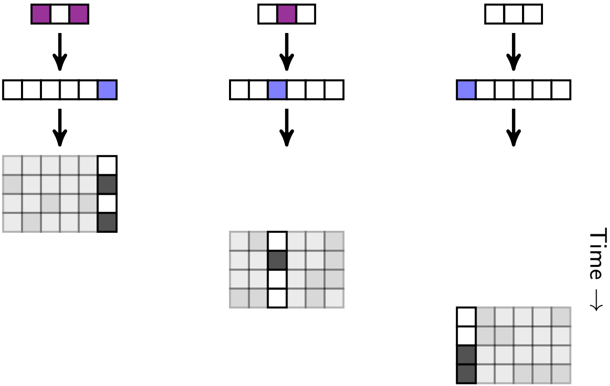
(Submitted on 18 Jan 2019)

This paper studies the optimal achievable performance of compressed sensing based unsourced random-access communication over the real AWGN channel. "Unsourced" means, that every user employs the same codebook. This paradigm, recently introduced by Polyanskiy, is a natural consequence of a very large number of potential users of which only a finite number is active in each time slot. The idea behind compressed sensing based schemes is that each user encodes his message into a sparse binary vector and compresses it into a real or complex valued vector using a random linear mapping. When each user employs the same matrix this creates an effective binary inner multiple-access channel. To reduce the complexity to an acceptable level the messages have to be split into blocks. An outer code is used to assign the symbols to individual messages. This division into sparse blocks is analogous to the construction of sparse regression codes (SPARCs), a novel type of channel codes, and we can use concepts from SPARCs to design efficient random-access codes. We analyze the asymptotically optimal performance of the inner code using the recently rigorized replica symmetric formula for the free energy which is achievable with the approximate message passing (AMP) decoder with spatial coupling. An upper bound on the achievable rates of the outer code is derived by classical Shannon theory. Together this establishes a framework to analyse the trade-off between SNR, complexity and achievable rates in the asymptotic infinite blocklength limit. Finite blocklength simulations show that the combination of AMP decoding, with suitable approximations, together with an outer code recently proposed by Amalladinne et. al. outperforms state of the art methods in terms of required energy-per-bit at lower decoding complexity.

Comments: 16 pages, 7 Figures
Subjects: **Information Theory (cs.IT)**
Cite as: **arXiv:1901.06234 [cs.IT]**
(or **arXiv:1901.06234v1 [cs.IT]** for this version)

- Connection between CCS indexing and sparse regression codes
- Circumvent slotting under CCS and dispersion effects
- Introduce denoiser tailored to CCS

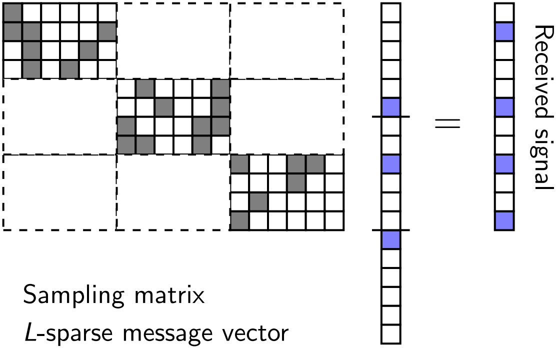
CCS Revisited



Columns are possible signals

- Bit sequence split into L fragments
- Each bit + parity block converted to index in $[0, 2^{m/L} - 1]$
- Stack sub-codewords into $(n/L) \times 2^{m/L}$ sensing matrices

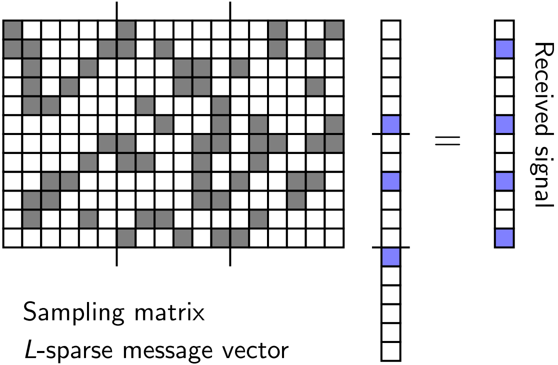
CCS Unified CS Analogy



- Initial non-linear indexing step
- Index vector is block sparse
- Connection to sparse regression codes

C. Rush, A. Greig, R. Venkataramanan. *Capacity-Achieving Sparse Superposition Codes via Approximate Message Passing Decoding*. IEEE IT Trans 2017

CCS-AMP



- Complexity management comes from dimensionality reduction
- Use full sensing matrix on sparse regression codes
- Decode inner code with low-complexity AMP
- Decode outer code with tree decoding

A. Fengler, P. Jung, and G. Caire. *SPARCs and AMP for Unsourced Random Access*. ISIT 2019

Approximate Message Passing Algorithm

Governing Equations

- AMP algorithm iterates through

$$\mathrm{z}^{(t)} = \mathrm{y} - \mathrm{A}\mathrm{D}\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big) + \underbrace{\frac{1}{n}\operatorname{div}\mathrm{D}\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big)}_{\text{Onsager correction}}$$

$$\mathrm{r}^{(t+1)} = \mathrm{A}^{\mathsf{T}}\mathrm{z}^{(t)} + \mathrm{D}\,\underbrace{\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big)}_{\text{Denoiser}}$$

Initial conditions $\mathrm{z}^{(0)} = 0$ and $\boldsymbol{\eta}_0\left(\mathrm{r}^{(0)}\right) = 0$

- Application falls within framework for non-separable functions

R. Berthier, A. Montanari, and P.-M. Nguyen. *State Evolution for Approximate Message Passing with Non-Separable Functions*. arXiv 2017

Marginal Posterior Mean Estimate (PME)

Proposed Denoiser (Fengler, Jung, and Caire)

- State estimate based on Gaussian model

$$\hat{s}^{\mathrm{OR}}(q, r, \tau) = \mathbb{E}\left[s \middle| \sqrt{P_\ell} s + \tau\zeta = r \right]$$
$$= \frac{q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}{(1-q)\exp\left(-\frac{r^2}{2\tau^2} \right) + q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}$$

  with prior $q = K/m$ fixed

- $\boldsymbol{\eta}_t\left(\mathrm{r}^{(t)}\right)$ is aggregate of PME values
- τ_t is obtained from state evolution or $\tau_t^2 = \|\mathrm{z}^{(t)}\|^2/n$

Performance is quite good!

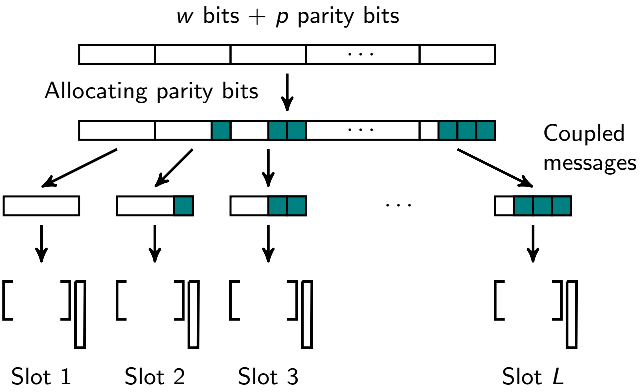
Pertinent References

- V. K. Amalladinne, J.-F. Chamberland, and K. R. Narayanan. An enhanced decoding algorithm for coded compressed sensing. In *International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, 2020.
- V. K. Amalladinne, K. R. Narayanan, J.-F. Chamberland, and D. Guo. Asynchronous neighbor discovery using coupled compressive sensing. In *International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, 2019.
- A. Fengler, P. Jung, and G. Caire. SPARCs and AMP for unsourced random access. In *International Symposium on Information Theory (ISIT)*, 2019.
- R. Venkataramanan, S. Tatikonda, and A. Barron. Sparse regression codes. *Foundations and Trends in Communications and Information Theory*, vol. 15, no. 1-2, pp. 1–195, 2019.
- C. Rush, A. Greig, and R. Venkataramanan. Capacity-achieving sparse superposition codes via approximate message passing decoding. *IEEE Transactions on Information Theory*, vol. 63, no. 3, pp. 1476–1500, 2017.

Part III

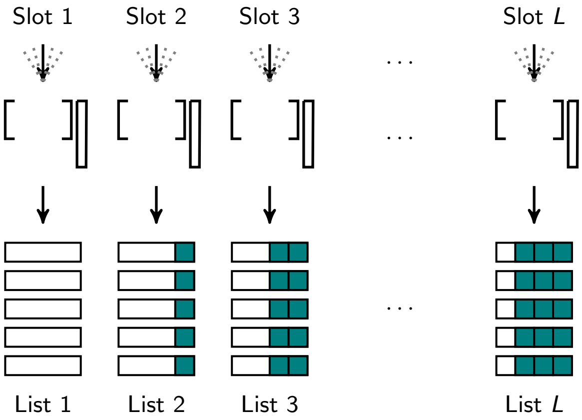
Connecting Coding and Compressed Sensing via Approximate Message Passing

Coded Compressive Sensing – Device Perspective



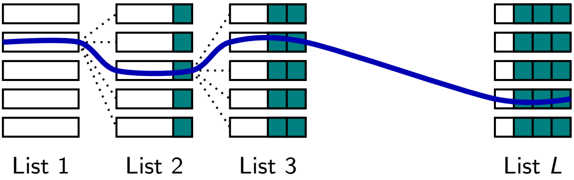
- Divide and conquer produces slots
- Parity bits determined by tree code

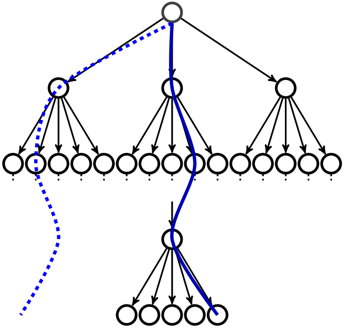
Coded Compressive Sensing – Multiple Access



- L instances of compressed sensing
- Produces L lists of K decoded sub-packets
- Must piece sub-packets together using tree decoder

Coded Compressive Sensing – Stitching Process

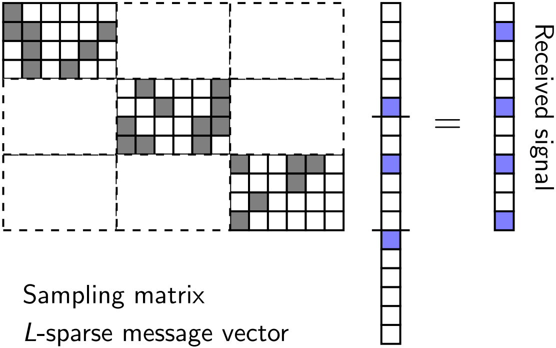




Original Tree Decoding

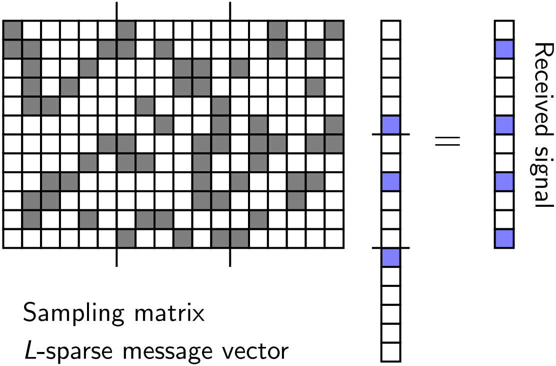
- Random parity bits
- Control girth of tree
- Decoded from root to leafs

Coded Compressed Sensing – Uniform View



- Slots produce block diagonal (unified) matrix
- Message is one-sparse per section
- Width of A is smaller: $2^{m/L}$ instead of 2^m

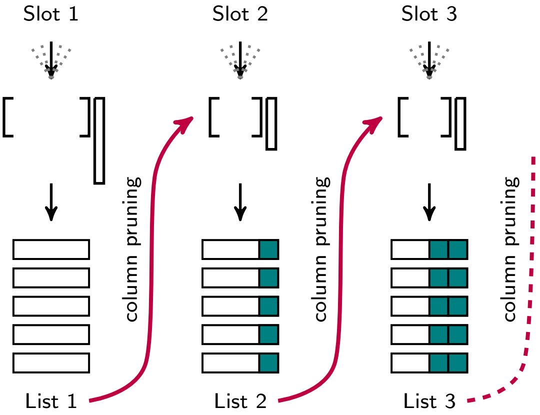
Idea 1: CCS-AMP



- Complexity reduction due to narrower A
- Use full sensing matrix A
- Decode inner code with low-complexity AMP

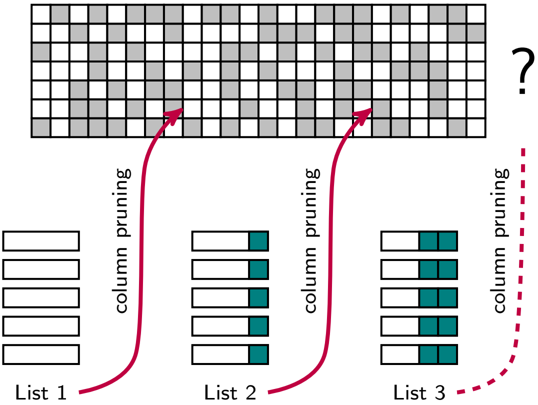
A. Fengler, P. Jung, and G. Caire. *SPARCs and AMP for Unsourced Random Access*. ISIT 2019

Idea 2: Dimensionality Reduction



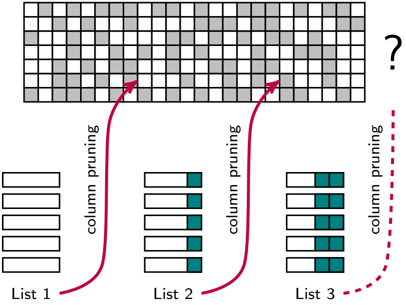
- Incorporate tree code structure into CS recovery task
- Structure from outer code can be integrated into denoiser
- Column pruning acts as side information from neighbors

Combining Idea 1 and Idea 2



Why Not Both?

Issues in Combining Ideas 1 and 2



No columns to prune!

Decisions not in sequence!

Approximate Message Passing Algorithm

Governing Equations

- AMP algorithm iterates through

$$z^{(t)} = y - AD\boldsymbol{\eta}_t\big(r^{(t)}\big) + \underbrace{\frac{1}{n}\operatorname{div} D\boldsymbol{\eta}_t\big(r^{(t)}\big)}_{\text{Onsager correction}}$$

$$r^{(t+1)} = A^{T}z^{(t)} + D\,\underbrace{\boldsymbol{\eta}_t\big(r^{(t)}\big)}_{\text{Denoiser}}$$

Initial conditions $z^{(0)} = 0$ and $\boldsymbol{\eta}_0\left(r^{(0)}\right) = 0$

- Application falls within framework for non-separable functions

Task

- Define denoiser and derive correction term

R. Berthier, A. Montanari, and P.-M. Nguyen. *State Evolution for Approximate Message Passing with Non-Separable Functions.* arXiv 2017

Marginal Posterior Mean Estimate (PME)

Proposed Denoiser (Fengler, Jung, and Caire)

- State estimate based on Gaussian model

$$\hat{s}^{\mathrm{OR}}(q, r, \tau) = \mathbb{E}\left[s \middle| \sqrt{P_\ell} s + \tau \zeta = r \right]$$

$$= \frac{q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}{(1-q) \exp\left(-\frac{r^2}{2\tau^2} \right) + q \exp\left(-\frac{\left(r - \sqrt{P_\ell}\right)^2}{2\tau^2} \right)}$$

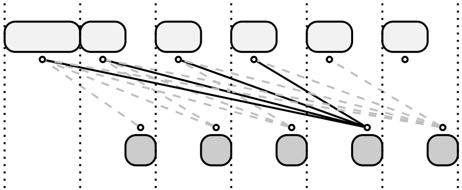
  with (essentially) uninformative prior $q = K/m$ fixed
- $\boldsymbol{\eta}_t\left(\mathrm{r}^{(t)}\right)$ is aggregate of PME values
- τ_t is obtained from state evolution or $\tau_t^2 = \|\mathrm{z}^{(t)}\|^2/n$

A. Fengler, P. Jung, and G. Caire. *SPARCs and AMP for Unsourced Random Access*. ISIT 2019

Redesigning Outer Code

Properties of Original Tree Code

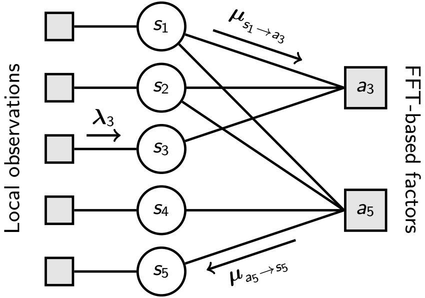
- Aimed at stitching message fragments together
- Works on short lists of K fragments
- Parities allocated to control growth and complexity



Challenges to Integrate into AMP

1. Must compute beliefs for all possible 2^{v} fragments
2. Must provide pertinent information to AMP
3. Should maintain ability to stitch outer code

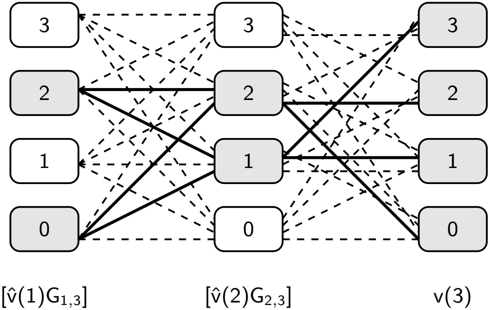
Factor Graph Interpretation of Tree Code



- Tree code with circular convolution structure

$$\boldsymbol{\mu}_{a_p \to s_\ell}\left([\hat{\mathrm{v}}(\ell)]_2\right) \propto \frac{1}{\left\| \mathrm{g}_{\ell,p}^{(g)} \right\|_0} \left(\mathsf{FFT}^{-1} \left(\prod_{s_j \in N(a_p) \setminus s_\ell} \mathsf{FFT}\left(\boldsymbol{\lambda}_{j,p}\right) \right) \right)(g)$$

Factor Graph Interpretation of Tree Code

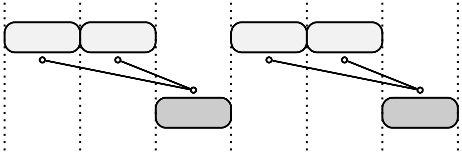


- Multiple devices on same graph
- Parity factor mix concentrated values
- Suggests triadic tree structure

Redesigning Outer Code

Solutions to Integrate into AMP

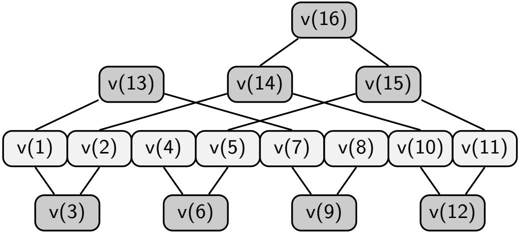
- Parity bits are generated over Abelian group amenable to Hadamard transform (original) or FFT (modified)
- Discrimination power proportional to # parities



New Design Strategy

1. Information sections with parity bits interspersed in-between
2. Parity over two blocks (triadic dependencies)

Message Passing Rules



- Marginal beliefs on message from generic device
- Message from check node a_p to variable node $s \in N(a_p)$:

$$\boldsymbol{\mu}_{a_p \to s}(k) = \sum_{\mathsf{k}_{a_p} : k_p = k} \mathcal{G}_{a_p}\left(\mathsf{k}_{a_p}\right) \prod_{s_j \in N(a_p) \backslash s} \boldsymbol{\mu}_{s_j \to a_p}(k_j)$$

- Message from variable node s_ℓ to check node $a \in N(s)$:

$$\boldsymbol{\mu}_{s_\ell \to a}(k) \propto \boldsymbol{\lambda}_\ell(k) \prod_{a_p \in N(s_\ell) \backslash a} \boldsymbol{\mu}_{a_p \to s_\ell}(k)$$

Approximate Message Passing Algorithm

Updated Equations

AMP two-step algorithm

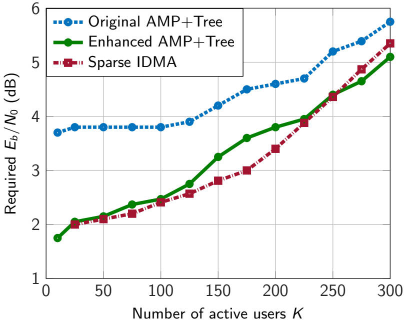
$$\mathrm{z}^{(t)} = \mathrm{y} - \mathrm{AD}\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big) + \underbrace{\frac{1}{n}\operatorname{div}\mathrm{D}\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big)}_{\text{Correction}}$$

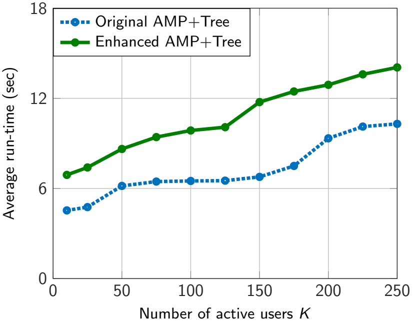
$$\mathrm{r}^{(t+1)} = \mathrm{A}^{\mathsf{T}}\mathrm{z}^{(t)} + \mathrm{D}\,\underbrace{\boldsymbol{\eta}_t\big(\mathrm{r}^{(t)}\big)}_{\text{Denoiser}}$$

Initial conditions $\mathrm{z}^{(0)} = 0$ and $\boldsymbol{\eta}_0\left(\mathrm{r}^{(0)}\right) = 0$

- Denoiser is BP estimate from factor graph
- Message passing uses fresh effective observation r
- Fewer rounds than shortest cycle on factor graph
- Close to PME, but incorporating beliefs from tree code

Preliminary Performance Enhanced CCS



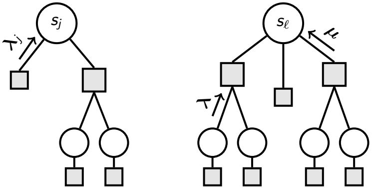


- Performance improves significantly with enhanced CCS-AMP decoding
- Computational complexity is approximately maintained
- Reparametrization may offer additional gains in performance?

CCS and AMP

Summary

- New connection between CCS and AMP
- Natural application of BP on factor graph as denoiser
- Tree code design depends on sparsity
  1. Degree distributions (small graph)
  2. Message size (birthday problem)
  3. Final step is tree decoding
- Many theoretical and practical challenges/opportunities exist



Coding plays increasingly central role in large-scale CS

Pertinent References

- V. K. Amalladinne, A. K. Pradhan, C. Rush, J.-F. Chamberland, K. R. Narayanan. On approximate message passing for unsourced access with coded compressed sensing. In *International Symposium on Information Theory (ISIT)*, 2020.
- V. Amalladinne, J.-F. Chamberland, and K. R. Narayanan. A coded compressed sensing scheme for uncoordinated multiple access. *Accepted for publication in the IEEE Transactions on Information Theory*, 2020.
- A. Fengler, P. Jung, and G. Caire. SPARCs and AMP for Unsourced Random Access. In *International Symposium on Information Theory (ISIT)*, 2019.
- A. Fengler, P. Jung, and G. Caire. Unsourced Multiuser Sparse Regression Codes In *International Symposium on Information Theory (ISIT)*, 2020.
- R. Berthier, A. Montanari, and P.-M. Nguyen. State Evolution for Approximate Message Passing with Non-Separable Functions. *Information and Inference: A Journal of the IMA*, 2020.

Takeaways

Main points from this part

- Traditional GMAC channel model is not suitable for modeling IoT
- Many access channel models number of users growing with n
- Finite Block Length achievability bounds serve as a good benchmark
- Existing coding schemes for GMAC need to be modified

Rest of the talk

- Designing schemes to get close to the FBL benchmark
- Connections between Unsourced MAC and Compressed sensing

Algorithms for Coded Random Access and Inference in Large Dimensional Spaces – Bridging Fundamental Limits and Practical Schemes
Part IV

J.-F. Chamberland and Krishna Narayanan
Vamsi Amalladinne, Avinash Vem, Asit Pradhan

Electrical and Computer Engineering
Texas A&M University

IISc, India
2020

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What Part IV is about

- Review of Slotted ALOHA with interference cancellation
- Extension to the Unsourced Gaussian MAC
- Sparse IDMA for Unsourced multiple access

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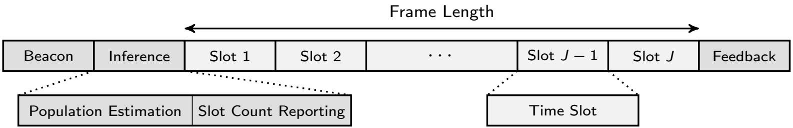
Uncoordinated MAC Frame Structure

- K active devices out of many, many devices



- Beacon employed for coarse synchronization
- Same devices transmit within frame
- Focus is on what happens within the Frame Length
- Each device may or may not use slots within the frame

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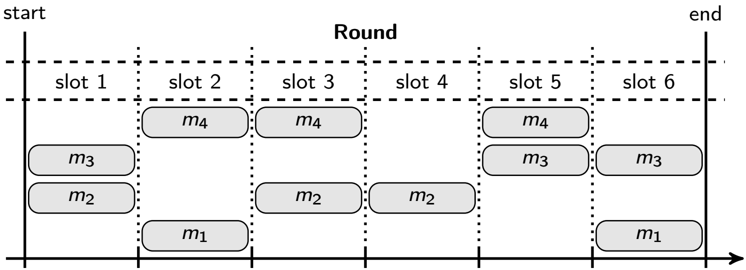
Coded Slotted ALOHA[1]



Leveraging Prior Work on Uncoordinated Access

- K **uncoordinated** devices, each with one packet to send
- Time is **slotted**; transmissions occur within slots
- No power constraint and no Gaussian noise - focus on interference
- Successive interference cancellation

[1] E Paolini, G Liva, M Chiani. *Coded slotted ALOHA: A graph-based method for uncoordinated multiple access*. IEEE Trans on Info Theory, 2015

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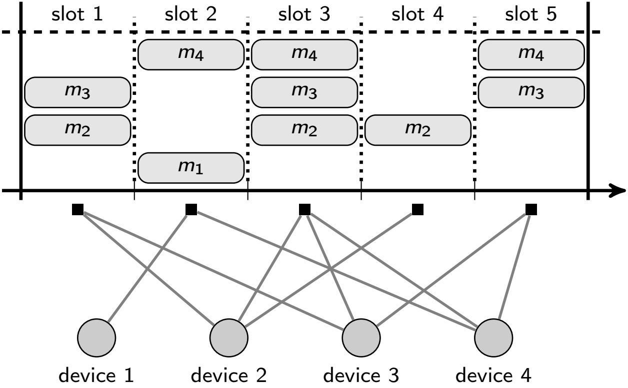
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Instance of Random Access

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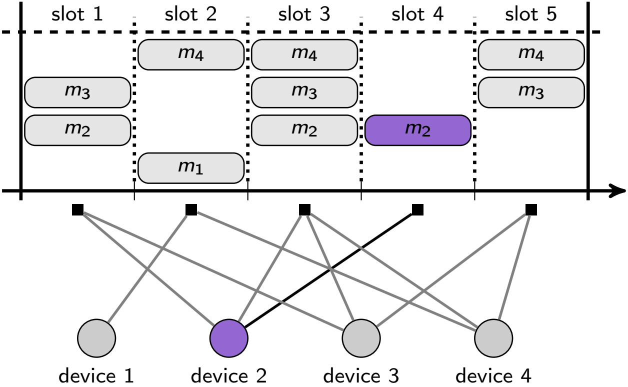
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 1

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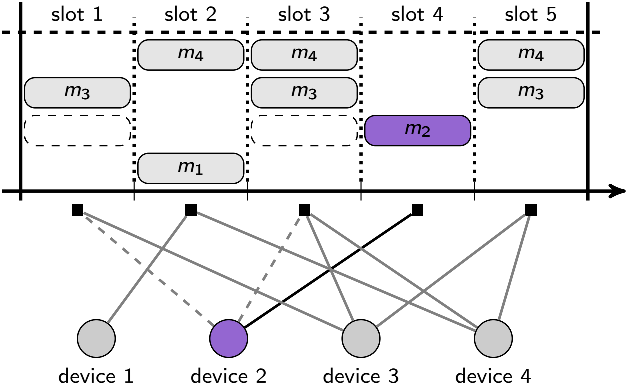
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 1

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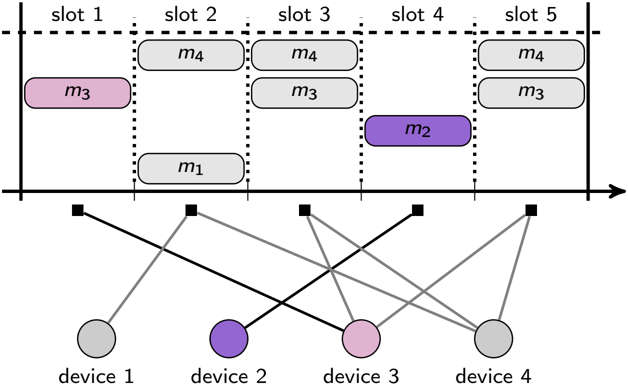
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation

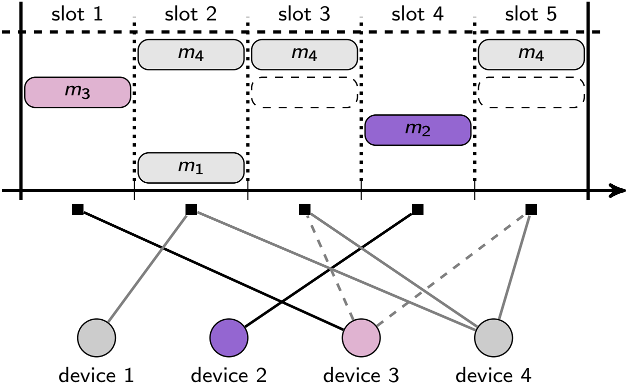


Step 2

5 / 24

Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 2

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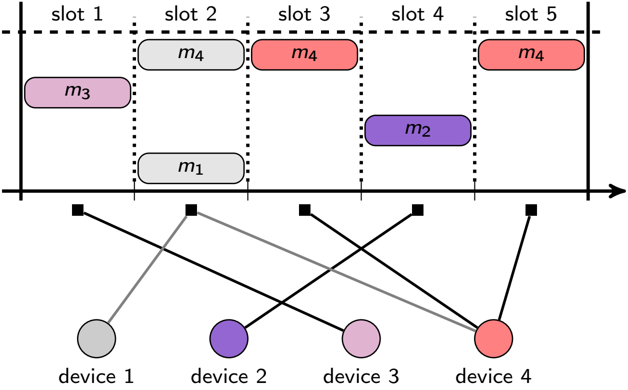
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 3

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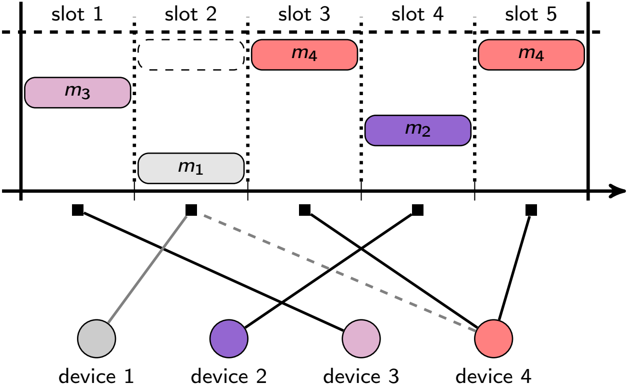
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 3

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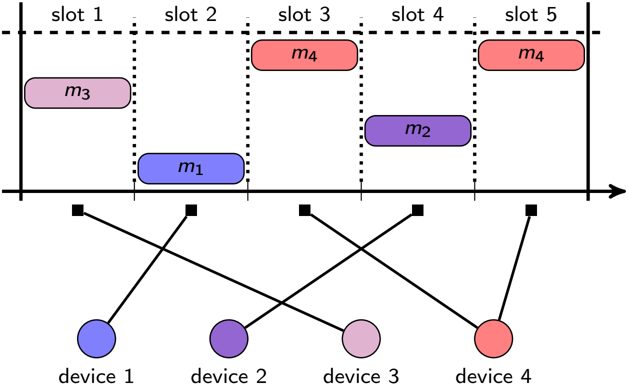
Decoder – Peeling Algorithm

Joint decoding via successive interference cancellation



Step 4

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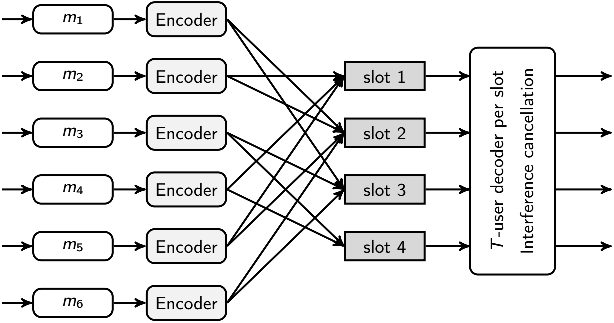
Unsourced MAC – SIC UGMAC Scheme

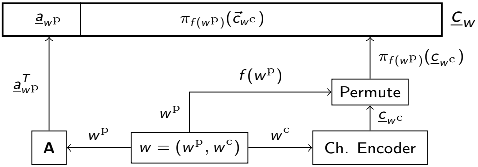


Key Features

- Schedule selected based on **message bits**
- Devices can transmit in multiple sub-blocks
- Scheme facilitates peeling decoder

A. Vem, K. Narayanan, J. Cheng, JFC. ***A User-Independent Successive Interference Cancellation Based Coding Scheme for the Unsourced Random Access Gaussian Channel*****. IEEE Trans on Comm, 2019**

What Really Happens within Slot?



Implementation Notes

- Message is partitioned into two parts $w = (w_p, w_c)$
- Every device uses identical codebook built from LDPC-type codes tailored to T-user real-adder channel
- w_p dictate permutation on encoder and recovered through CS
- Non-negative ℓ_1-regularized LASSO

A. Vem, K. Narayanan, J. Cheng, JFC. ***A User-Independent Successive Interference Cancellation Based Coding Scheme for the Unsourced Random Access Gaussian Channel*****. IEEE Trans on Comm, 2019**

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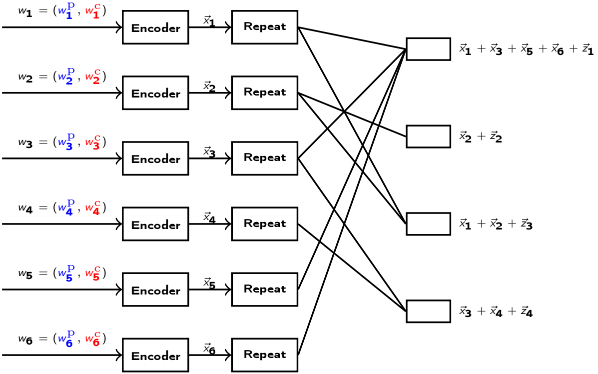
Unsourced MAC – SIC UGMAC Scheme for $T = 2$



Key Features

- Devices repeat codewords in multiple slots based on w_p
- Schedule selected based on **message bits**
- Scheme facilitates peeling decoder

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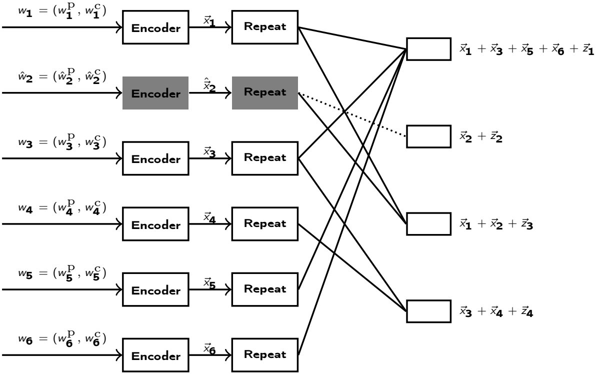
Unsourced MAC – SIC UGMAC Scheme for $T = 2$



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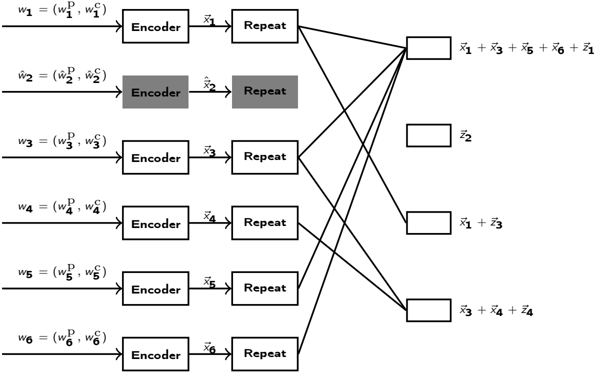

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8 / 24

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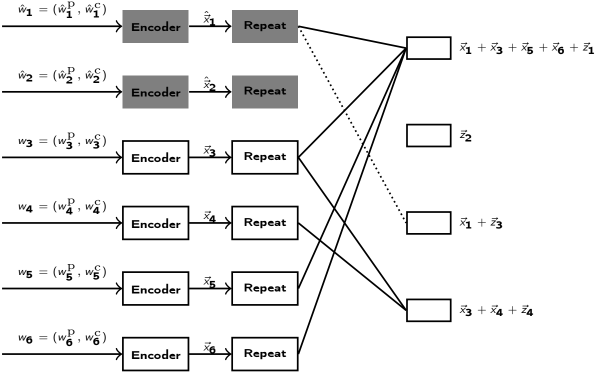

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8 / 24

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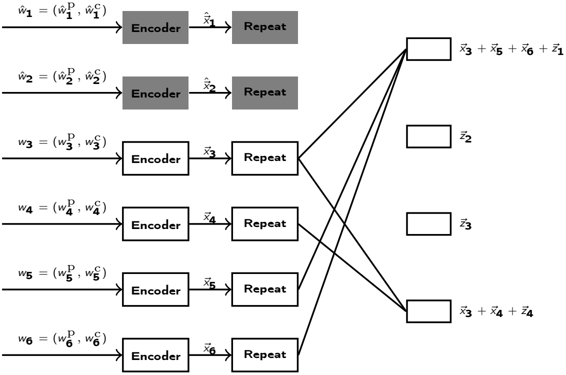

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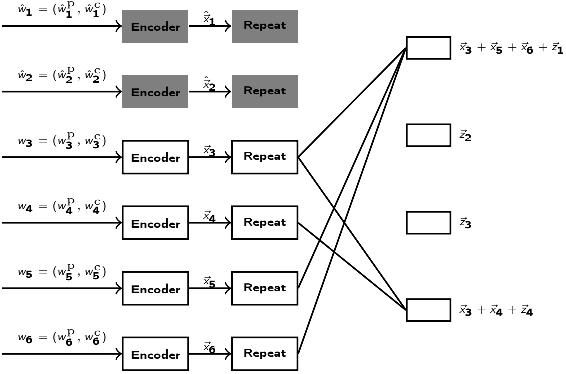

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8 / 24

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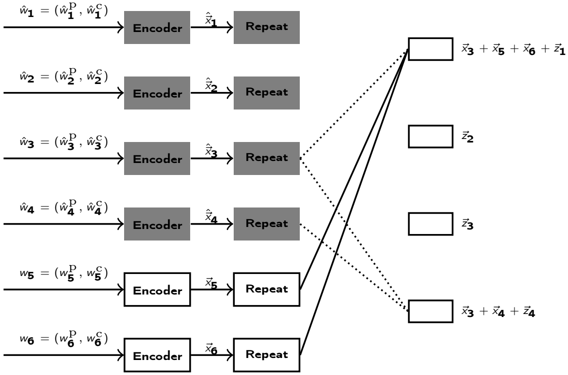

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8 / 24

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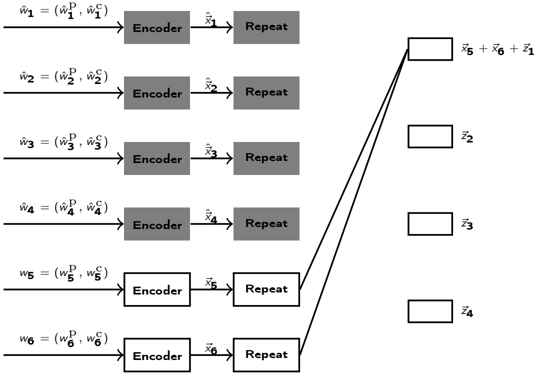

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8 / 24

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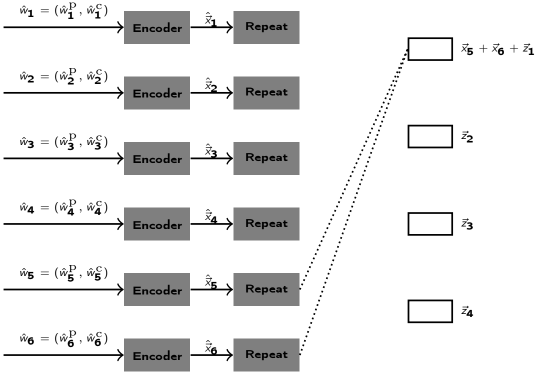

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8 / 24

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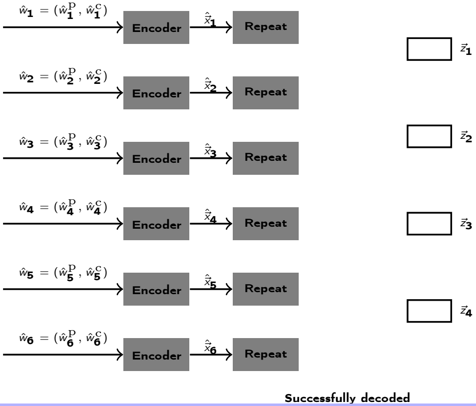

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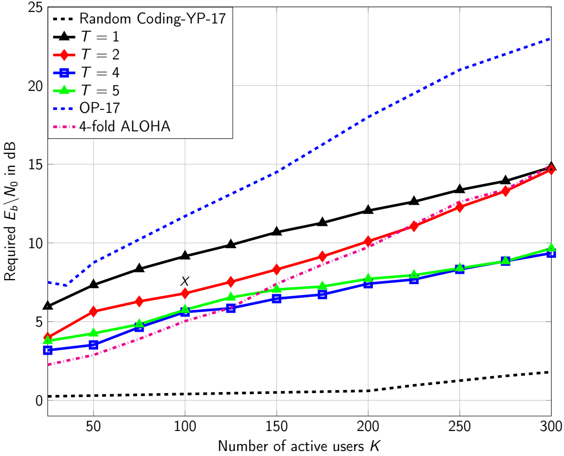
8 / 24



Random Coding-YP-17
$T = 1$
$T = 2$
$T = 4$
$T = 5$
OP-17
4-fold ALOHA
Required $E_b \backslash N_0$ in dB
Number of active users K
25
20
15
10
5
0
50
100
150
200
250
300
X

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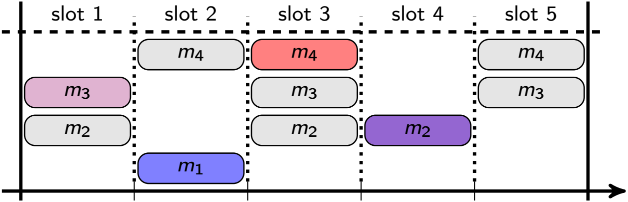
Limitations of Sparsifying Collisions

Drawbacks of Slots

- Second order dispersion effects comes into play in FBL
- Energy expended solely to resolving collisions
- Gray slots are discarded during decoding process (60%)



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Limitations of Sparsifying Collisions

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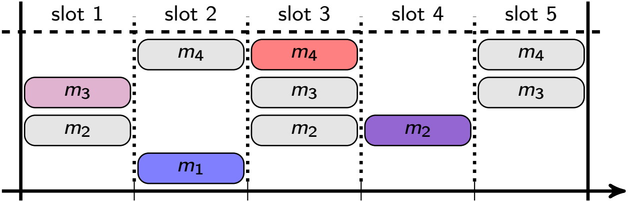

To fix this - Sparse IDMA

An IDMA like scheme which does not divide the number of channel uses into slots

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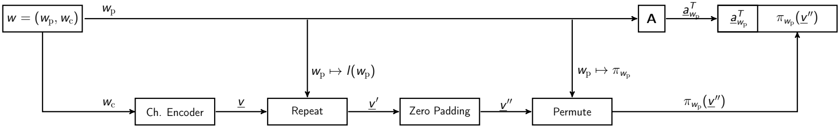
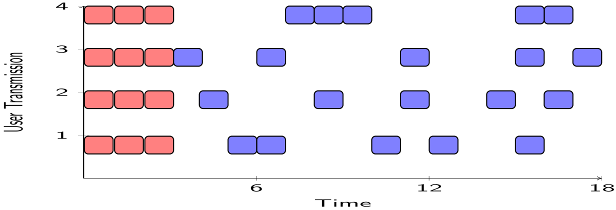
Sparse IDMA - Encoding



- Divide the message into two parts: w_p, w_c
- w_p is transmitted using compressed sensing
- w_c is transmitted using a channel code
- Based on w_p a repetition pattern and permutation pattern is chosen for the channel coding part



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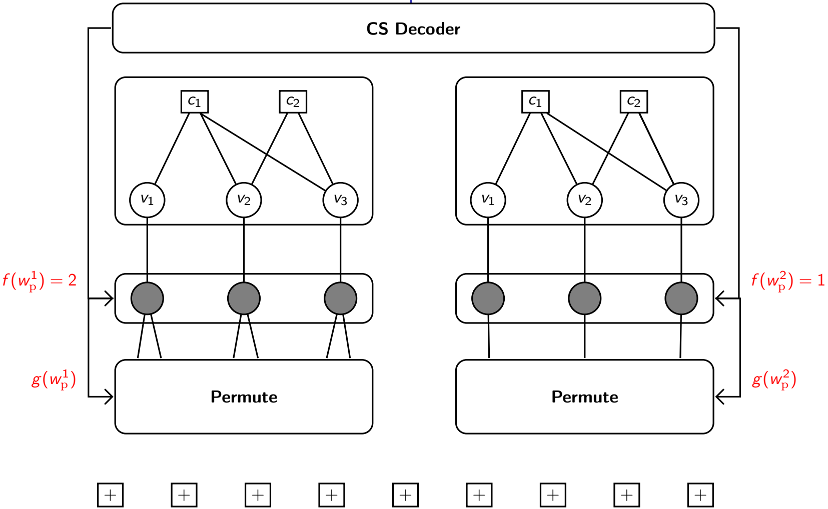
CS Decoder and the Joint Graph



- Decode the first part using non-negative least square
- Recover the permutation patterns from the first part

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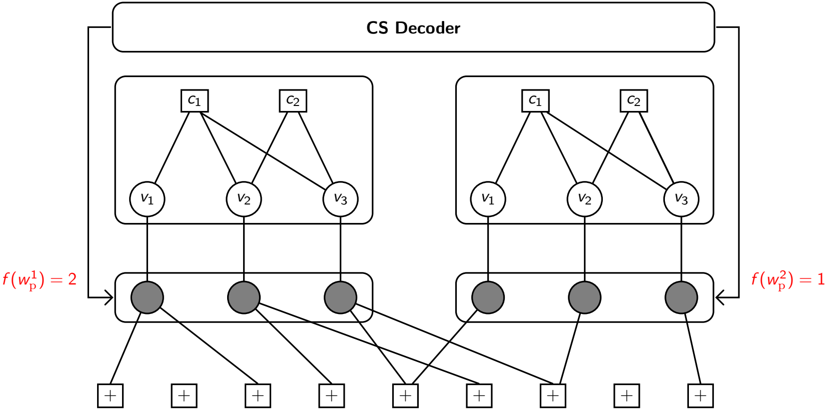
CS Decoder and the Joint Graph



- Decode the first part using non-negative least square
- Recover the permutation patterns from the first part
- Use the permutation patterns to decode the second part of the message by using message passing decoder

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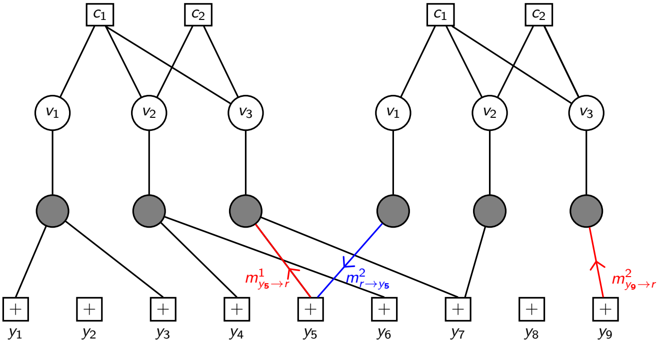
Message Passing: MAC to Repetition



$$m^1_{y_5 \to r} = \frac{2\left(y_5 - \tanh\left(\frac{m^2_{r \to y_5}}{2}\right)\right)}{\sigma^2 + \left(1 - \tanh\left(\frac{m^2_{r \to y_5}}{2}\right)\right)^2}$$

$$m^2_{y_9 \to r} = \frac{2y_9}{\sigma^2}$$

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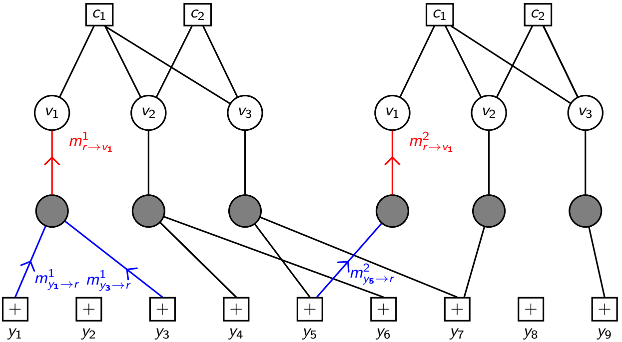
Message Passing: Repetition to Variable



$$m^1_{r\to v_1} = m^1_{y_1\to r} + m^1_{y_3\to r}$$

$$m^2_{r\to v_1} = m^2_{y_5\to r}$$

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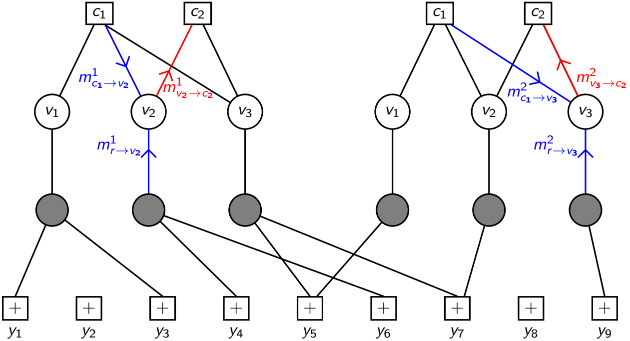
Message Passing: Variable to Check



$$m^1_{v_2 \to c_2} = m^1_{r \to v_2} + m^1_{c_1 \to v_2}$$

$$m^2_{v_3 \to c_2} = m^2_{r \to v_3} + m^2_{c_1 \to v_3}$$

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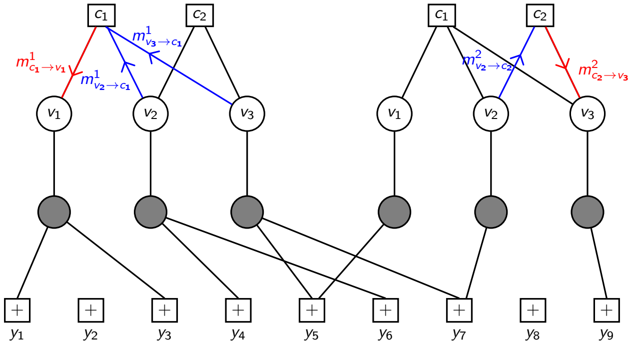
Message Passing: Check to Variable



$$m^1_{c_1 \to v_1} = 2\tanh^{-1}\left(\tanh\left(\frac{m^1_{v_2 \to c_1}}{2}\right)\tanh\left(\frac{m^1_{v_3 \to c_1}}{2}\right)\right)$$

$$m^2_{c_2 \to v_3} = 2\tanh^{-1}\left(\tanh\left(\frac{m^2_{v_2 \to c_2}}{2}\right)\right)$$

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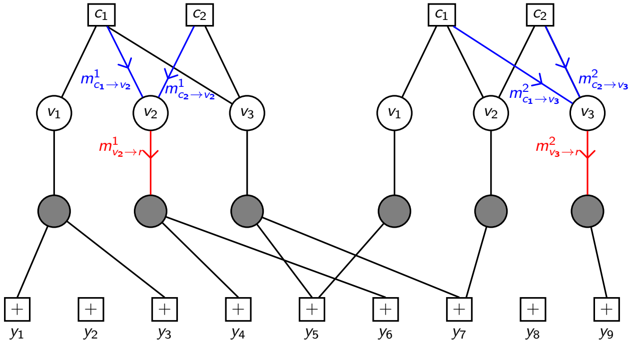
Message Passing:Variable to Repetition



$$m^1_{v_2\to r} = m^1_{c_1\to v_2} + m^1_{c_2\to v_2} \qquad m^2_{v_3\to r} = m^2_{c_1\to v_3} + m^2_{c_2\to v_3}$$

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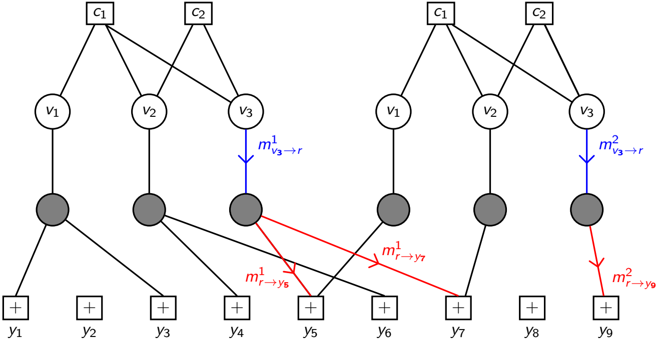
Message Passing: Repetition to MAC



$$m^1_{r \to y_5} = m^1_{v_3 \to r}$$
$$m^1_{r \to y_7} = m^1_{v_3 \to r}$$

$$m^2_{r \to y_9} = m^2_{v_3 \to r}$$

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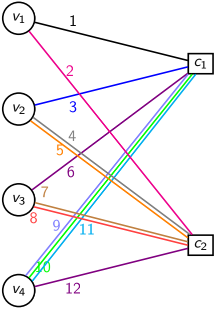
Protograph LDPC Code



- Single user codes are not optimal
- Protograph: small graph used to generate larger Tanner graph
- Actual Tanner graph: expanded or lifted version of protograph
    1. **Copy** protograph z times and **permute** edges of same color

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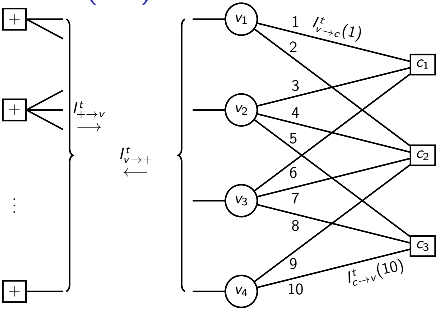
Mutual Information (MI) over Iterations



- $I^t_{c\to v}(e)$: MI between the message from check node to variable node along edge type e and codeword bit
- $I^t_{+\to v}$: Average MI between the message from MAC nodes to variable nodes and the associated codeword bits
- MAC node degree distribution $\sim poi(q)$, where $q = \frac{Nl}{N_c}$

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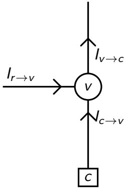
Mutual Information (MI) over Iterations

Variable to check node



[2]D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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Mutual Information (MI) over Iterations

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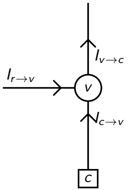

- $m_{v\to c} = m_{c\to v} + m_{r\to v}$
- $m_{c\to v} \sim \mathcal{N}(\frac{\sigma^2_{c\to v}}{2}, \sigma^2_{c\to v})$ and $m_{r\to v} \sim \mathcal{N}(\frac{\sigma^2_{r\to v}}{2}, \sigma^2_{r\to v})$
- $J(\sigma)$: MI between message m and associated codeword bit, where $m \sim \mathcal{N}(\frac{\sigma^2}{2}, \sigma^2)$

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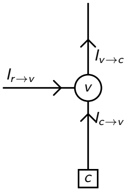
Mutual Information (MI) over Iterations

Variable to check node



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$$\begin{aligned} I_{v\to c} = &= J(\sqrt{\sigma^2_{c\to v} + \sigma^2_{r\to v}}) \\ &= J\left(\sqrt{[J^{-1}(I_{c\to v})]^2 + [J^{-1}(I_{r\to v})]^2}\right) \end{aligned}$$

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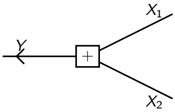
Mutual Information (MI) over Iterations

MAC to variable node



Two user adder channel

[2]D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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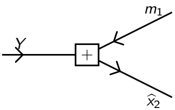
Mutual Information (MI) over Iterations

MAC to repetition node



Soft interference cancellation

- $\widehat{X}_2 = Y - \tanh\left(m_1/2\right)$
- $X_2 = \widehat{X}_2 + Z_\mathsf{I}$, where $Z_\mathsf{I} \sim \mathcal{N}(0, \sigma_\mathsf{I}^2)$

[2]D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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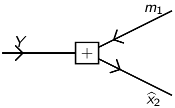
Mutual Information (MI) over Iterations

MAC to repetition node



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[2]D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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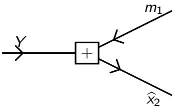

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- $I_{+\to r} = J(\frac{2}{\sigma_I})$

[2] D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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Mutual Information (MI) over Iterations

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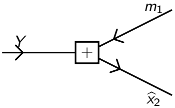

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- $I_{+\to r} = J(\frac{2}{\sigma_I})$
- $I_{+\to r} = \sum_k \gamma_k J(\frac{2}{\sqrt{(k-1)\sigma_I^2 + \sigma^2}})$

[2] D. Guo, S. Shamai, and S. Verdu, "Mutual information and minimum mean-square error in Gaussian channel"

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Density Evolution and Threshold

Density Evolution

Compute $I^{t}_{+\to v}, I^{t}_{v\to +}, I^{t}_{v\to c}(i), I^{t-1}_{c\to v}(i)$ from $I^{t-1}_{+\to v}, I^{t-1}_{v\to +} I^{t-1}_{v\to c}(i), I^{t-1}_{c\to v}(i)$ for $t = 1, 2, \cdots, \infty$

[4] R. Storn and K. Price, "Differential evolution a simple and efficient heuristic for global optimization over continuous spaces," Journal of Global Optimization.

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Density Evolution and Threshold

Density Evolution

Compute $I^t_{+\to v}, I^t_{v\to +}, I^t_{v\to c}(i), I^{t-1}_{c\to v}(i)$ from $I^{t-1}_{+\to v}, I^{t-1}_{v\to +} I^{t-1}_{v\to c}(i), I^{t-1}_{c\to v}(i)$ for $t = 1, 2, \cdots, \infty$

Threshold

Threshold σ^*= maximum σ such that $I_{v\to c}(i) \to 1$ for each $i \in E$

[4] R. Storn and K. Price, "Differential evolution a simple and efficient heuristic for global optimization over continuous spaces," Journal of Global Optimization.

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Density Evolution and Threshold

Density Evolution

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Threshold σ^*= maximum σ such that $I_{v\to c}(i) \to 1$ for each $i \in E$

Optimization

Optimize the protograph and repetition factor to maximize the threshold using differential evolution[4]

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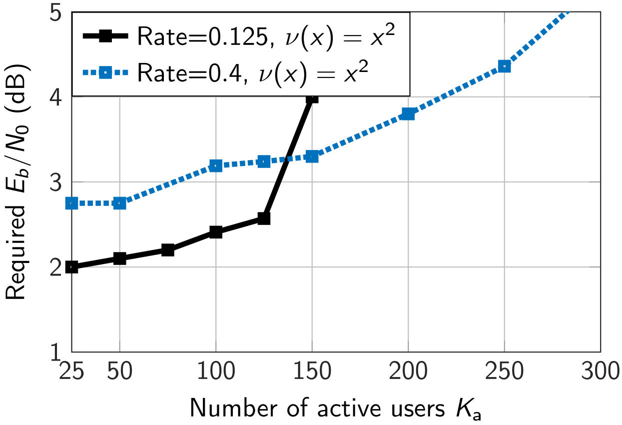
Rate of the LDPC Code vs K



- Optimal rate changes with K

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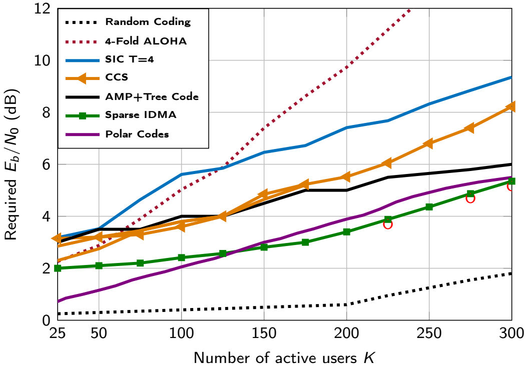
Performance Comparison



- $B = 100$, $N = 30000$
- Only 3.2 dB away from Polyanksiy's achievability result

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Takeaways

- Slotted ALOHA - interference cancellation for handling interference
- Proposed an IDMA like scheme for using the dimensions better
- Sparse IDMA vs. IDMA
  - Sparsity allows us to control interference
  - Makes it easier to design LDPC like codes
- Low complexity scheme for large number of users

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Algorithms for Coded Random Access and Inference in Large Dimensional Spaces – Bridging Fundamental Limits and Practical Schemes
Part V - Polar coding and Spreading

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Vamsi Amalladinne, Avinash Vem, Asit Pradhan

Electrical and Computer Engineering
Texas A&M University

IISc, India
2020

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What Part V is about

- (Non-orthogonal) spreading sequences for controlling interference
- Spreading + Polar codes + list decoding

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Encoding

$$\boxed{\underline{\mathsf{w}} = (\underline{\mathsf{w}}_{\mathrm{s}}, \underline{\mathsf{w}}_{\mathrm{c}})} \xrightarrow{\underline{\mathsf{w}}_{\mathrm{s}} \in \mathbb{F}_2^{B_{\mathrm{s}}}} \boxed{h(\cdot)} \xrightarrow{j} \boxed{\mathbf{S} = \begin{bmatrix} | & \cdots & | & \cdots & | \\ \underline{\mathsf{s}}_1 & \cdots & \underline{\mathsf{s}}_j & \cdots & \underline{\mathsf{s}}_{2^{B_{\mathrm{s}}}} \\ | & \cdots & | & \cdots & | \end{bmatrix}}$$

- Divide the message into two parts: $w_{\mathrm{s}}, w_{\mathrm{c}}$
- Based on w_{s} a spreading sequence is chosen from the set $\mathbf{S}$

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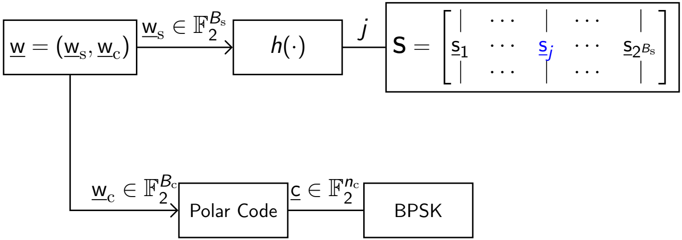
Encoding



- Divide the message into two parts: w_s, w_c
- Based on w_s a spreading sequence is chosen from the set $\mathbf{S}$
- w_c is encoded using a polar code

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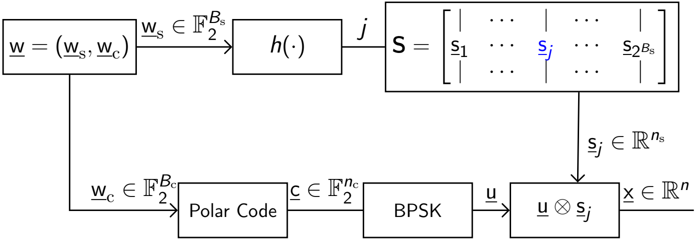
Encoding



- Divide the message into two parts: $w_{\mathrm{s}}, w_{\mathrm{c}}$
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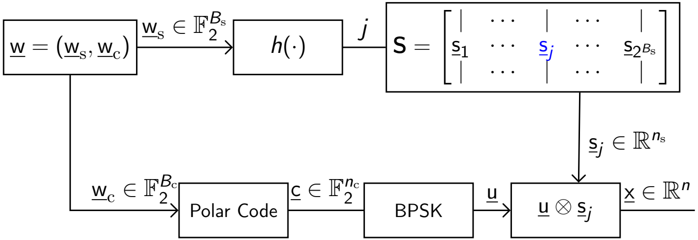
- w_{c} is encoded using a polar code
- Coded bits are spread using the spreading sequence $\underline{s}_j$

3 / 17

Encoding



- Divide the message into two parts: w_s, w_c
- Based on w_s a spreading sequence is chosen from the set $\mathbf{S}$
- w_c is encoded using a polar code
- Coded bits are spread using the spreading sequence $\underline{s}_j$
- 2^{B_s} is not too large
- With non-trivial probability, multiple users will choose the same $\underline{s}_j$

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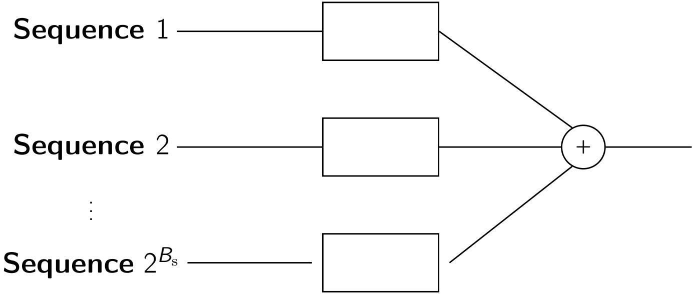
Transmitter from the Spreading Sequence Perspective



- $\mathcal{M}_j$: set of active users who choose $\underline{\mathsf{s}}_j$

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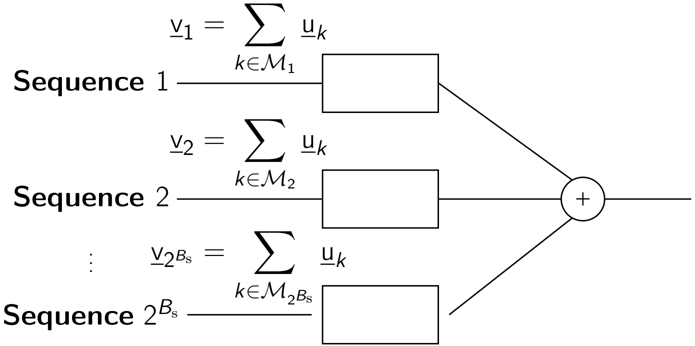
Transmitter from the Spreading Sequence Perspective



- $\mathcal{M}_j$: set of active users who choose $\underline{s}_j$
- Sum of the codewords associated with sequence $\underline{s}_j$: $\underline{v}_j = \sum_{k \in \mathcal{M}_j} \underline{u}_k$

4 / 17

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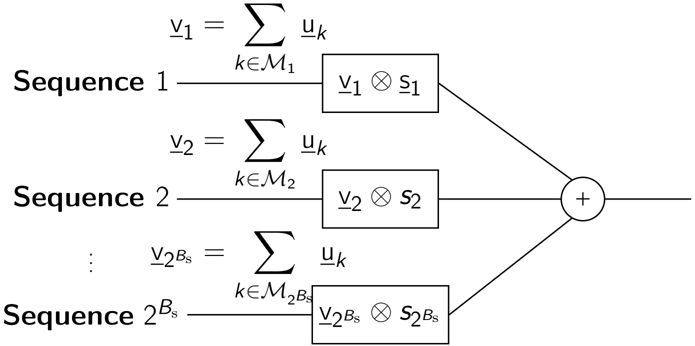

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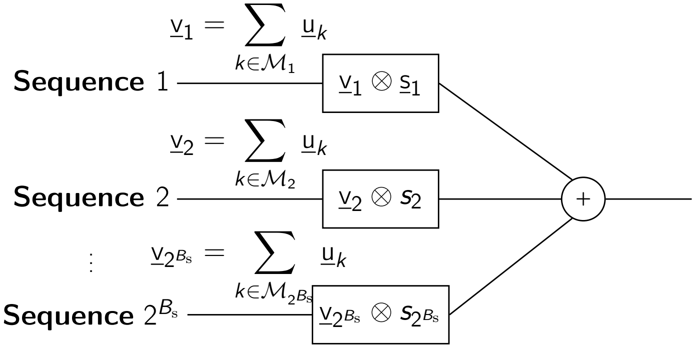

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- $\mathbf{V} := \begin{bmatrix} \underline{v}_1^{\mathsf{T}} & \underline{v}_2^{\mathsf{T}} & \cdots & \underline{v}_{2^{B_s}}^{\mathsf{T}} \end{bmatrix}^{\mathsf{T}}$

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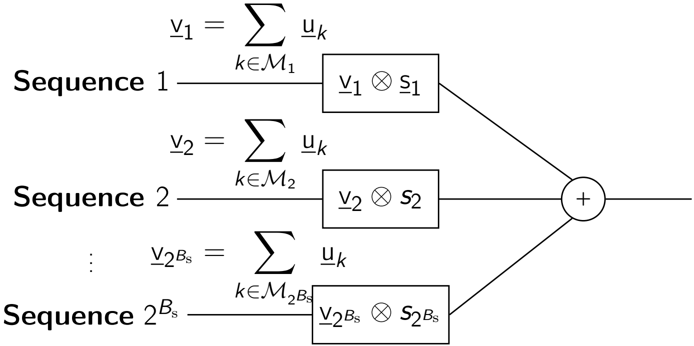

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- $\underline{\mathrm{y}} = \underbrace{\underline{\mathrm{y}}(1 : n_\mathrm{s})}_{\underline{\mathrm{y}}_1^\mathsf{T}} \underbrace{\underline{\mathrm{y}}(n_\mathrm{s}+1 : 2n_\mathrm{s})}_{\underline{\mathrm{y}}_2^\mathsf{T}} \cdots \underbrace{\underline{\mathrm{y}}((i-1)n_\mathrm{s}+1 : in_\mathrm{s})}_{\underline{\mathrm{y}}_i^\mathsf{T}} \cdots \underbrace{\underline{\mathrm{y}}(N - n_\mathrm{s}+1 : n_c)}_{\underline{\mathrm{y}}_{n_\mathrm{c}}^\mathsf{T}}$

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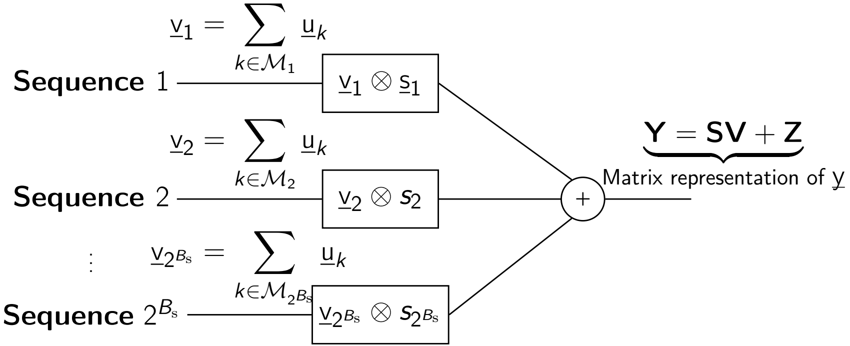
Transmitter from the Spreading Sequence Perspective



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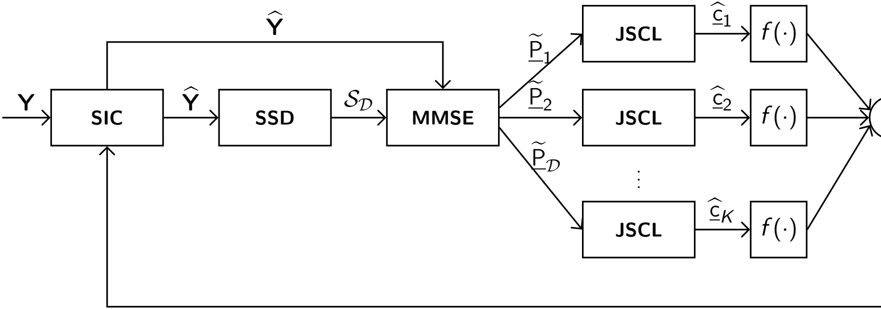
Main Components of the Receiver



- Blind Spreading Sequence detector (SSD)
- Soft Output MMSE Multi-user Detector
- Joint successive cancellation list (JSCL) decoder of polar codes + CRC
- Successive interference canceller (SIC)

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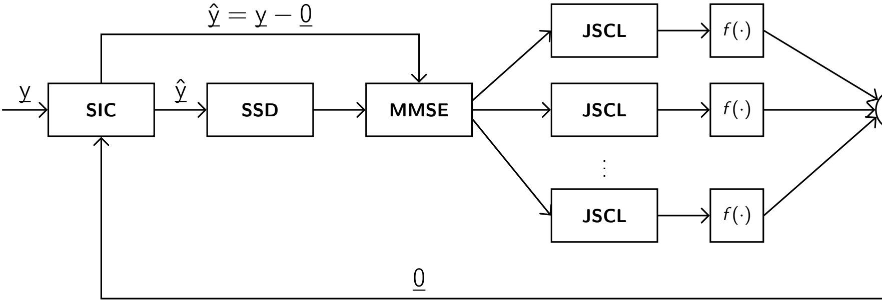
Illustration of Decoding: $K = 3$

- User 1 picks $\underline{s}_5$, $\underline{v}_5 = \underline{u}_1$
- Users 2 and 3 pick $\underline{s}_1$, $\underline{v}_1 = \underline{u}_2 + \underline{u}_3$



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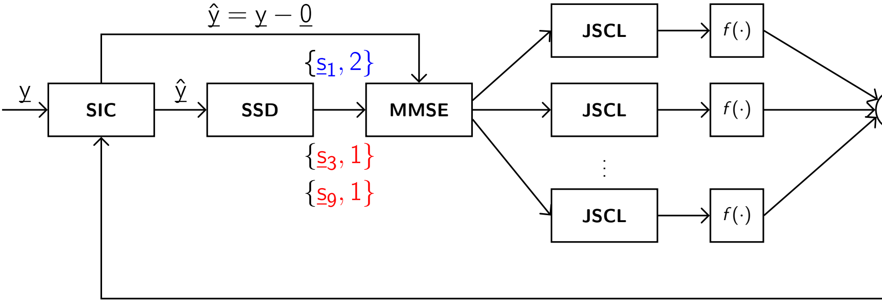

Iteration 1

- $S_{\mathcal{D}} = \{\underline{s}_1, \underline{s}_3, \underline{s}_9\}$.

6 / 17

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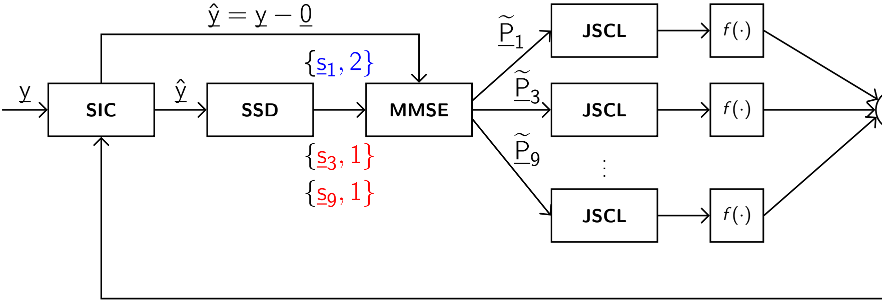

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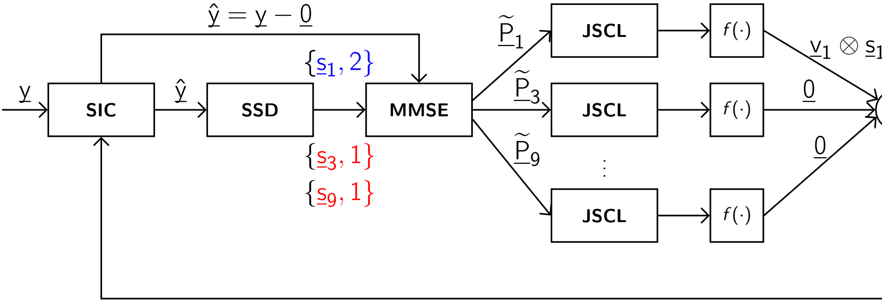

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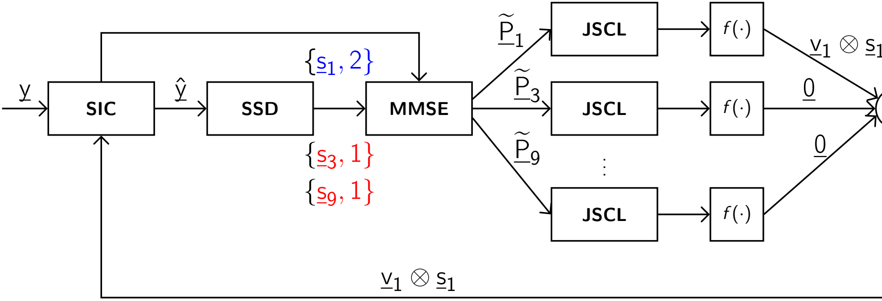

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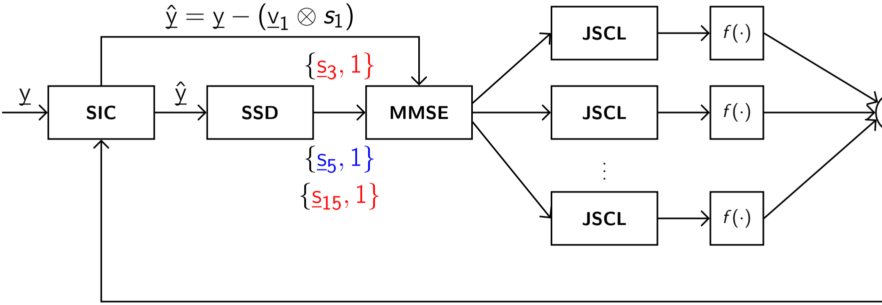

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- $S_{\mathcal{D}} = \{\underline{s}_3, \underline{s}_5, \underline{s}_{15}\}$.

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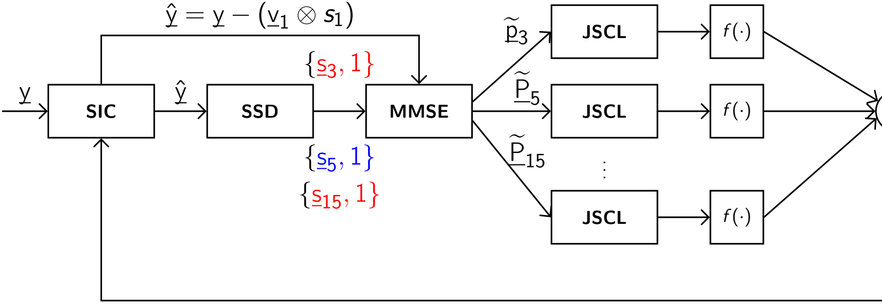

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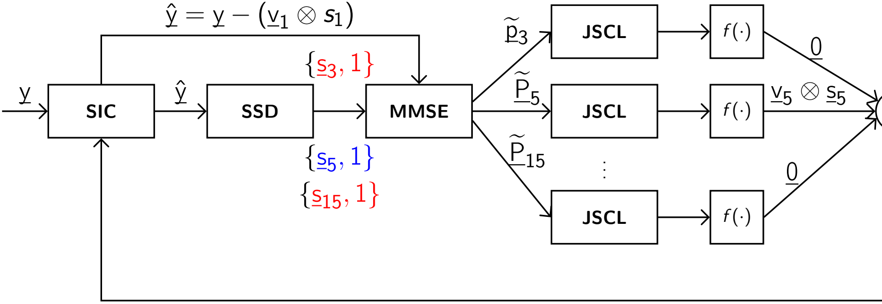

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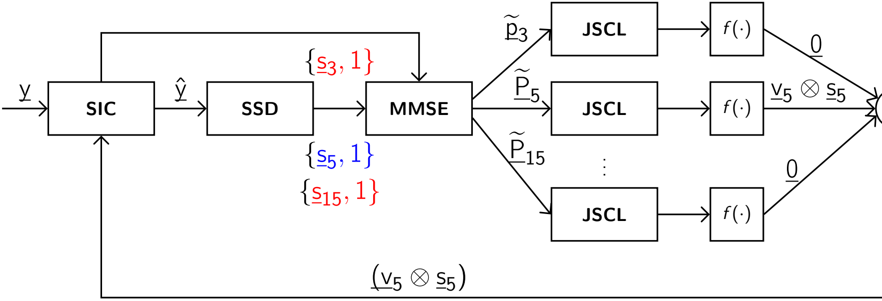

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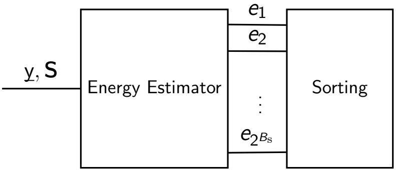
(Blind) Spreading Sequence Detector



- For each $\underline{s}_j \in \mathbf{S}$ compute the statistic $e_j = \sum_{i=1}^{n_\mathrm{c}} \left(\underline{y}_i^\mathsf{T} \underline{s}_j\right)^2$

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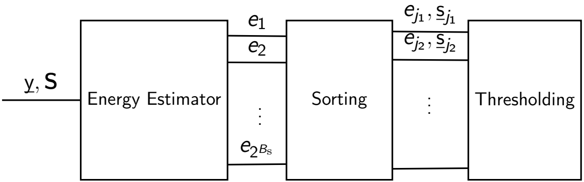
(Blind) Spreading Sequence Detector



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- Sort sequences in descending order of their statistics

7 / 17

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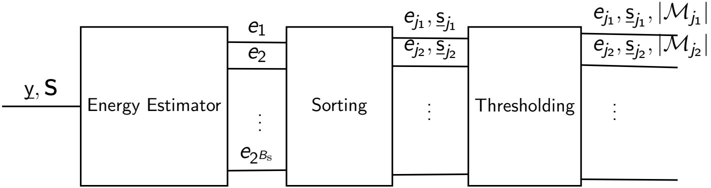

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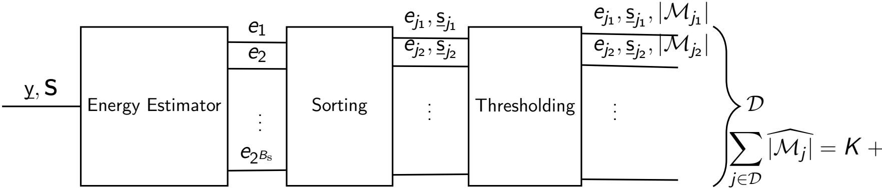

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- Sort sequences in descending order of their statistics
- Based on e_j compute estimate $\widehat{|\mathcal{M}_j|}$ of $|\mathcal{M}_j|$
- Output first $|\mathcal{D}|$ sequences from the sorted list
- Define $\widehat{\mathbf{M}} := \mathrm{diag}(|\widehat{\mathcal{M}}_1|, |\widehat{\mathcal{M}}_2|, \ldots, |\widehat{\mathcal{M}}_{|\mathcal{D}|}|)$

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MMSE Estimator

- The received signal is hypothesized as

$$\mathbf{Y} = \mathbf{S}_{\mathcal{D}}\mathbf{V}_{\mathcal{D}} + \mathbf{Z}$$

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MMSE Estimator

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$$\mathbf{Y} = \mathbf{S}_{\mathcal{D}}\mathbf{V}_{\mathcal{D}} + \mathbf{Z}$$

- Pass $\mathbf{Y}$ through a MMSE filter to obtain an estimate $\widetilde{\mathbf{V}}$

$$\widetilde{\mathbf{V}} = \begin{bmatrix} \widetilde{\underline{\mathsf{v}}}_1 \\ \widetilde{\underline{\mathsf{v}}}_2 \\ \vdots \\ \widetilde{\underline{\mathsf{v}}}_{|\mathcal{D}|} \end{bmatrix} = \underbrace{\widehat{\mathbf{M}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}}(\mathbf{S}_{\mathcal{D}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}} + I_{n_{\mathrm{s}}})^{-1}}_{\text{Linear MMSE filter}}\mathbf{Y}$$

8 / 17

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- The error covariance matrix is given by

$$\Sigma = I_{|\mathcal{D}|} - \widehat{\mathbf{M}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}}(\mathbf{S}_{\mathcal{D}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}} + I_{n_{\mathrm{s}}})^{-1}\widehat{\mathbf{M}}\mathbf{S}_{\mathcal{D}}$$

- We convert $\widetilde{v}_j$ and Σ_{jj} into LLRs to be fed to Polar decoder

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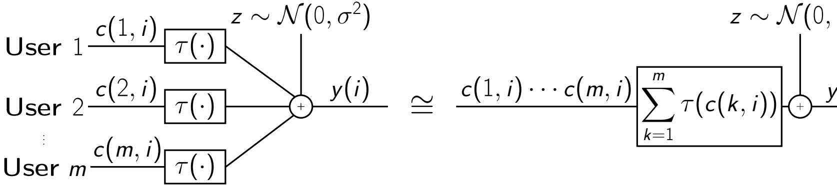
JSCL Decoding of Polar Codes

- Recall that multiple users can pick the same spreading sequence
- m-user GMAC over $\mathbb{F}_2$ is equivalent to single user AWGN over $\mathbb{F}_2^m$.



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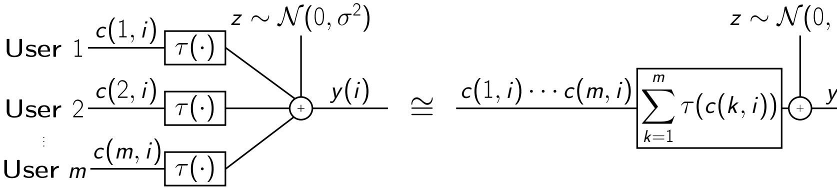

- $\underline{c}(:,i) = \begin{bmatrix} \underline{c}(1,i) & \underline{c}(2,i) & \cdots & \underline{c}(m,i) \end{bmatrix}$
- $\Pr(\underline{c}(:,i) = \mathrm{g} | y(i)) \propto \exp\left(-\frac{(y(i)-\tau(\mathrm{g}))^2}{2\sigma^2}\right)$, for $\mathrm{g} \in \mathbb{F}_2^m$

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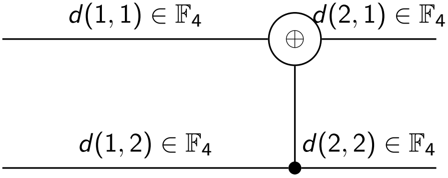
Example: JSC Decoding of Polar Codes

- $m = 2, n_\mathrm{c} = 2$



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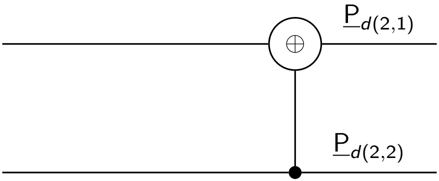
Example: JSC Decoding of Polar Codes

- $m = 2, n_{\mathrm{c}} = 2$
- $$\underline{\mathrm{P}}_{d(2,1)} = \Pr\left(d(2,1)|y(1)\right) = \{\Pr\left(00|y(1)\right), \Pr\left(01|y(1)\right), \Pr\left(10|y(1)\right), \Pr\left(11|y(1)\right)\}$$
- $$\underline{\mathrm{P}}_{d(2,2)} = \Pr\left(d(2,2)|y(2)\right) = \{\Pr\left(00|y(2)\right), \Pr\left(01|y(2)\right), \Pr\left(10|y(2)\right), \Pr\left(11|y(2)\right)\}$$



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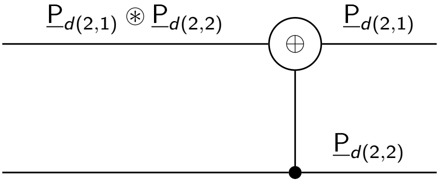

- $\underline{\mathrm{P}}_{d(1,1)} = \underline{\mathrm{P}}_{d(2,1)} \circledast \underline{\mathrm{P}}_{d(2,2)}$

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Example: JSC Decoding of Polar Codes

- $m = 2, n_{\mathrm{c}} = 2$
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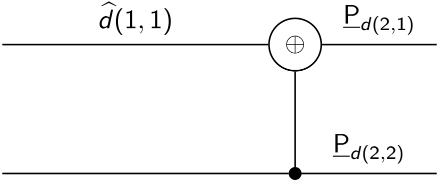
- $$\underline{\mathrm{P}}_{d(2,2)} = \Pr\left(d(2,2)|y(2)\right) = \{\Pr\left(00|y(2)\right), \Pr\left(01|y(2)\right), \Pr\left(10|y(2)\right), \Pr\left(11|y(2)\right)\}$$



- $\underline{\mathrm{P}}_{d(1,1)} = \underline{\mathrm{P}}_{d(2,1)} \circledast \underline{\mathrm{P}}_{d(2,2)}$
- Based on $\underline{\mathrm{P}}_{d(1,1)}$ make a hard decision $\widehat{d}(1,1)$ on $d(1,1)$

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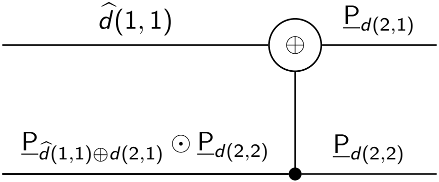

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- $\underline{\mathrm{P}}_{d(1,2)} = \underline{\mathrm{P}}_{\widehat{d}(1,1)+d(2,1)} \odot \underline{\mathrm{P}}_{d(2,2)}$

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Successive Interference Cancellation

- If the decoding is successful, remove $\underline{\widetilde{v}}_j$ from $\underline{y}$

$$\underline{y} = \underline{y} - \underline{v}_j \otimes \underline{s}_j$$

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Choice of Parameters

Parameters to choose

- Spreading sequence length
- Rate of the code
- Number of spreading sequences in the master list

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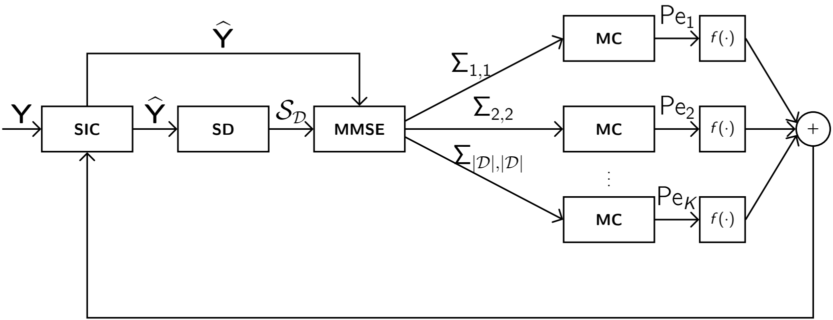
Density Evolution Using Meta-Converse (MC) Bound



- $\Sigma = I_{|\mathcal{D}|} - \widehat{\mathbf{M}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}}(\mathbf{S}_{\mathcal{D}}\mathbf{S}_{\mathcal{D}}^{\mathsf{T}} + I_{n_s})^{-1}\widehat{\mathbf{M}}\mathbf{S}_{\mathcal{D}}$

- $f(\mathrm{Pe}_j) = \begin{cases} \underline{\mathrm{v}}_1 \otimes s_1, \text{ with probability } 1 - \mathrm{Pe}_j \\ 0, \text{ with probability } \mathrm{Pe}_j \end{cases}$

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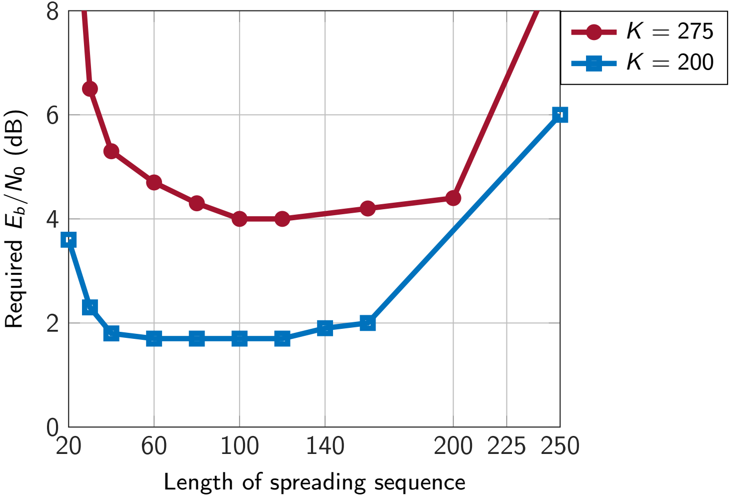
SNR versus Length of Spreading Sequences



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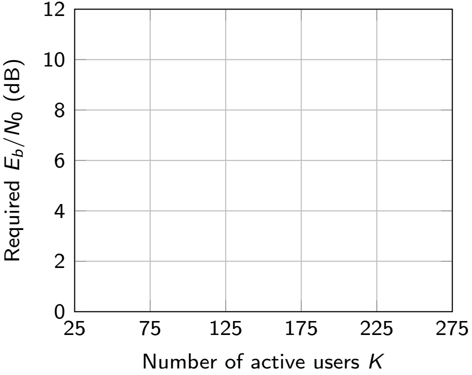
Comparison



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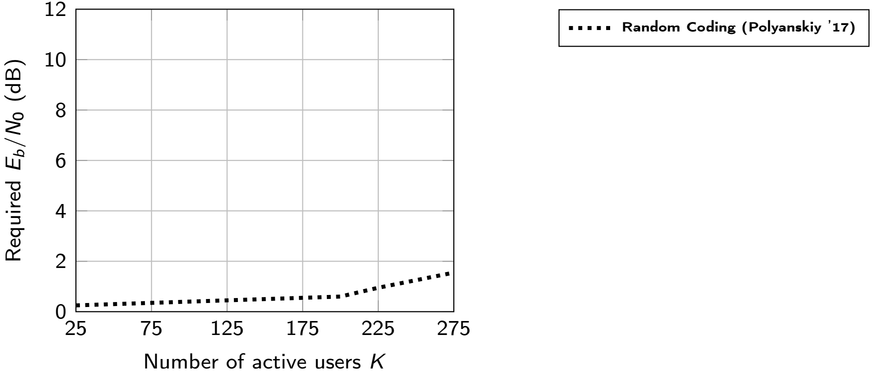
Comparison



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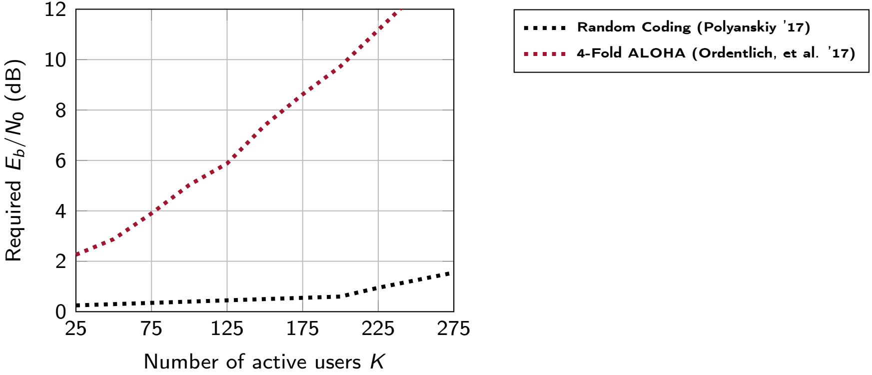

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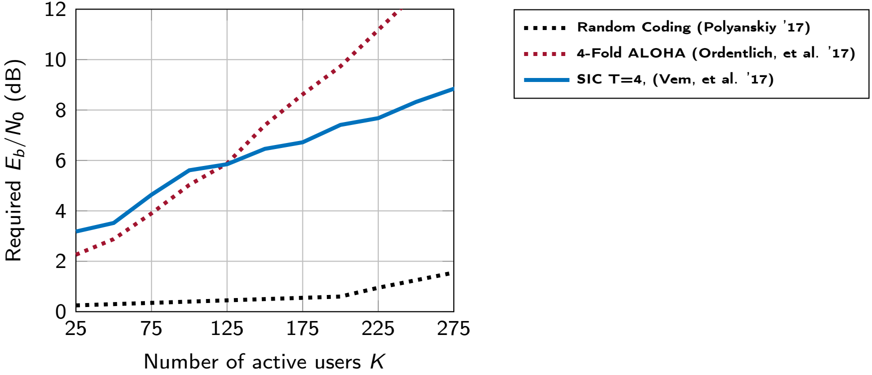

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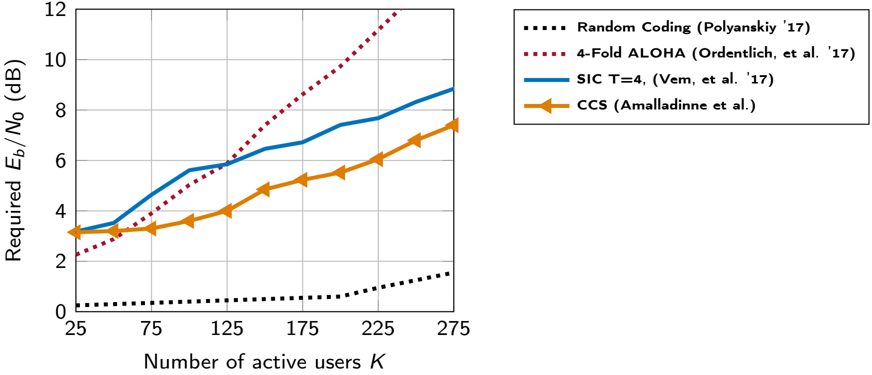
Comparison



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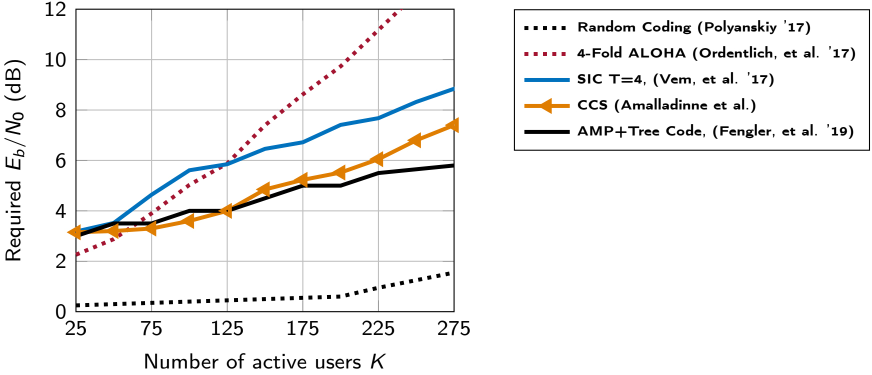
Comparison



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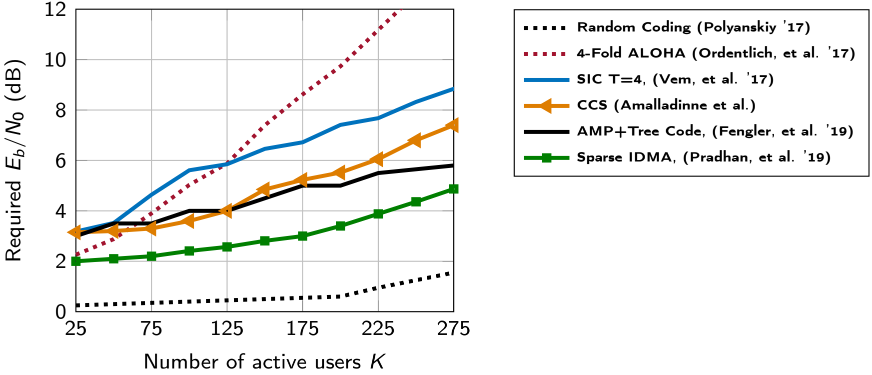
Comparison



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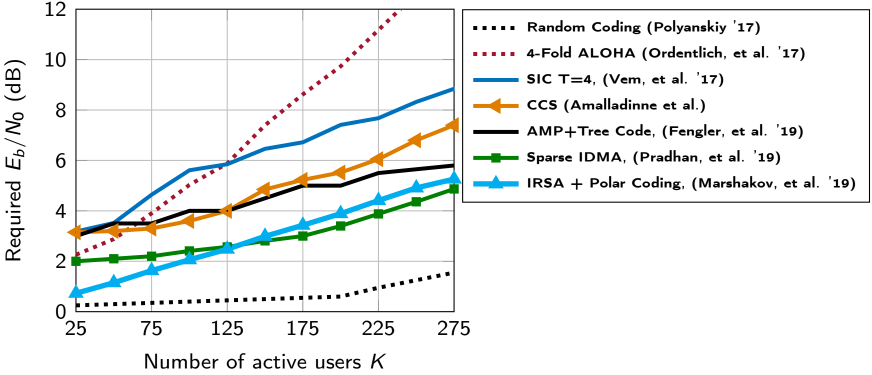
Comparison



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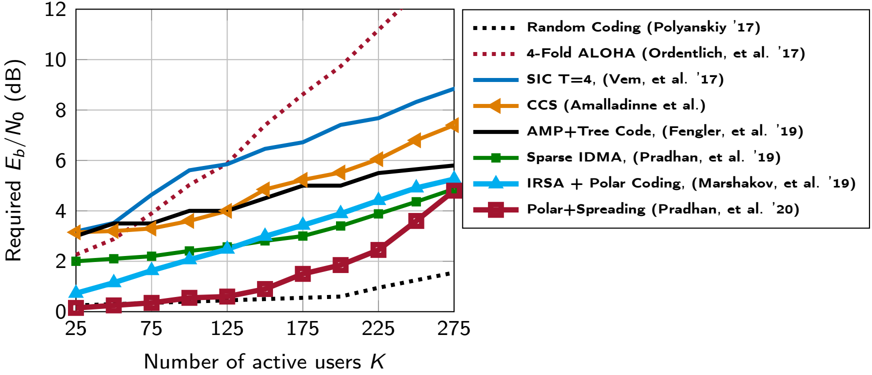
Comparison



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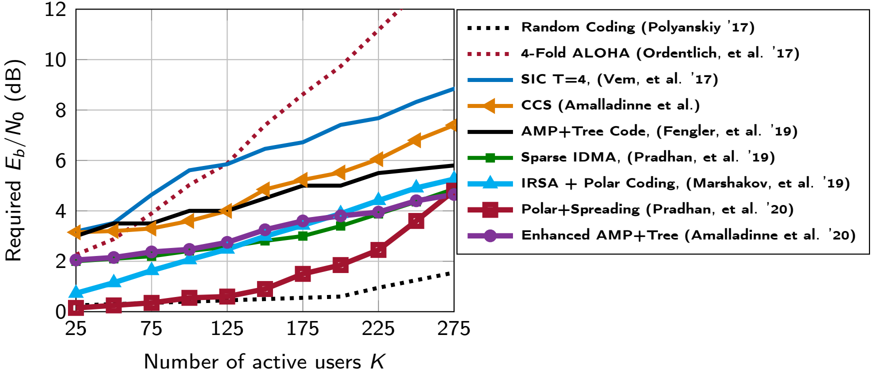
Comparison



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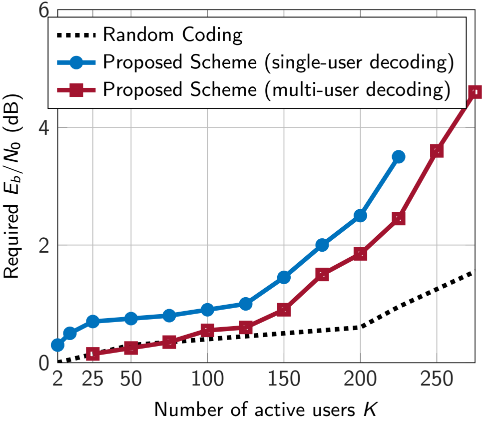
Simulation Results



- List size - 32
- m - 4
- CRC length - 16 bits

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Take Aways

- Proposed a receiver with complexity $O(K^3)$ (can be reduced)
- Blind sequence detection + classical SIC+MMSE receivers
- Near finite length bound achieving codes are required (CRC+Polar+List)
- All these are standard components of a 5G system
- Scaling with the number of users should be improved

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Algorithms for Coded Random Access and Inference in Large Dimensional Spaces – Bridging Fundamental Limits and Practical Schemes Part VI - Designing Sensing Matries for Heavy Hitters Based on Multiple Access Codes

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2020

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This material is also based upon work support, in part, by Qualcomm Technologies, Inc., through their University Relations Program

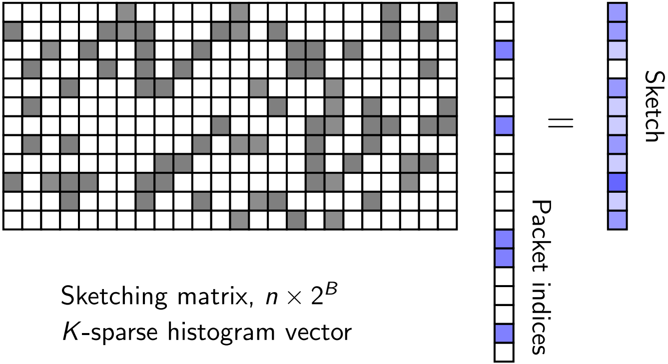
Outline of Part 6

- In Part 3, we introduced a compressed sensing view of unsourced multiple access (UMAC)
- In Parts 4&5, we designed codes for the Unsourced MAC
- Here, we take an unsourced MAC view of compressed sensing
- We will design sensing matrices based on UMAC codes

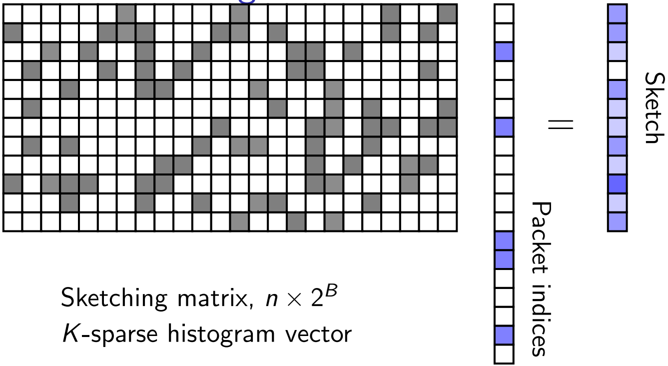
Heavy hitters in data stream computing

Problem - consider a router in a large network

- Count the number of packets from source i to destination j
- Data vector $\mathbf{x}$ is huge, $N = 2^{64}$
- Heavy hitters - only K of them are significant
- Sketch: $\mathbf{y} = \mathbf{Ax}$ is of much lower dimension
- Recovery of $\mathbf{x}$ from $\mathbf{y}$ should happen in sub-linear time



Computational Challenge



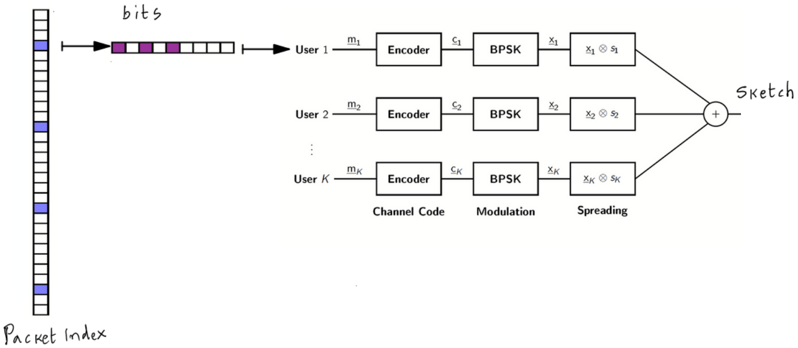
Sketching matrix, $n \times 2^B$

K-sparse histogram vector

Challenge: Very large dimension - K out of $N = 2^B$ sparse

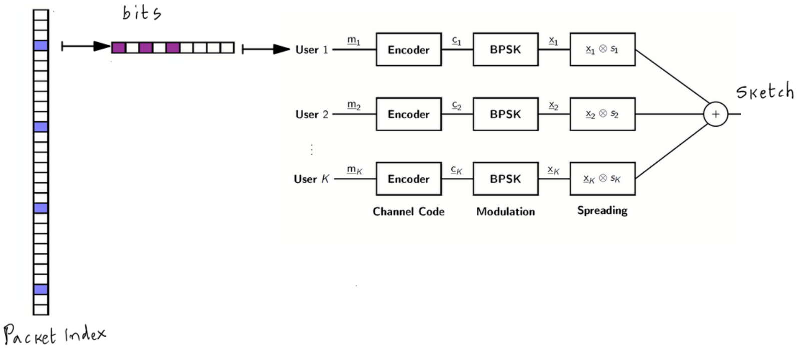
- Computational complexity of commodity CS solvers: $\mathcal{O}(2^B)$
- Sub-linear time algortihms with structured matrices
  - Chaining pursuit (Gilbert et al' 07) - $\mathcal{O}(KB)$
  - Sparse FFT based algorithm (Chen and Guo'17) - - $\mathcal{O}(KB)$
  - Peeling (Li, Ramchandran '16) - $\mathcal{O}(KB)$, $K = 2^{\delta B}, 0 < \delta < 1$
  - UMAC codes - better sample complexity at higher computational complexity

UMAC Codes for Heavy Hitters



- Encode the IP address using UMAC code (Polar+Spreading)
- Sketch $\longleftrightarrow$ received signal in UMAC
- Recovering heavy hitters $\longleftrightarrow$ decoding users

UMAC Codes for Heavy Hitters



- Encode the IP address using UMAC code (Polar+Spreading)
- Sketch $\longleftrightarrow$ received signal in UMAC
- Recovering heavy hitters $\longleftrightarrow$ decoding users
- Different no. of packets $\longleftrightarrow$ Fading
- Codewords of non-heavy hitters $\longleftrightarrow$ noise

Simulation Results, K = 25, $P(HH) = 0.4$