Spatial Sigma-Delta ADCs:
A Low Complexity Architecture for Massive MIMO

Lee Swindlehurst

Center for Pervasive Communications and Computing
University of California Irvine
Hans Fischer Senior Fellow, Institute for Advanced Study
Technical University of Munich

Spatial Sigma-Delta ADCs: A Low Complexity Architecture for Massive MIMO

Collaborators:

Amine Mezghani, Yongzhi Li, Hessam Pirzadeh, Shilpa Rao (UCI)

Hela Jedda, Josef Nossek (TU Munich, Germany)

Gonzalo Seco Granados (Univ. Autònoma de Barcelona, Spain)

Duy Nguyen, Ly V. Nguyen (San Diego State Univ.)

Ken Ma (CUHK), Christos Masouros (Univ. College London), etc.

The Road to Gigabit Wireless (5G and Beyond)

- How do we get to Gb/s wireless links?

- Three symbiotic trends emerging:
  - Deployment of pico- and femto-cells (5-10x decrease in cell size)
  - Millimeter wave frequencies (10x increase in bandwidth)
  - Massive MIMO (10x increase in antennas)

- Putting it all together, there is the potential for 500-1000x increase in throughput

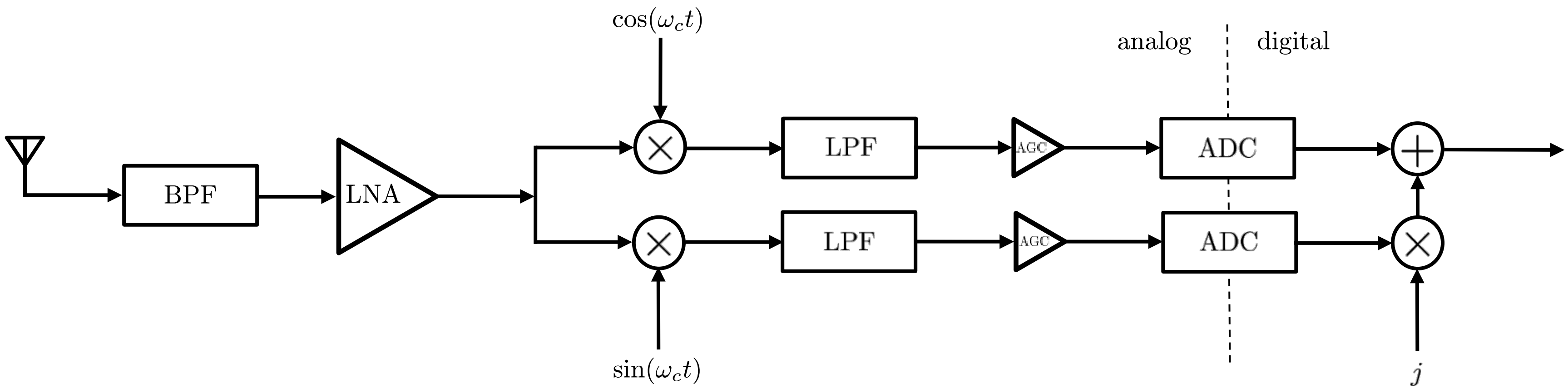
A Symbiotic Relationship

- **<u>Millimeter wave frequencies</u>**
  - short wavelengths
  - larger propagation losses, shorter range operation
  - little multipath, line-of-sight (LOS) or near-LOS
  - low SNR
  - larger Doppler shifts, more sensitive to mobility

- **<u>Massive antenna arrays</u>**
  - large array gain
  - size proportional to wavelength
  - narrow, focused beamforming

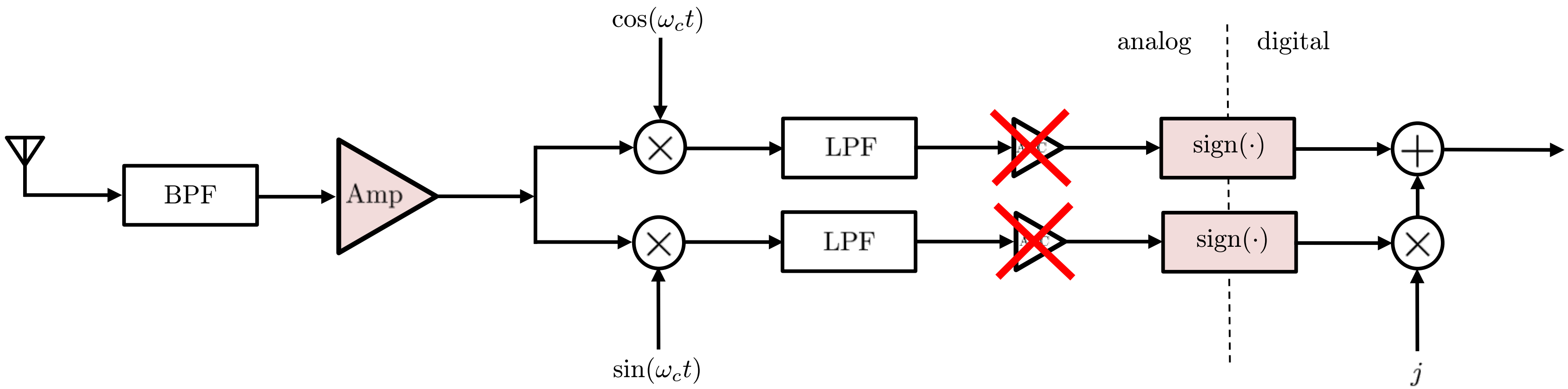
- **<u>Small cells</u>**
  - short range
  - lower power
  - low mobility
  - interference-limited

Standard Receiver Implementation



- Full precision ADC requires linear, low-noise amplifiers and AGC
- ADC power consumption grows exponentially with resolution
- A commercial TI 1 Gs/s 12-bit ADC requires $>$ 1W
- For 100 antennas, 500 Msamp/sec, RRH data rate is more than 1 Tb/sec!
- Not practical for ideal massive MIMO

A One-Bit Receiver



- One-bit ADC $\Rightarrow$ simple RF, no AGC or high cost LNA
- Operates at a fraction of the power (mW)
- Reduce data flow from RRH by 10x
- Performance degradation can be offset by adding more (cheap) antennas
- Compensate for coarse quantization with signal processing
- Unlike hybrid schemes, all antenna outputs available for full digital flexibility

Single Antenna Theoretical Analysis

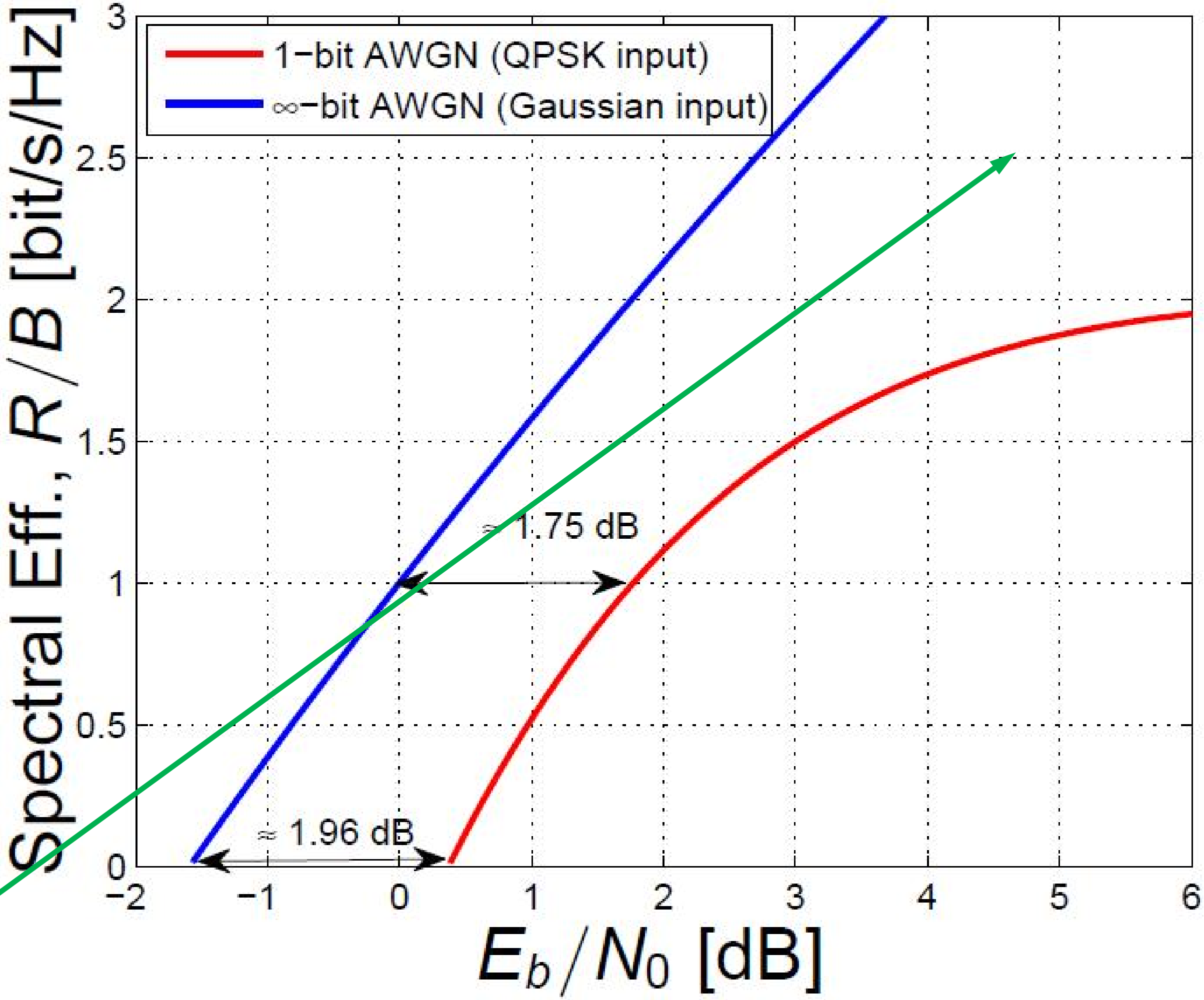
AWGN Channel Capacity

$$B \log_2(1 + \mathrm{SNR})$$

1-Bit AWGN Capacity

$$2B\left(1 - H_b(\Phi(\sqrt{\mathrm{SNR}}))\right)$$

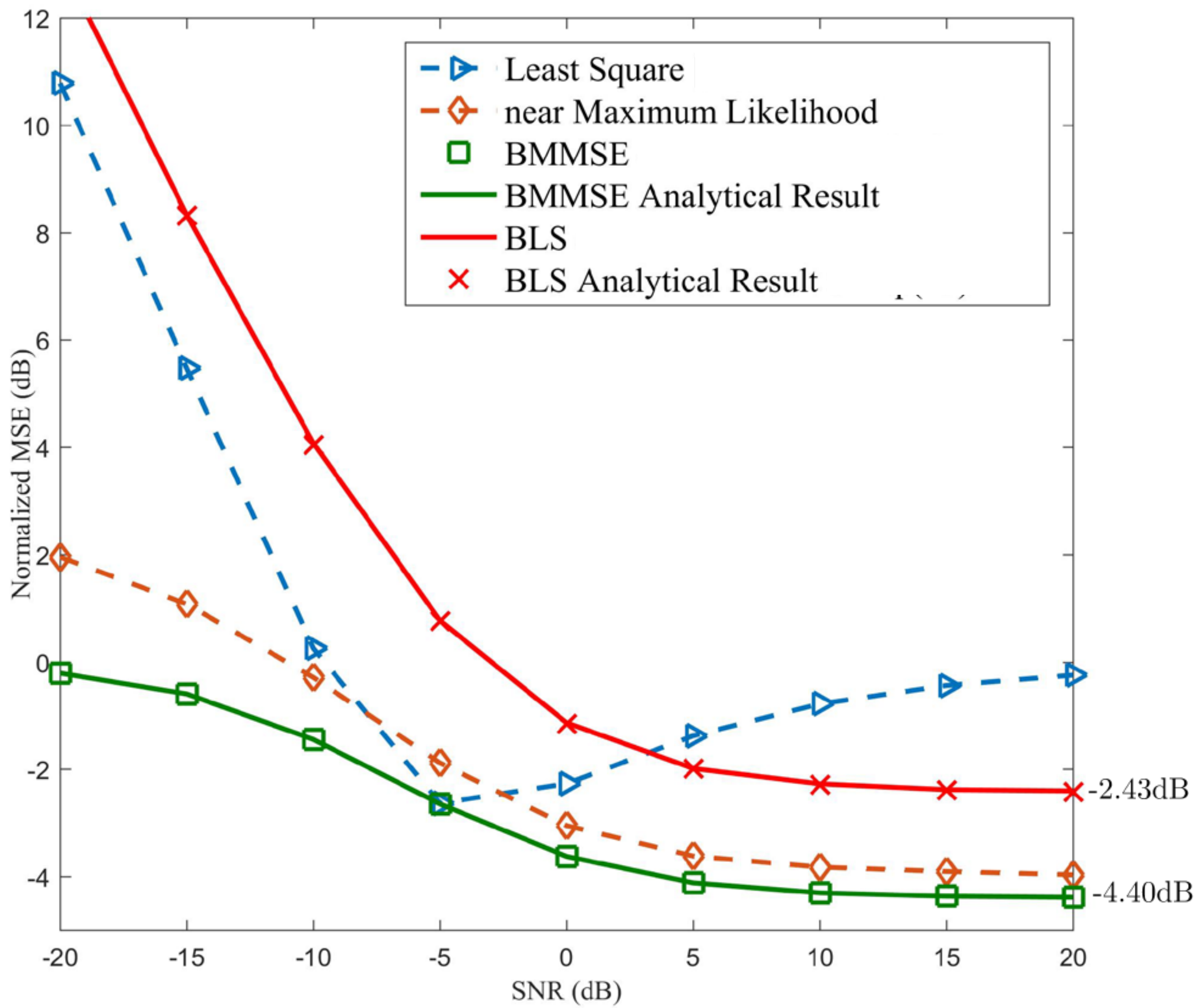
loss in power
efficiency < 2dB
when SE < 1.4 bpcu



large performance gap
(error floor) due to
coarse quantization at
moderate-to-high SNR

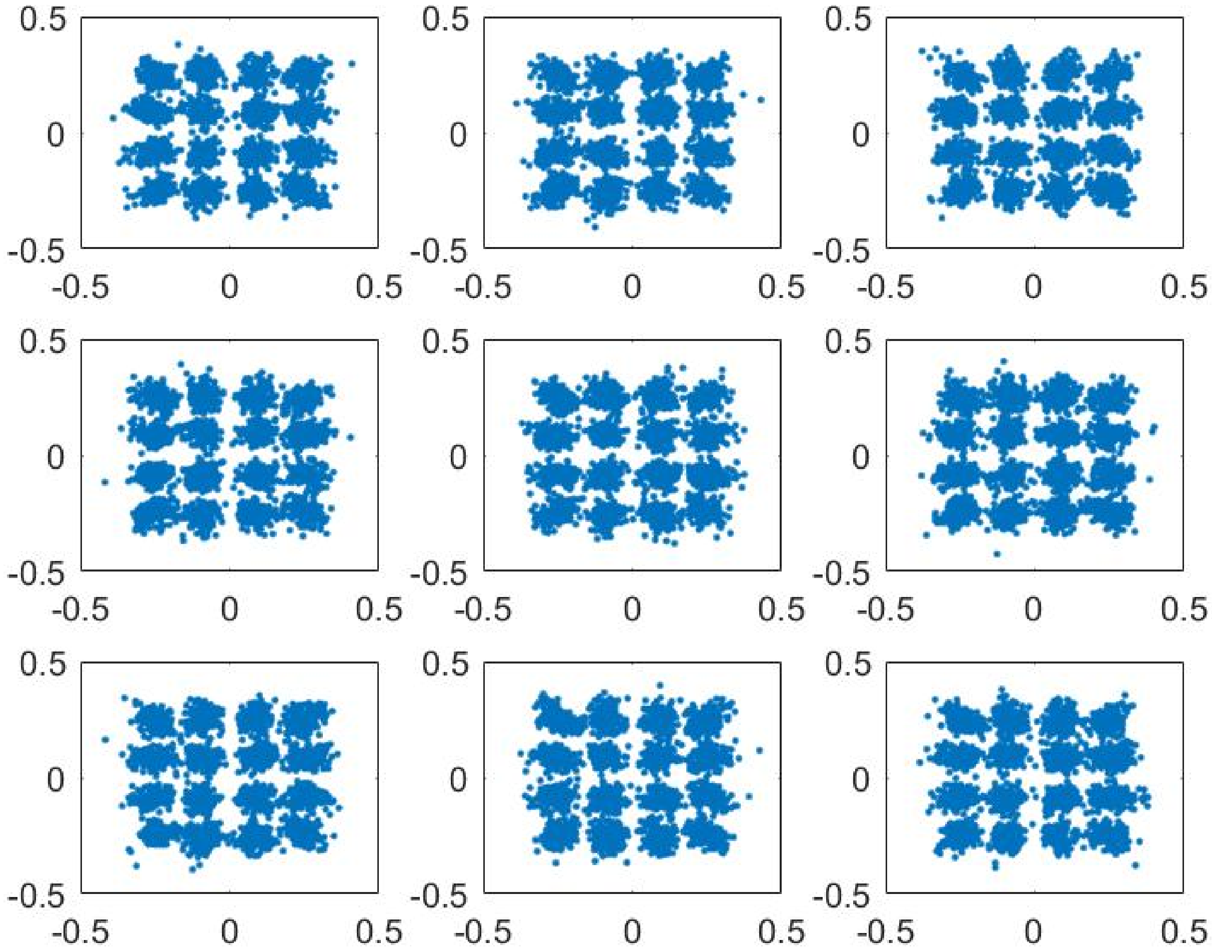
Channel Estimation with One-Bit Receivers

- 128 antennas
- 8 users
- 8 training samples
- Rayleigh fading



16-QAM Example

- 400 antennas
- 9 users
- 5dB SNR
- Rayleigh fading
- LMMSE channel estimation followed by ZF detection



How to Close the Gap?

- Use more bits (studies show 3-5 bits provide good spectral/energy efficiency trade-off)

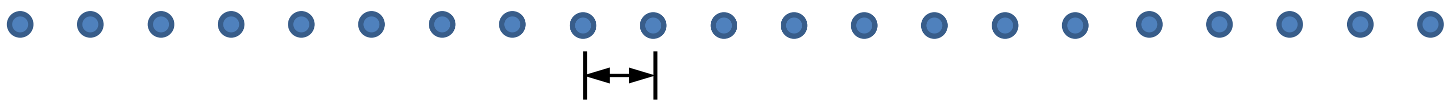
- Sample faster, e.g., using Sigma-Delta ($\Sigma\Delta$) ADCs

- Still, the above methods consume more energy and do not solve the data bottleneck problem

Idea: Use $\Sigma\Delta$ sampling in space = low power, low data rate!

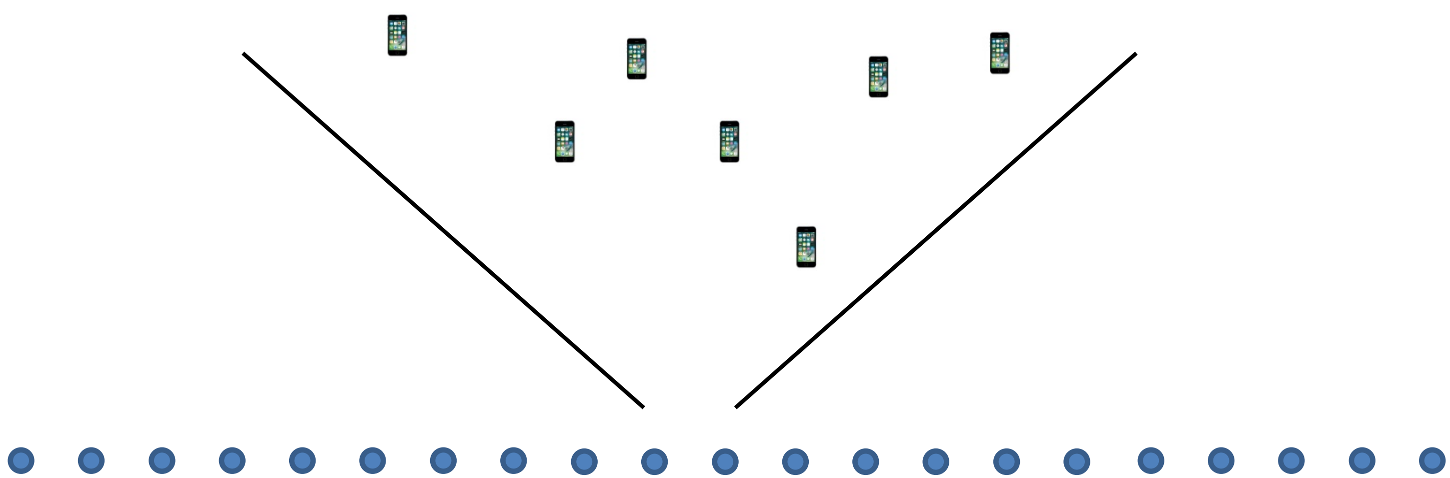
Motivation for Spatial ΣΔ Sampling

In space-constrained scenarios, massive MIMO ⇒ closely spaced antennas (spatial oversampling)



$d < \lambda/2$

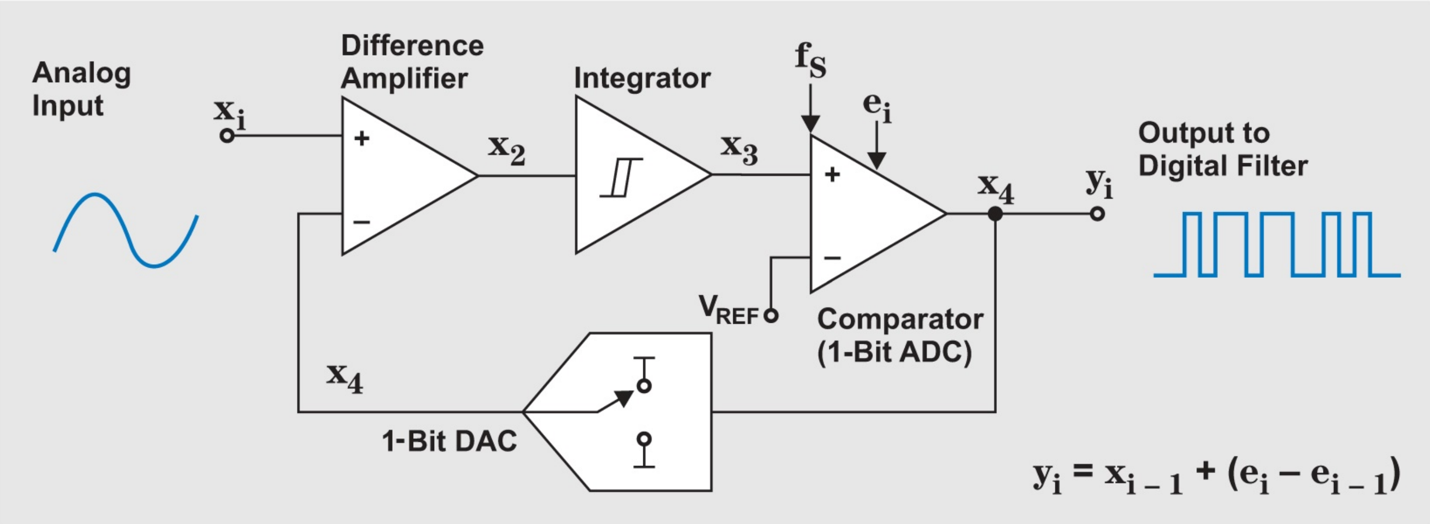
Cellular users are sectorized (either by design or due to environment)



⇒ **Ideal setting for SPATIAL ΣΔ sampling!**

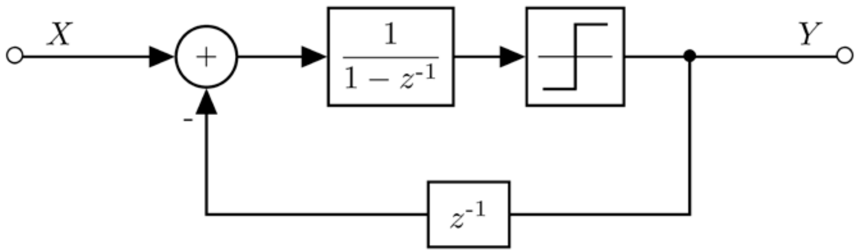
Temporal Oversampling with ΣΔ ADCs

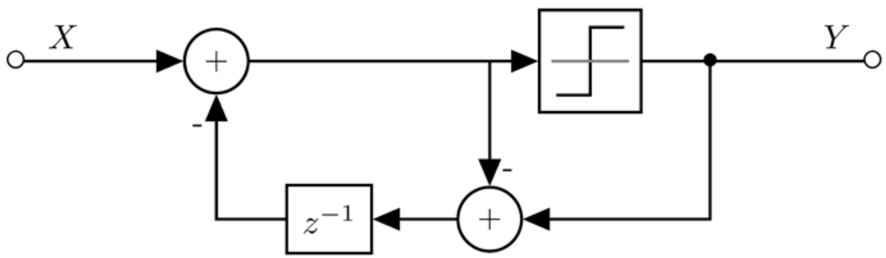
- Oversampling makes desired signal temporally correlated
- Exploit temporal correlation via feedback, quantization of the error signal
- Requires simple additional analog circuitry

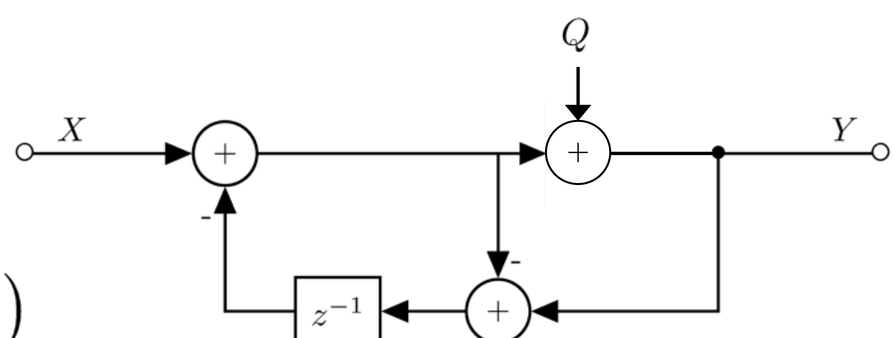


*From Texas Instruments *Analog Applications Journal*

Temporal ΣΔ ADC Discrete-Time Equivalent Models





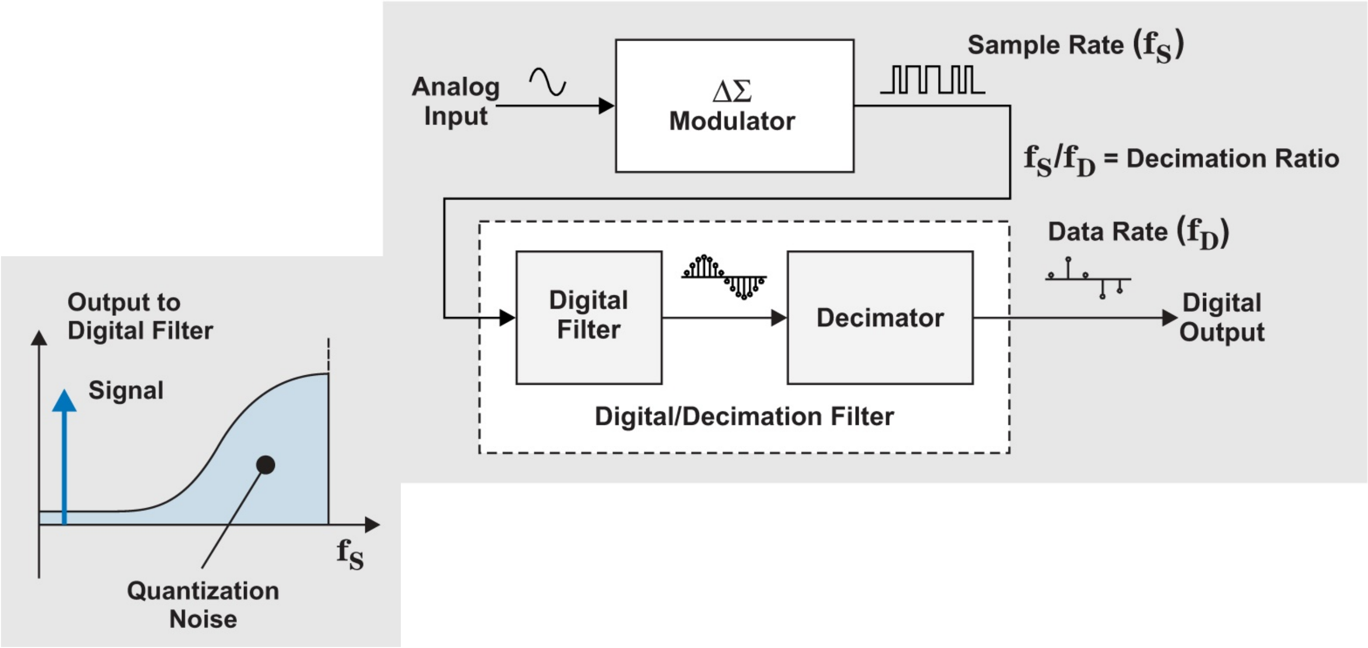


$$Y = X + Q\left(1 - z^{-1}\right)$$

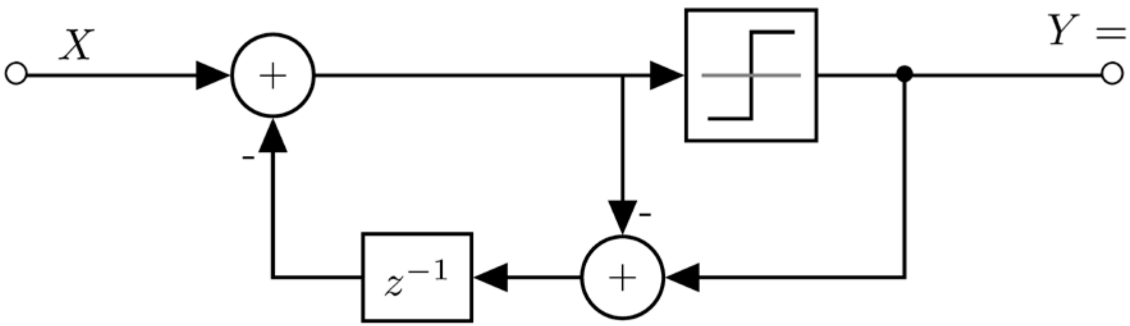
Temporal Oversampling with ΣΔ ADCs

- Desired signal pushed to lower frequencies due to oversampling, quantization noise pushed to higher frequencies (noise shaping)

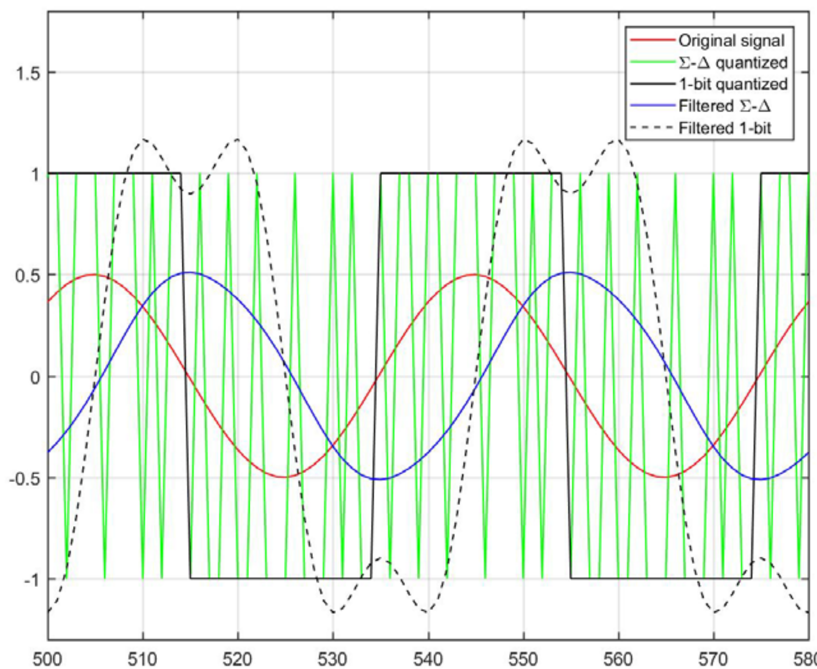
- post-processing low-pass filter and decimation used to recover desired samples



Temporal ΣΔ ADC Example



$$Y = X + Q\left(1 - z^{-1}\right)$$



input $X = \sin(\pi n/20)$

$\Sigma - \Delta$ output

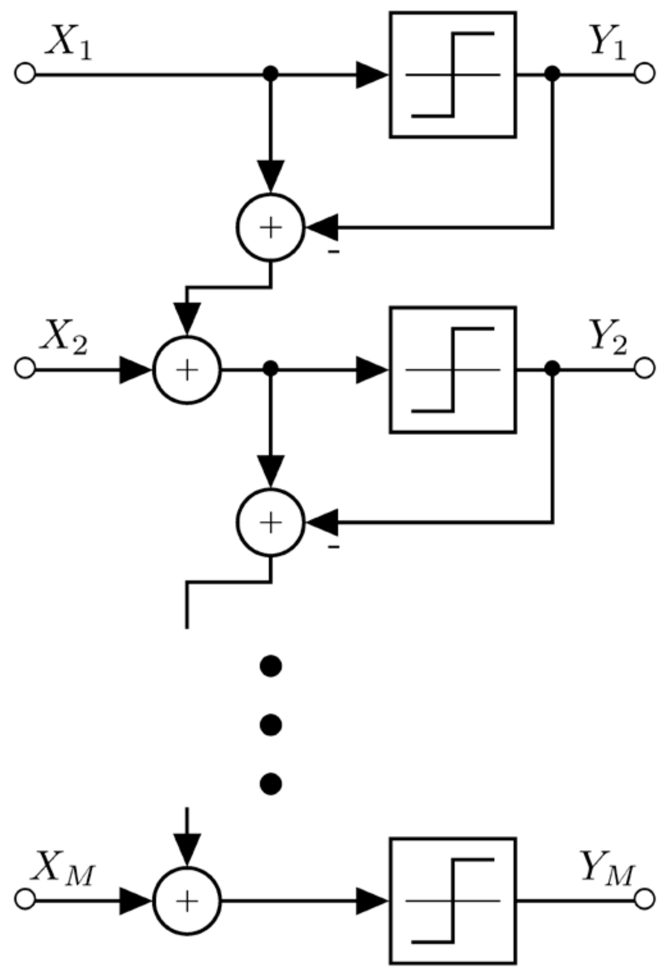
1-bit quantized output

$\Sigma - \Delta$ + LPF, $\omega_c = \frac{\pi}{6}$

1-bit + LPF

Spatial ΣΔ Quantization

Instead of delayed feedback in time, feedback to adjacent antenna:



Spatial ΣΔ Quantization, cont.

- Exploits oversampling in *space* – however mutual coupling and physical dimensions of antennas limit this

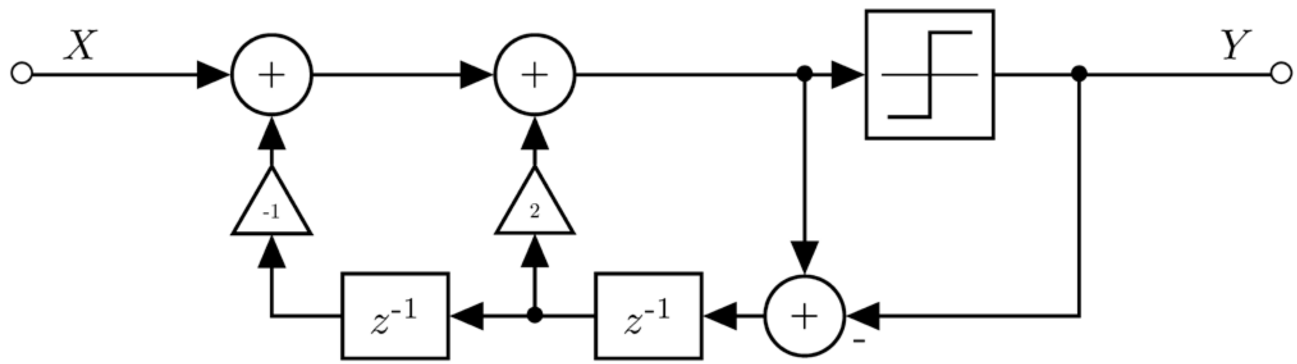
- Alternatively, users may already have low spatial frequency due to sectorization

- Quantization noise pushed to higher spatial frequencies, so lowpass spatial filtering (beamforming) can reduce quantization impact

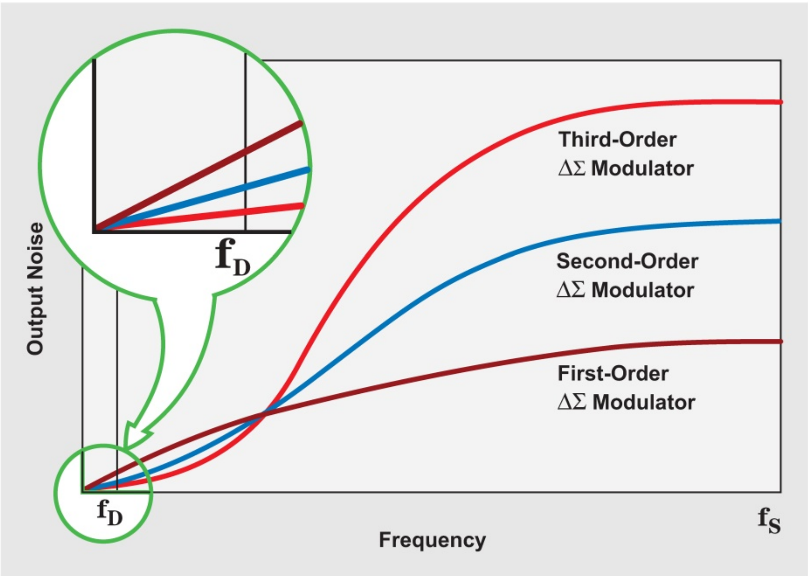
- Center of angular sector can be controlled

- Second- or higher-order spatial ΣΔ quantization for further noise shaping also possible

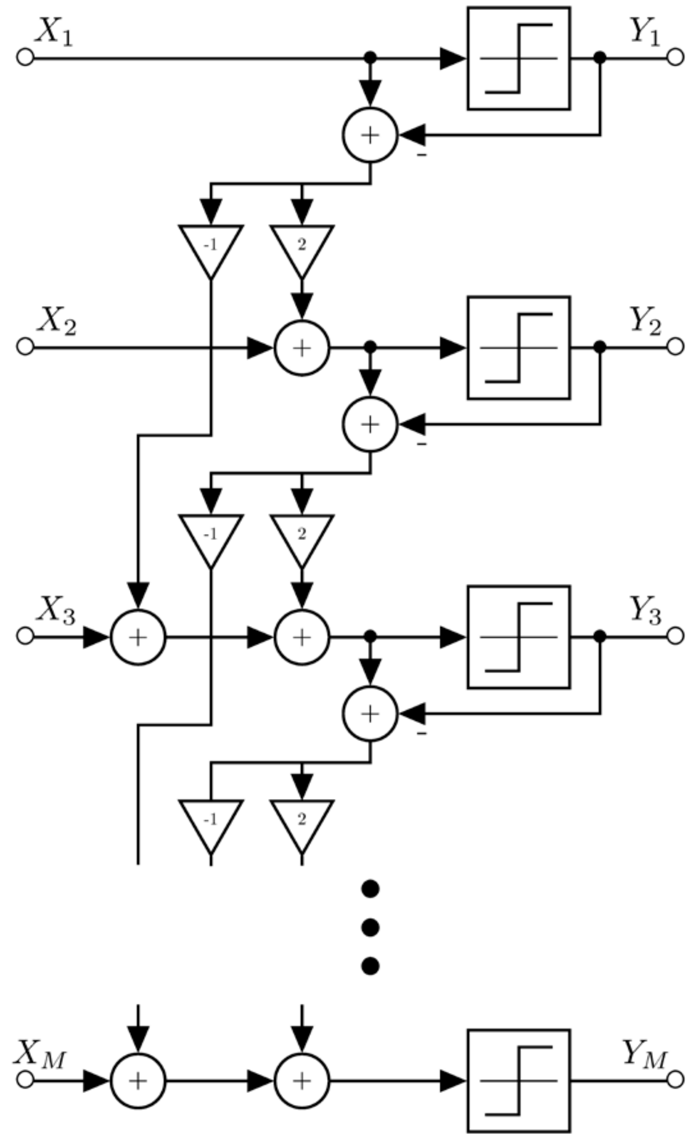
2nd Order ΣΔ ADC



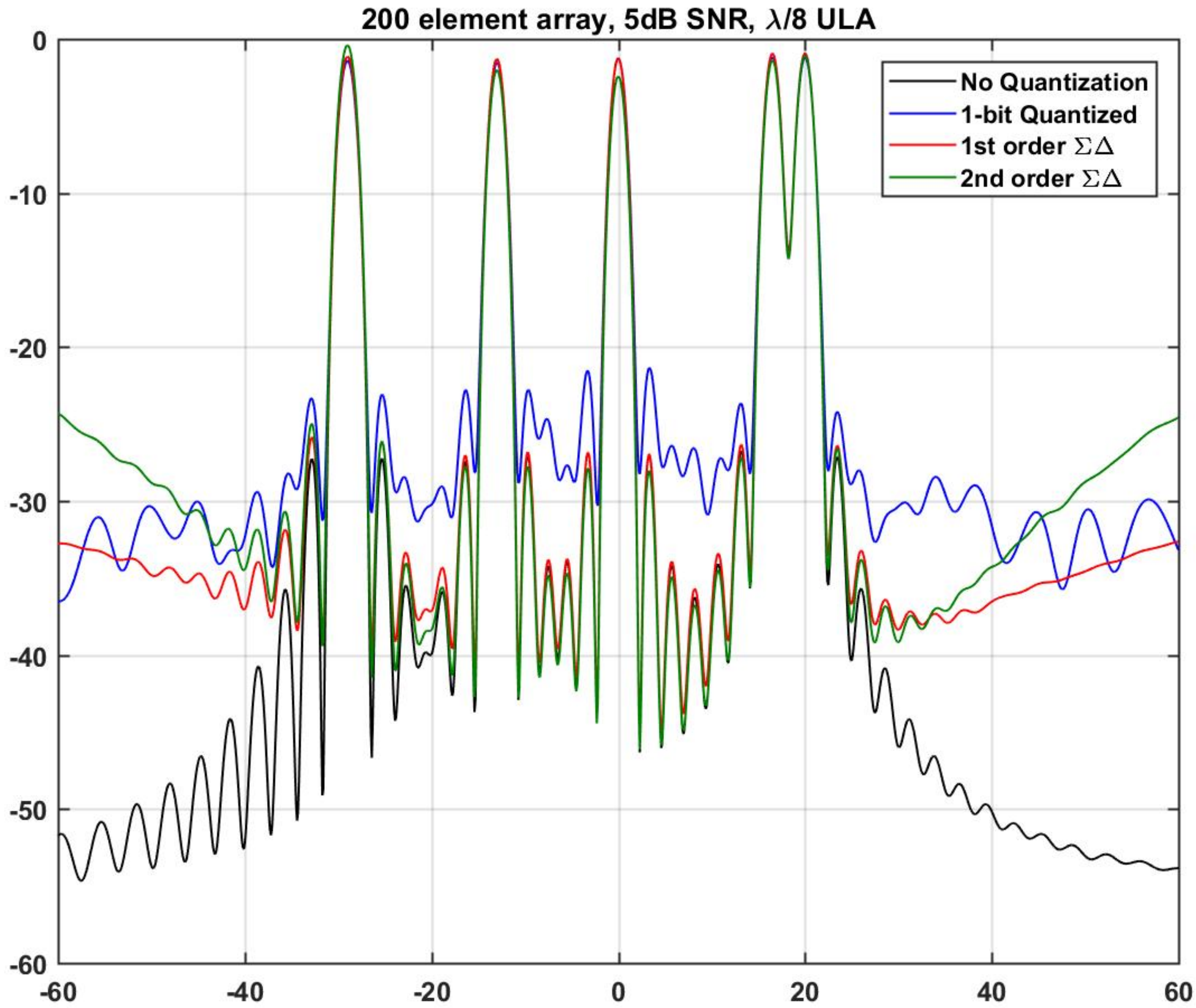
provides further shaping of the quantization noise: $Y = X + Q\left(1 - z^{-1}\right)^2$



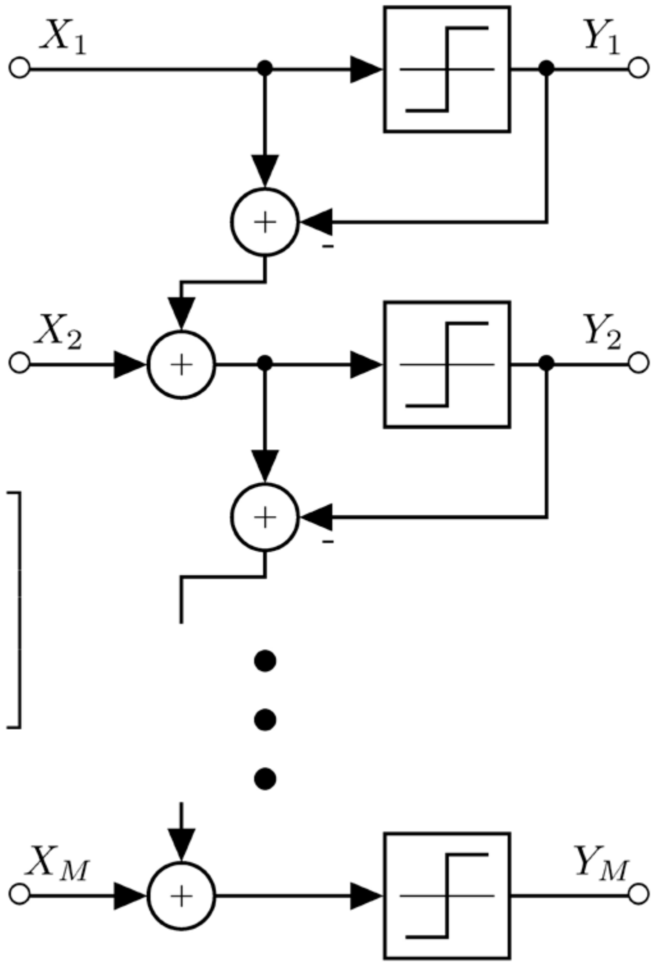
2nd Order Spatial ΣΔ ADC Architecture



Beampatterns Obtained with Spatial ΣΔ ADCs



Channel Estimation



$$\mathbf{x} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix}$$

Use $K \times \tau$ uplink training data $\mathbf{\Phi}_t$

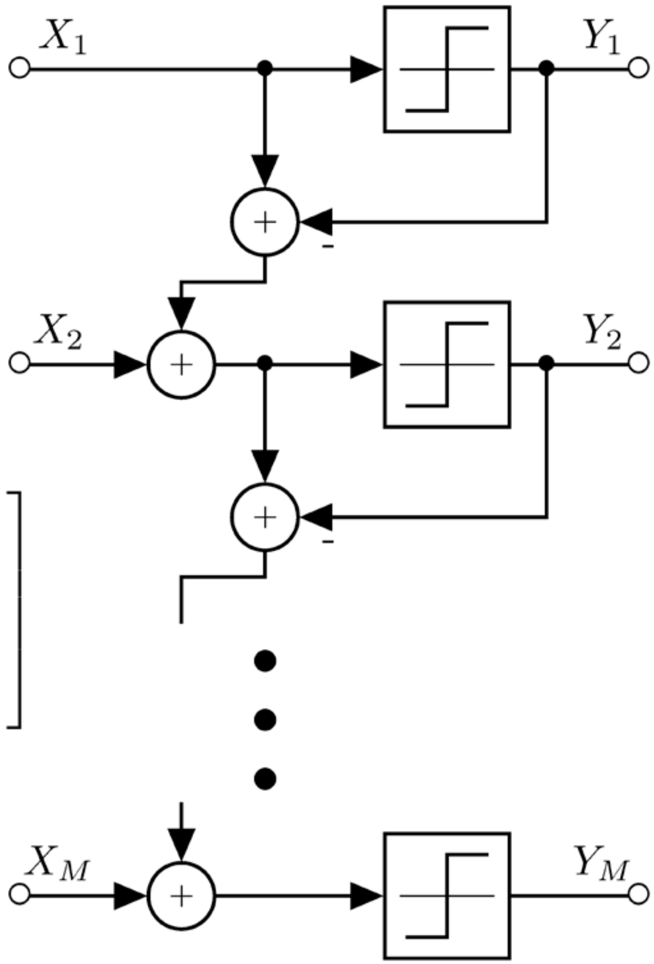
$$\mathbf{X} = \sqrt{\rho}\mathbf{H}\mathbf{\Phi}_t + \mathbf{N}$$

Vectorized model

$$\begin{aligned} \mathbf{x} &= \mathrm{vec}(\mathbf{X}) \\ &= \sqrt{\rho}\left(\mathbf{\Phi}_t^T \otimes \mathbf{I}\right) \mathrm{vec}(\mathbf{H}) + \mathrm{vec}(\mathbf{N}) \\ &= \mathbf{\Phi}\mathbf{h} + \mathbf{n} \end{aligned}$$

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_M \end{bmatrix} = \mathbf{y} = \mathcal{S}_\Delta(\mathbf{x})$$

Channel Estimation



$$\mathbf{x} = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix}$$

$$\mathbf{y} = \mathcal{S}_{\Delta}(\mathbf{x}) = \mathcal{Q}\left(\mathbf{U}\mathbf{x} - \underbrace{(\mathbf{U} - \mathbf{I})}_{\mathbf{V}}\mathbf{y}\right)$$

for first-order $\Sigma\Delta$:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 1 & 1 & 0 & 0 & & 0 \\ 1 & 1 & 1 & 0 & & 0 \\ & & & & \vdots & \\ 1 & 1 & 1 & 1 & \cdots & 1 \end{bmatrix}$$

for second-order $\Sigma\Delta$:

$$\mathbf{U} = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 2 & 1 & 0 & 0 & & 0 \\ 3 & 2 & 1 & 0 & & 0 \\ & & & & \vdots & \\ M & M-1 & M-2 & M-3 & \cdots & 1 \end{bmatrix}$$

Bussgang Analysis – Equivalent Linear Model

Bussgang Theorem: For zero-mean Gaussian $r(t)$ and nonlinearity $\mathcal{Q}(\cdot)$,

$$y(t) = \mathcal{Q}\left(r(t)\right) \quad \Rightarrow \quad r_{yr}(\tau) = \alpha r_{zz}(\tau)$$

Suggests an equivalent linear model with quantization "noise":

$$\mathbf{y} = \mathcal{S}_{\Delta}(\mathbf{x}) = \mathcal{Q}\left(\underbrace{\mathbf{U}\mathbf{x} - \mathbf{V}\mathbf{y}}_{\mathbf{r}}\right)$$

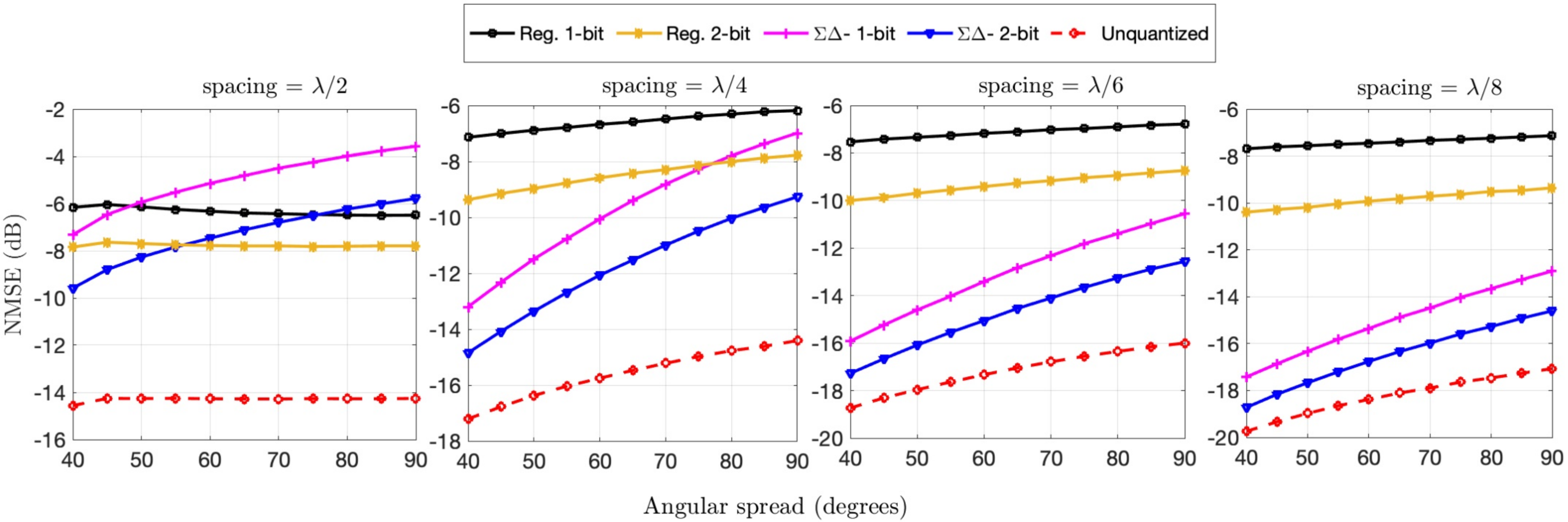
$$= \mathbf{\Gamma}\mathbf{r} + \mathbf{q}$$

There are in infinite number of such models; we choose the one for which $\mathcal{E}(r_i q_i) = 0$, and compute the LMMSE channel estimate:

$$\hat{\mathbf{h}} = \mathbb{E}\left[\mathbf{h}\mathbf{y}^H\right]\left(\mathbb{E}\left[\mathbf{y}\mathbf{y}^H\right]\right)^{-1}\mathbf{y} = \mathbf{C}_{hy}\mathbf{C}_y^{-1}\mathbf{y}$$

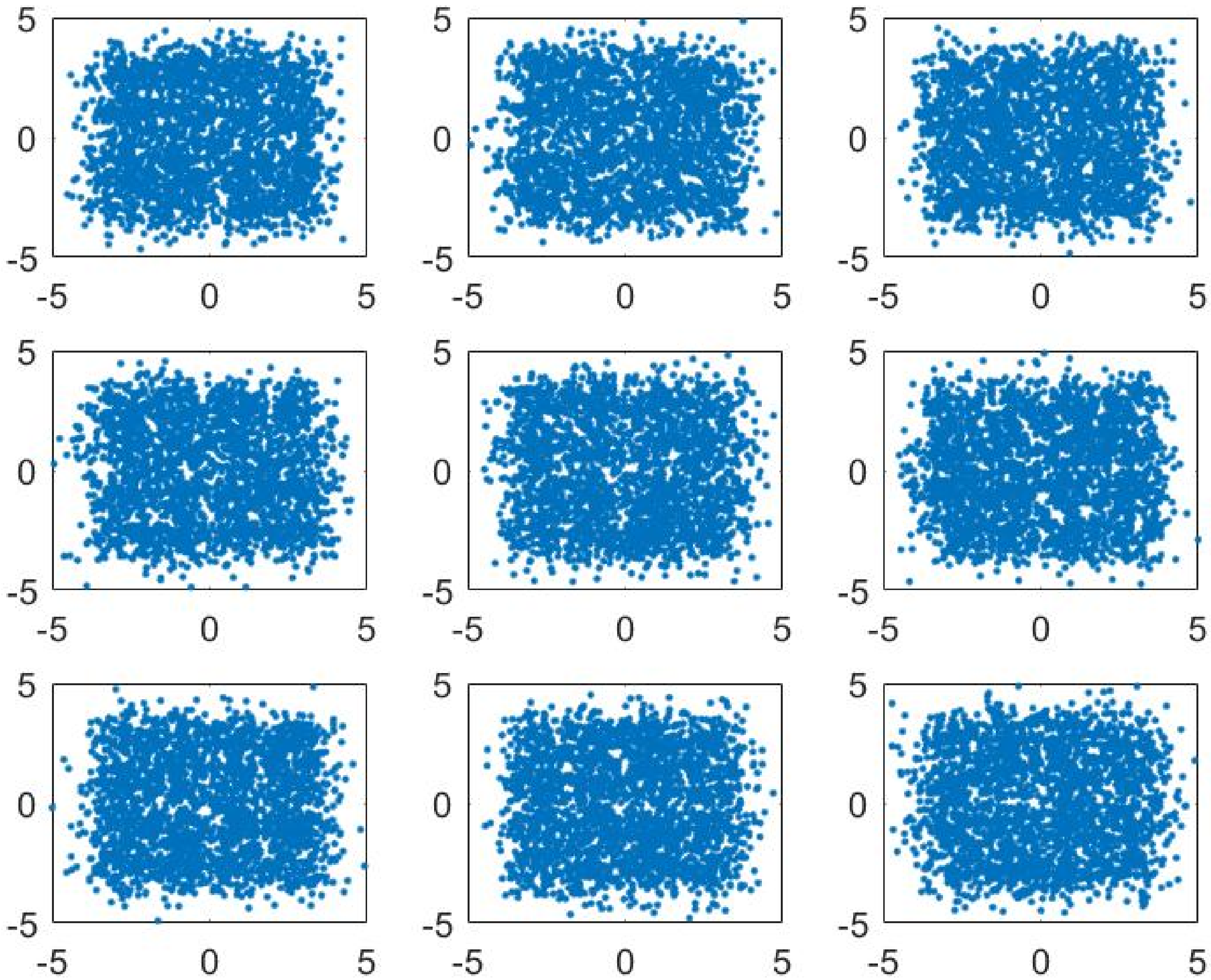
Effect of Antenna Spacing & ADC Resolution

- 128 antenna ULA
- 10 users with multiple angles of arrival uniformly distributed in $[-30°, 30°]$
- LMMSE channel estimate obtained using orthogonal pilots of duration 10 symbols



Standard 1-Bit Receiver vs. 1-Bit ΣΔ

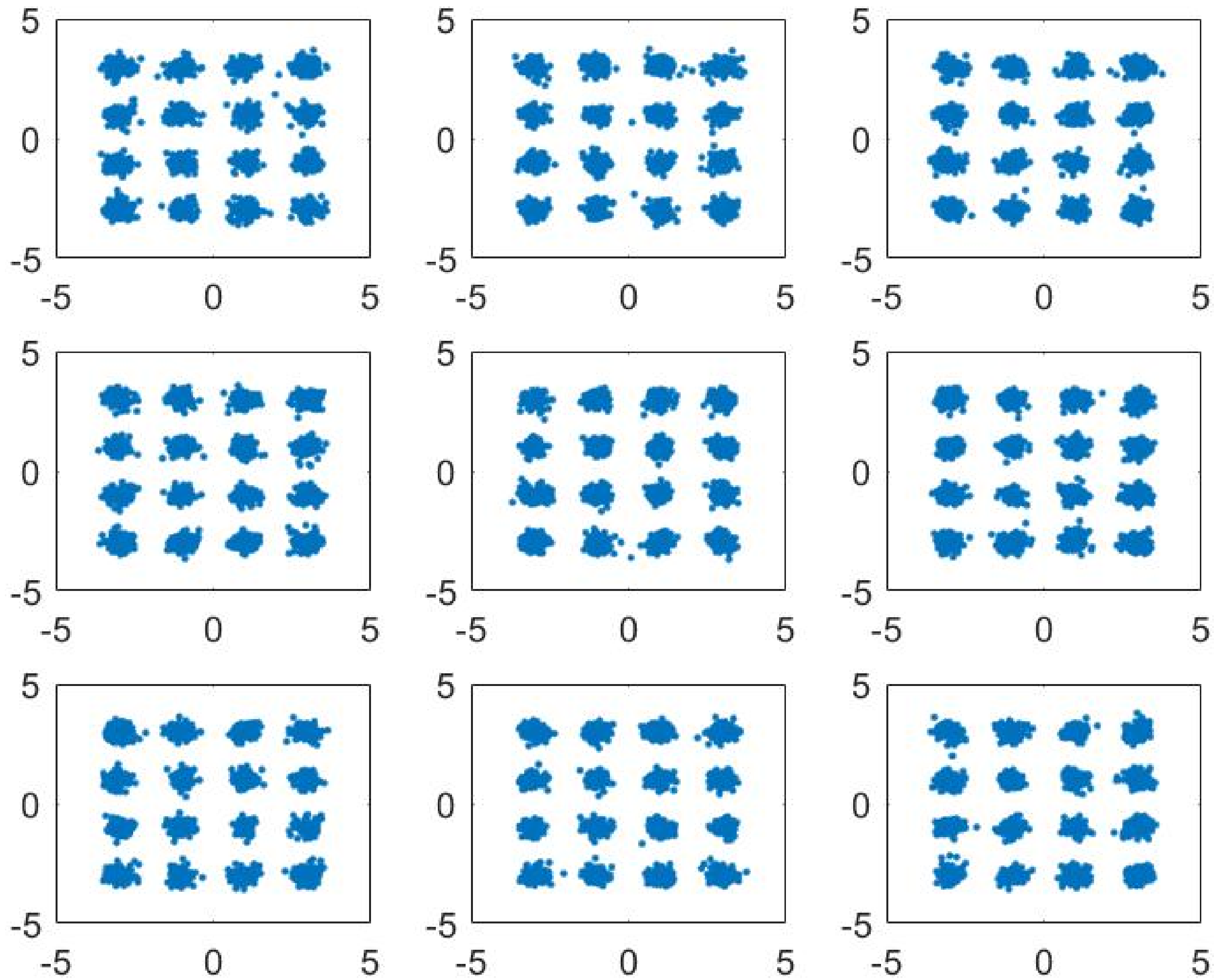
- 400 antennas
- $d = \lambda/6$
- 9 users
- user DoAs $\in [-25°, 25°]$
- 10dB SNR
- Rayleigh fading
- LMMSE channel estimation followed by ZF detection



Standard 1-Bit, 16-QAM

Standard 1-Bit Receiver vs. 1-Bit ΣΔ

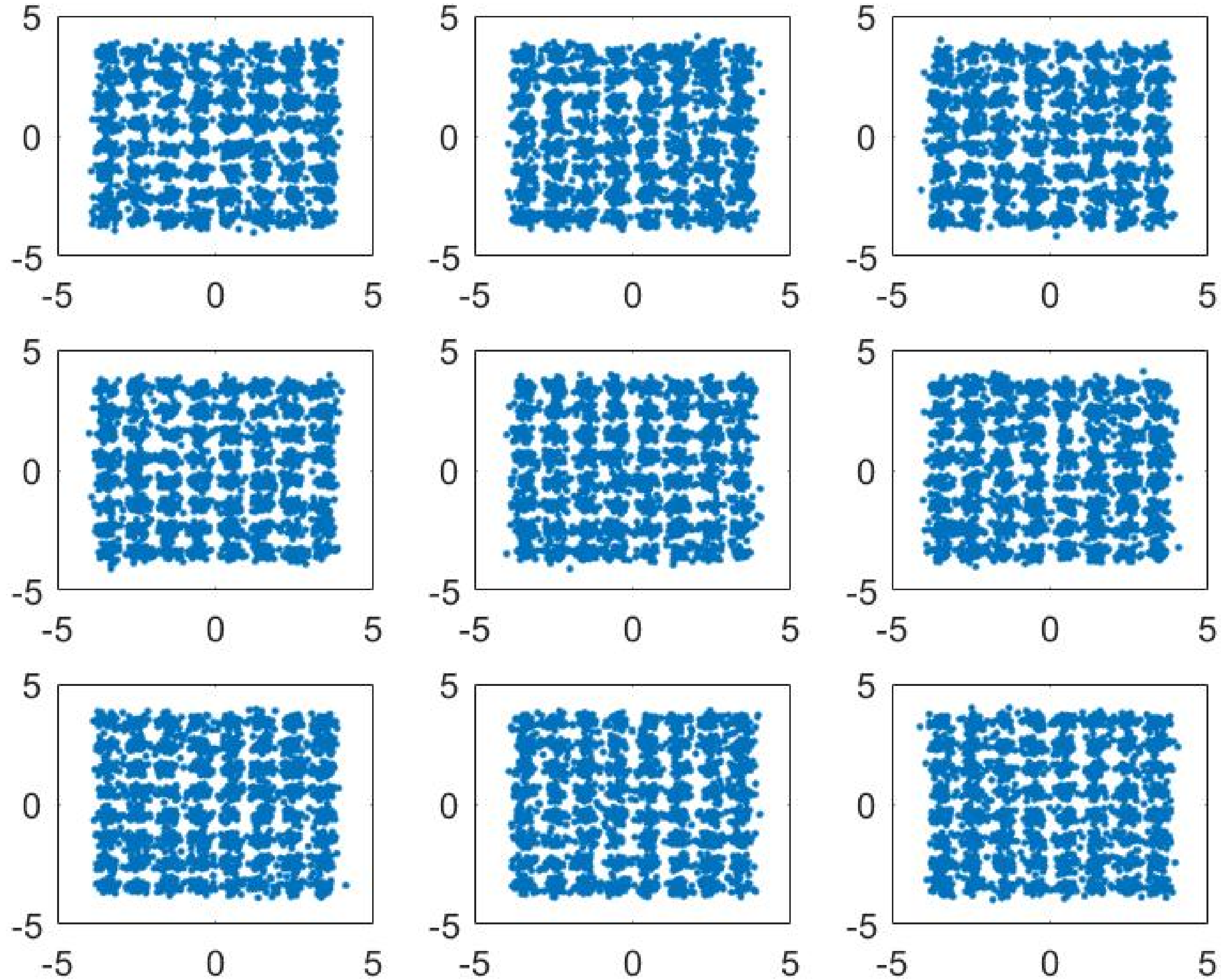
- 400 antennas
- $d = \lambda/6$
- 9 users
- user DoAs $\in [-25°, 25°]$
- 10dB SNR
- Rayleigh fading
- LMMSE channel estimation followed by ZF detection



1-Bit ΣΔ, 16-QAM

Standard 1-Bit Receiver vs. 1-Bit ΣΔ

- 400 antennas
- $d = \lambda/6$
- 9 users
- user DoAs $\in [-25°, 25°]$
- 10dB SNR
- Rayleigh fading
- LMMSE channel estimation followed by ZF detection

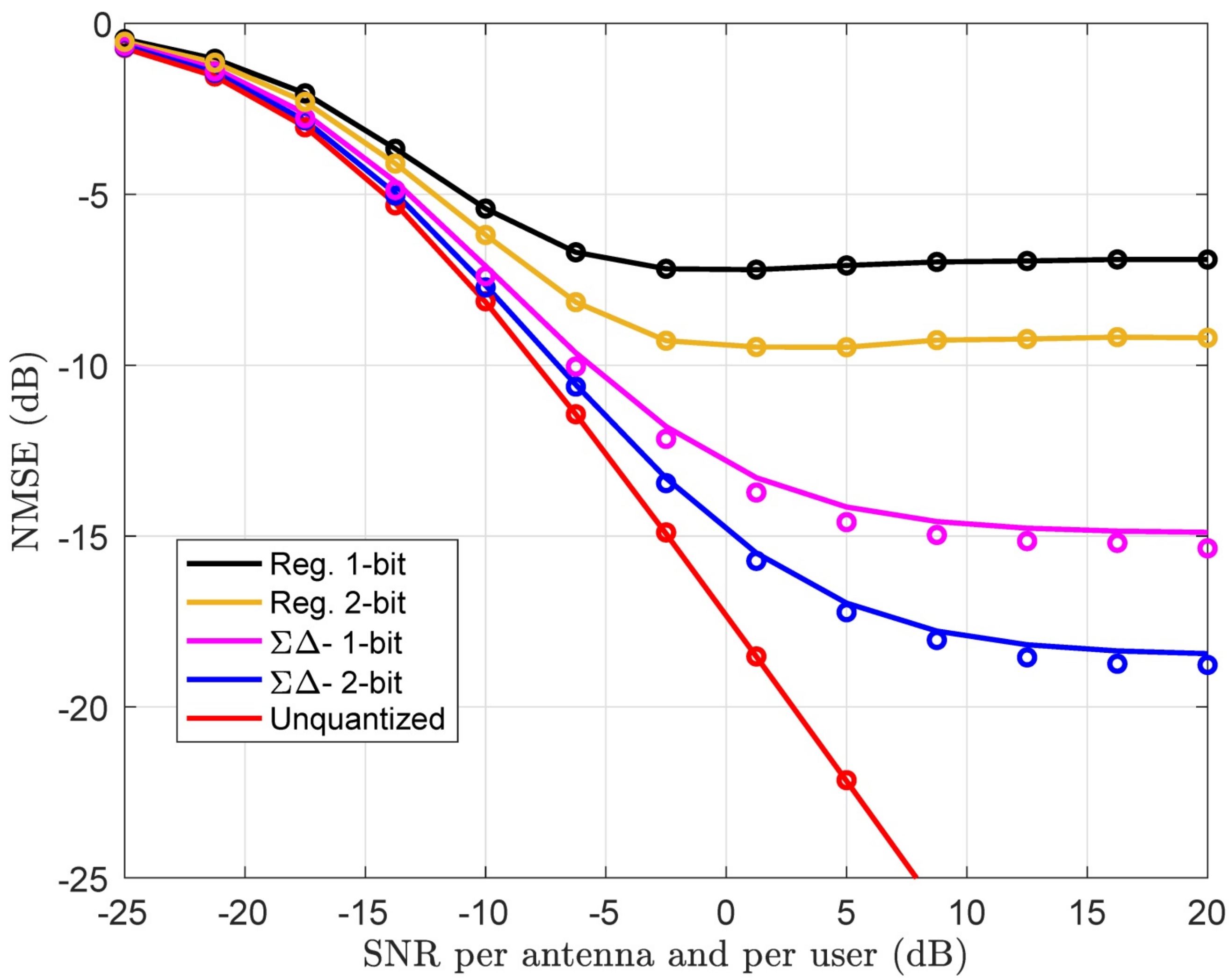


1-Bit ΣΔ, 64-QAM

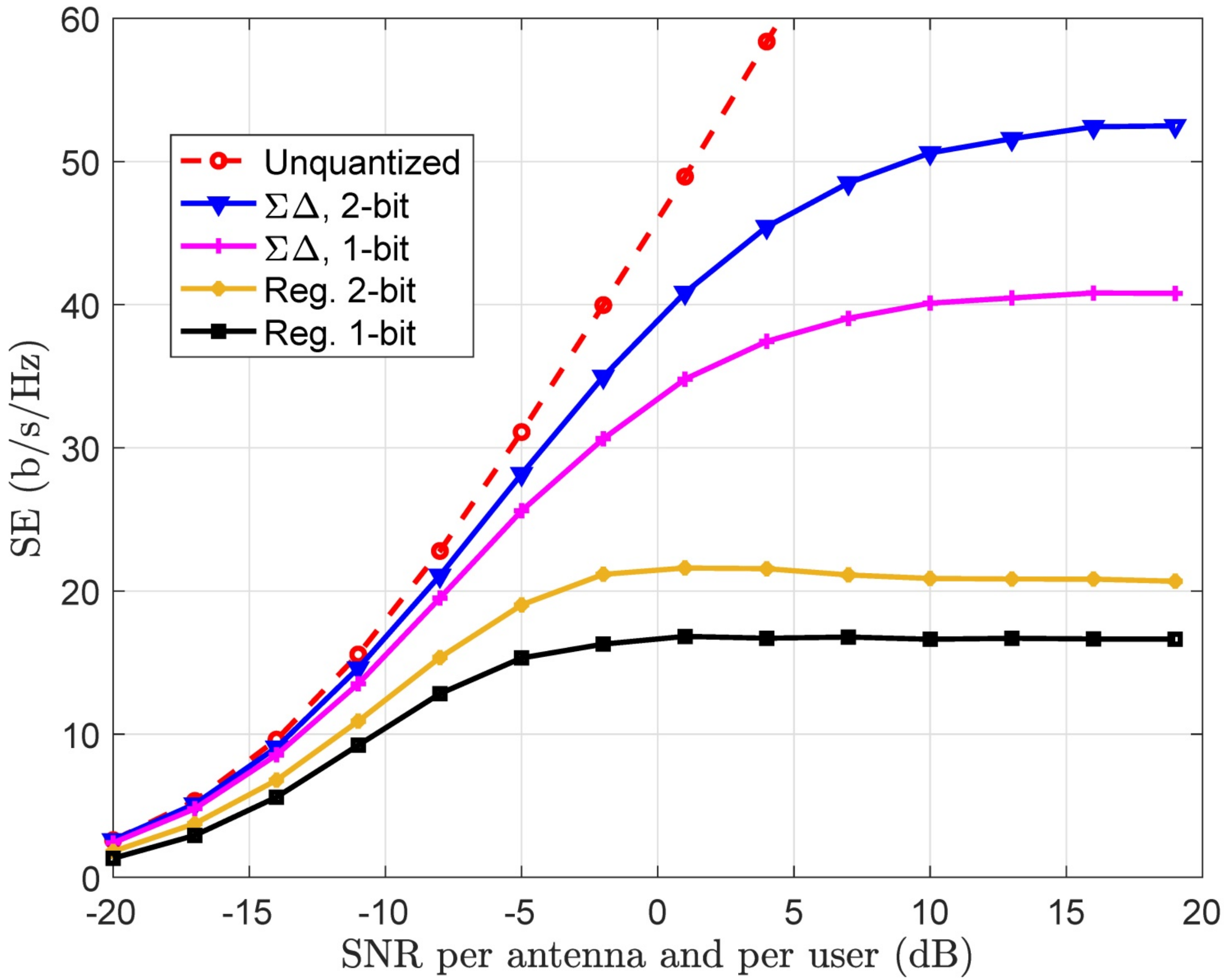
Uplink Simulation with Channel Estimation

- 128 antenna ULA with $\lambda/6$ element spacing
- 10 users with multiple angles of arrival uniformly distributed in $[-30°, 30°]$
- LMMSE channel estimate obtained using orthogonal pilots of duration 10 symbols
- Estimated channels used in ZF receiver to decode subsequent QPSK symbols
- Results compared with analytical predictions

Uplink Simulation with Channel Estimation



Spectral Efficiency Comparison



Impact of Mutual Coupling

<u>Assumed model</u>: ULA composed of thin dipoles
S. Schelkunoff, *Antennas: Theory and Practice*

steering vector: $\mathbf{T}\mathbf{a}(\theta)$

$$\mathbf{T} = \left(\mathbf{I} + \tfrac{1}{R}\mathbf{Z}\right)^{-1} \qquad \xi_{ij} = \pi\sqrt{1 + 4d_{ij}^2}$$

$$\mathbf{Z}_{ij} = 30\Big(2\mathrm{Ci}(2\pi d_{ij}) - \mathrm{Ci}(\xi_{ij} + \pi) - \mathrm{Ci}(\xi_{ij} - \pi)$$
$$+j\left(-2\mathrm{Si}(2\pi d_{ij}) + \mathrm{Si}(\xi_{ij} + \pi) + \mathrm{Si}(\xi_{ij} - \pi)\right)\Big),\ i \neq j$$

$$\mathbf{Z}_{ii} = 30\Big(\gamma + \log(2\pi) - \mathrm{Ci}(2\pi) + j\mathrm{Si}(2\pi)\Big)$$

$$\mathrm{Ci}(x) \triangleq \gamma + \log(x) + \int_0^x \frac{\cos(t) - 1}{t}dt$$

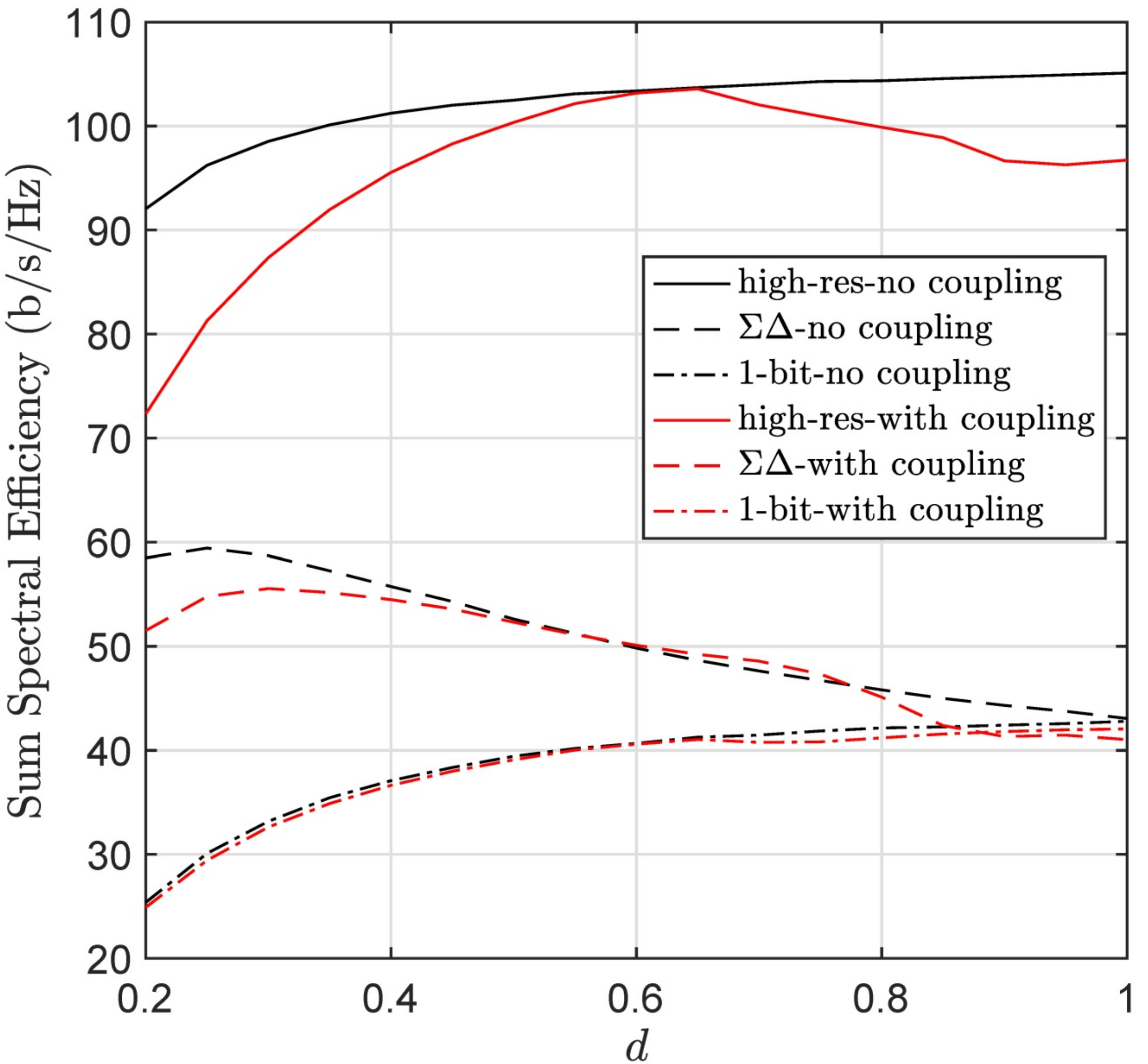
$$\mathrm{Si}(x) \triangleq \int_0^x \frac{\sin(t)}{t}dt$$

noise covariance also depends on $\mathbf{Z}$ and R

Uplink Simulation with Mutual Coupling

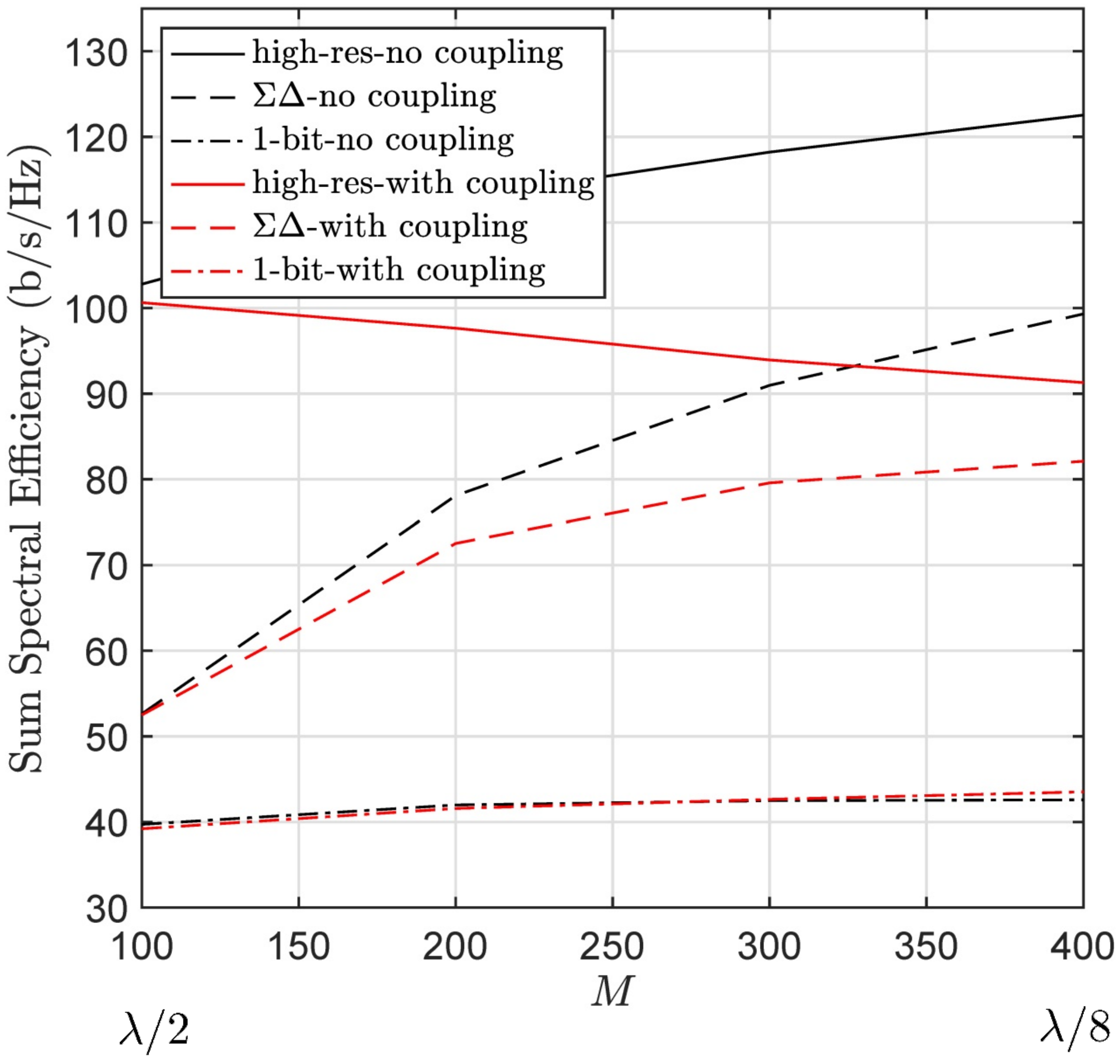
- 10 users with multiple angles of arrival uniformly distributed in $[-30^\circ, 10^\circ]$
- $\text{SNR} = 10\text{dB}$
- Spectral efficiency assuming CSI is known
- Case 1: 100 antenna ULA with variable antenna spacing (variable aperture)
- Case 2: ULA with 50λ fixed aperture, variable M and variable aperture

Impact of Mutual Coupling

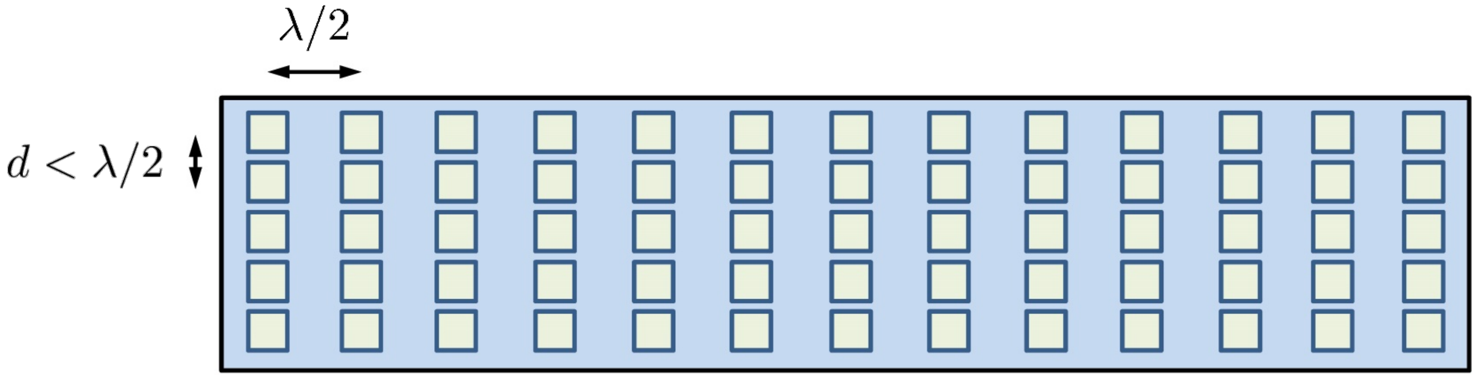


Impact of Mutual Coupling

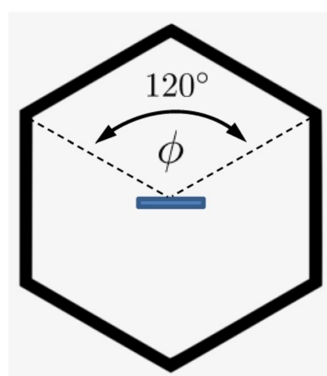
space-constrained scenario with total aperture of 50λ



ΣΔ Rectangular Array Geometry

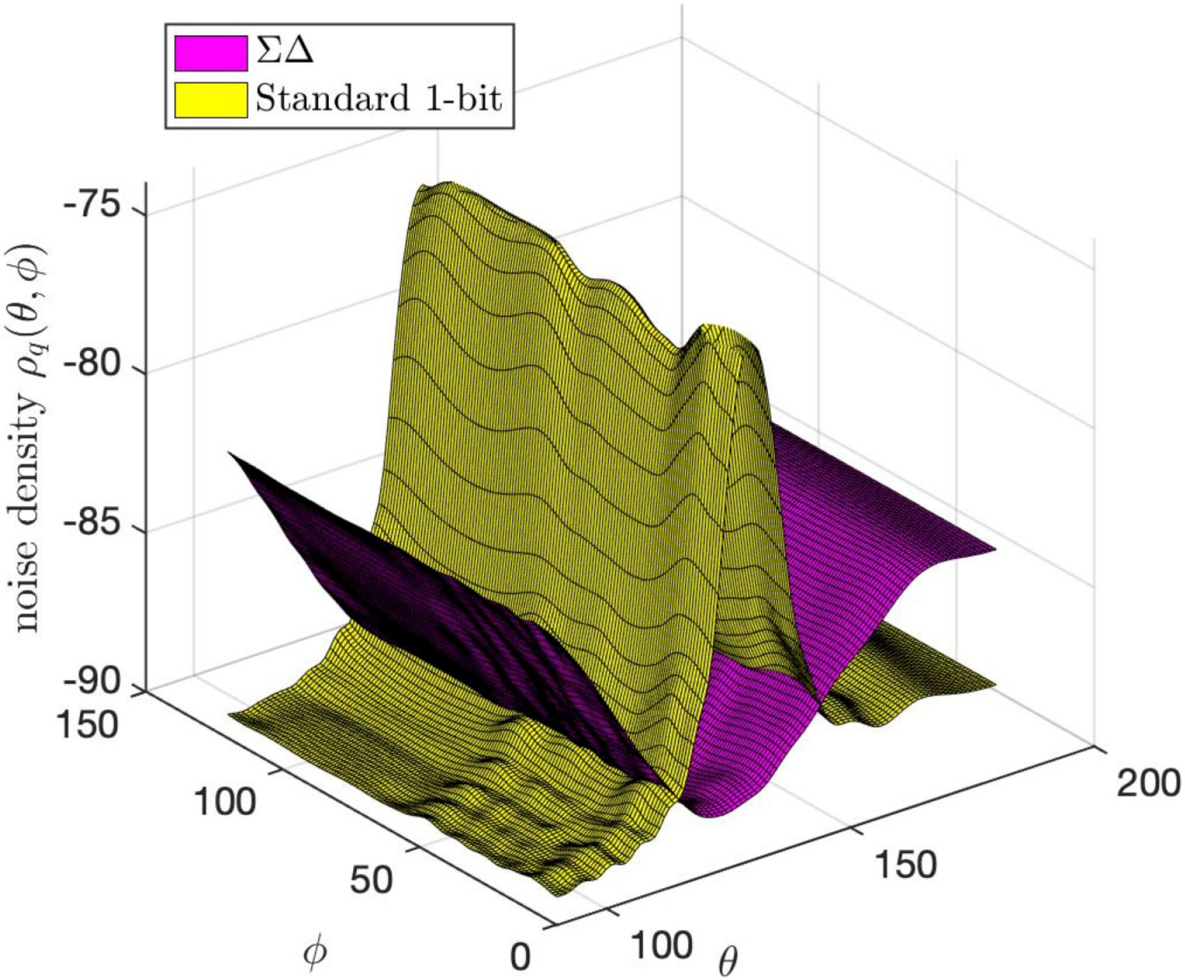


Use ΣΔ only in vertical direction, since elevation angle θ is focused on a narrow angular sector



Sector for azimuth angle ϕ is unconstrained

PSD of Quantization Noise for Rectangular Array



Conclusions

- Massive MIMO, small cells and mm-wave frequencies provide symbionic benefits for 5G
- Low-resolution (e.g., 1-bit) quantization can provide high spectral efficiency and significant energy savings, but there is a performance gap.
- One-bit $\Sigma\Delta$ ADC architectures provide gains in situations with spatial oversampling or where users have low spatial frequencies
- Similar benefits observed with $\Sigma\Delta$ implemented on the transmit side
- Current work: impact of mutual coupling, rectangular arrays