

Orthogonal Time Frequency Space (OTFS) Modulation and Applications

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Tharaj Thaj, Khoa T.Phan

1 Introduction

- Evolution of wireless
- High-Doppler wireless channels
- Conventional modulation schemes (e.g., OFDM)
- Effect of high Dopplers in conventional modulation

2 Wireless channel representation

- Time–frequency representation
- Time–delay representation
- Delay–Doppler representation

3 OTFS modulation

- Signaling in the delay–Doppler domain
- Compatibility with OFDM architecture

4 OTFS Input-Output Relation in Matrix Form

5 OTFS Signal Detection

- Vectorized formulation of the input-output relation
- Message passing based detection
- Other detectors

6 OTFS channel estimation

- Channel estimation in delay-Doppler domain
- Multiuser OTFS

7 OTFS applications

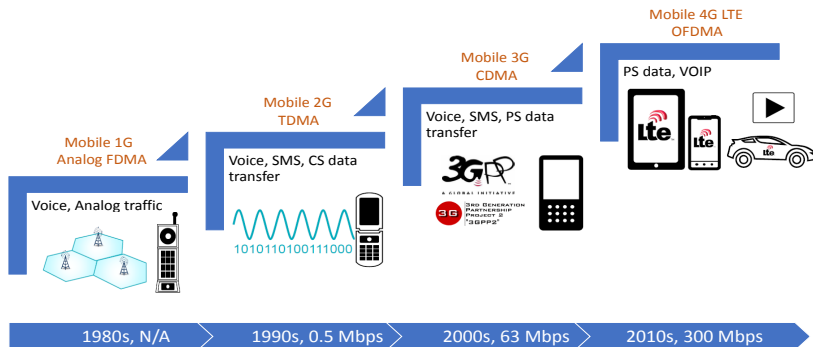
- SDR implementation of OTFS
- OTFS with static multipath channels

Link to download Matlab code:

https://ecse.monash.edu/staff/eviterbo/OTFS-VTC18/OTFS_sample_code.zip

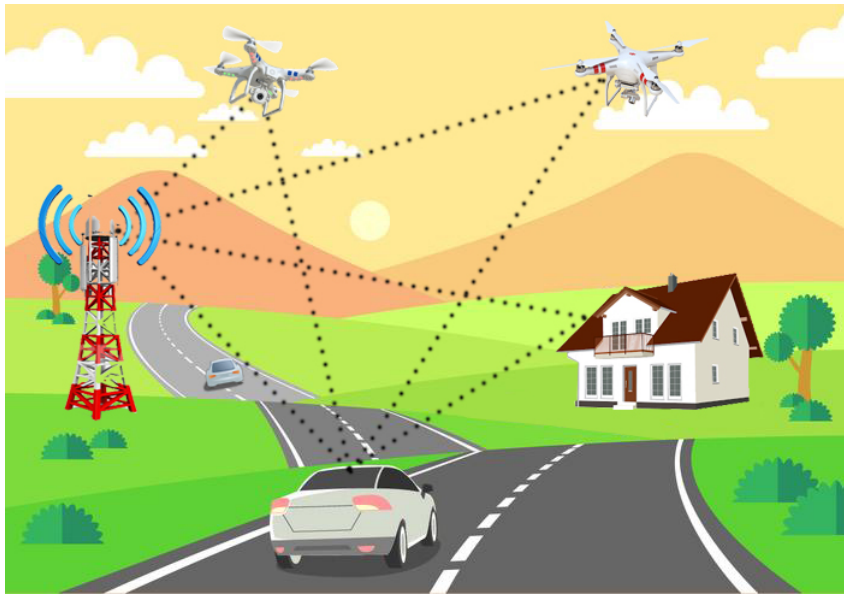
Introduction

Evolution of wireless

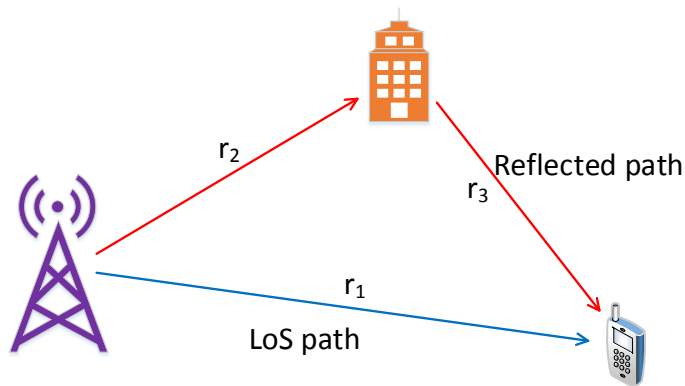


- Waveform design is the major change between the generations

High-Doppler wireless channels

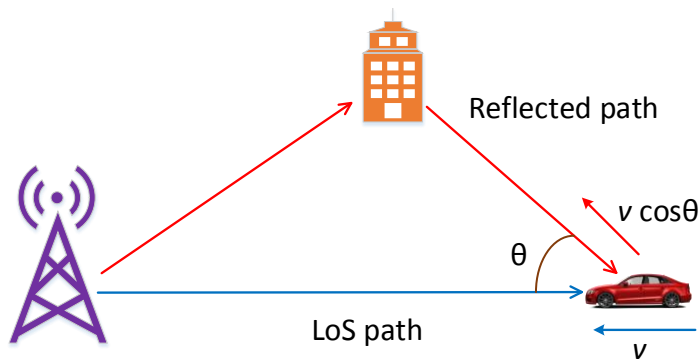


Wireless Channels - delay spread



- Delay of LoS path: $\tau_1 = r_1/c$
- Delay of reflected path: $\tau_2 = (r_2 + r_3)/c$
- Delay spread: $\tau_2 - \tau_1$

Wireless Channels - Doppler spread



- Doppler frequency of LoS path: $\nu_1 = f_c \frac{v}{c}$
- Doppler frequency of reflected path: $\nu_2 = f_c \frac{v \cos \theta}{c}$
- Doppler spread: $\nu_2 - \nu_1$

$$\text{TX: } s(t) \quad \text{RX: } r(t) = h_1 s(t - \tau_1) e^{-j2\pi\nu_1 t} + h_2 s(t - \tau_2) e^{-j2\pi\nu_2 t}$$

Typical delay and Doppler spreads

Delay spread ($c = 3 \cdot 10^8 \text{m/s}$)

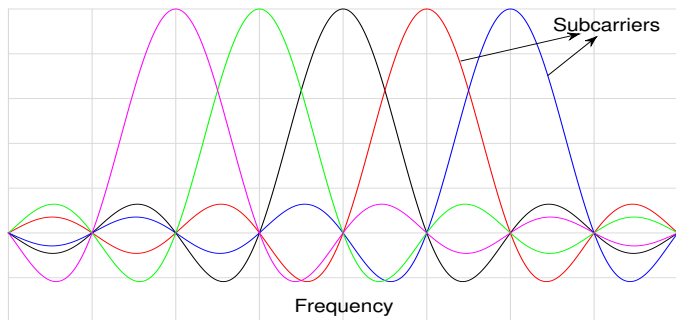
Δr_{\max}	Indoor (3m)	Outdoor (3km)
τ_{\max}	10ns	$10\mu\text{s}$

Doppler spread

ν_{\max}	$f_c = 2\text{GHz}$	$f_c = 60\text{GHz}$
$\nu = 1.5\text{m/s} = 5.5\text{km/h}$	10Hz	300Hz
$\nu = 3\text{m/s} = 11\text{km/h}$	20Hz	600Hz
$\nu = 30\text{m/s} = 110\text{km/h}$	200Hz	6KHz
$\nu = 150\text{m/s} = 550\text{km/h}$	1KHz	30KHz

Conventional modulation scheme – OFDM

- OFDM - Orthogonal Frequency Division Multiplexing



- OFDM divides the frequency selective channel into multiple parallel sub-channels

OFDM system model

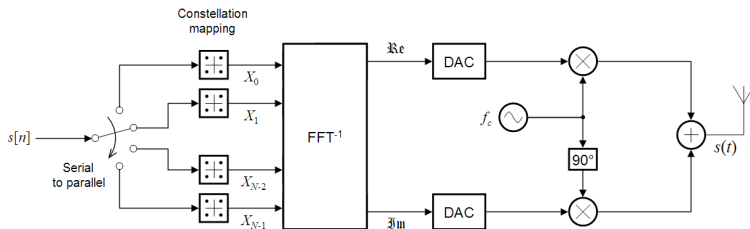


Figure: OFDM Tx

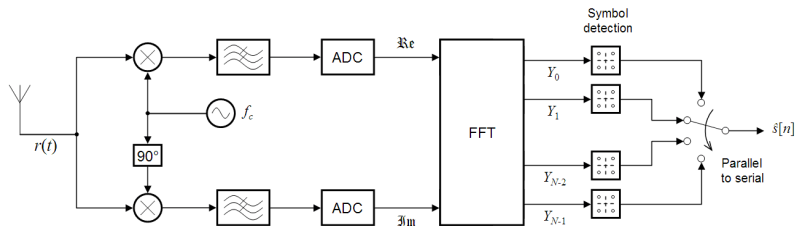


Figure: OFDM Rx

(*) From Wikipedia, the free encyclopedia

OFDM time domain input-output relation

- Received signal – channel is constant over OFDM symbol (no Doppler)
 $\mathbf{h} = (h_0, h_1, \dots, h_{P-1})$ – Path gains over P taps

$$\mathbf{r} = \mathbf{h} \circledast \mathbf{s} = \underbrace{\begin{bmatrix} h_0 & 0 & \cdots & 0 & h_{P-1} & h_{P-2} & \cdots & h_1 \\ h_1 & h_0 & \cdots & 0 & 0 & h_{P-1} & \cdots & h_2 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & h_{P-1} \\ h_{P-1} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & h_{P-1} & h_{P-2} & \cdots & h_1 & h_0 \end{bmatrix}}_{M \times M \text{ Circulant matrix } (\mathbf{H})} \mathbf{s}$$

- Eigenvalue decomposition property $\mathbf{H} = \mathbf{F}^H \mathbf{D} \mathbf{F}$ where $\mathbf{D} = \text{diag}[\text{DFT}_M(\mathbf{h})]$

OFDM frequency domain input-output relation

- At the receiver we have

$$\mathbf{r} = \mathbf{H}\mathbf{s} = \mathbf{F}^H \mathbf{D} \mathbf{F} \mathbf{s} = \sum_{i=0}^{P-1} h_i \mathbf{\Pi}^i \mathbf{s}$$

where $\mathbf{\Pi}$ is the permutation matrix

$$\begin{pmatrix} 0 & \cdots & 0 & 1 \\ 1 & & & 0 \\ \vdots & \ddots & & \vdots \\ 0 & \cdots & 1 & 0 \end{pmatrix}$$

(notation used later as alternative representation of the channel)

- At the receiver we have input-output relation in frequency domain

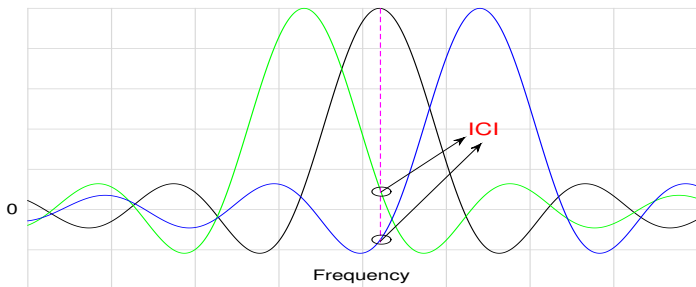
$$\mathbf{y} = \mathbf{F} \mathbf{r} = \underbrace{\mathbf{D}}_{\text{Diagonal matrix with subcarrier gains}} \mathbf{x} \quad \text{where} \quad \mathbf{x} = \mathbf{F} \mathbf{s} \quad \text{and} \quad \underbrace{\mathbf{s} = \mathbf{F}^H \mathbf{x}}_{\text{Tx IFFT}}$$

- OFDM Pros

- Simple detection (one tap equalizer)
- Efficiently combat the multi-path effects

Effect of high multiple Dopplers in OFDM

- \mathbf{H} matrix lost the circulant structure – decomposition becomes erroneous
- Introduces inter carrier interference (ICI)



• OFDM Cons

- multiple Dopplers are difficult to equalize
- Sub-channel gains are not equal and lowest gain decides the performance

Effect of high Dopplers in OFDM

- Orthogonal Time Frequency Space Modulation (OTFS)^(*)
 - Solves the two cons of OFDM
 - Works in Delay–Doppler domain rather than Time–Frequency domain

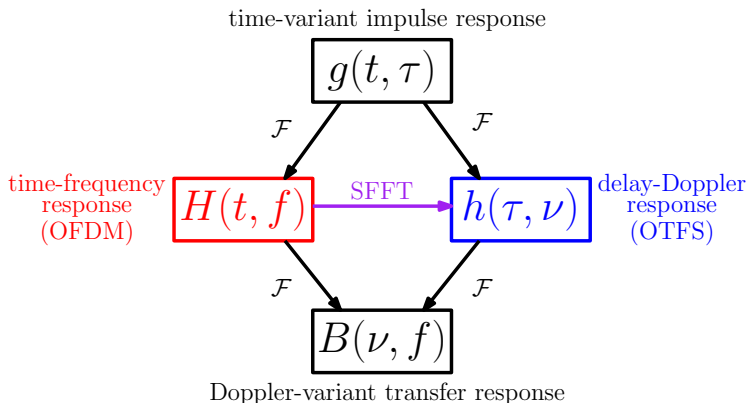
(*) R. Hadani, S. Rakib, M. Tsatsanis, A. Monk, A. J. Goldsmith, A. F. Molisch, and R. Calderbank, “Orthogonal time frequency space modulation,” in *Proc. IEEE WCNC*, San Francisco, CA, USA, March 2017.



Wireless channel representation

Wireless channel representation

- Different representations of linear time variant (LTV) wireless channels



Wireless channel representation

- The received signal in linear time variant channel (LTV)

$$r(t) = \int \underbrace{g(t, \tau)}_{\text{time-variant impulse response}} s(t - \tau) d\tau \rightarrow \text{generalization of LTI}$$

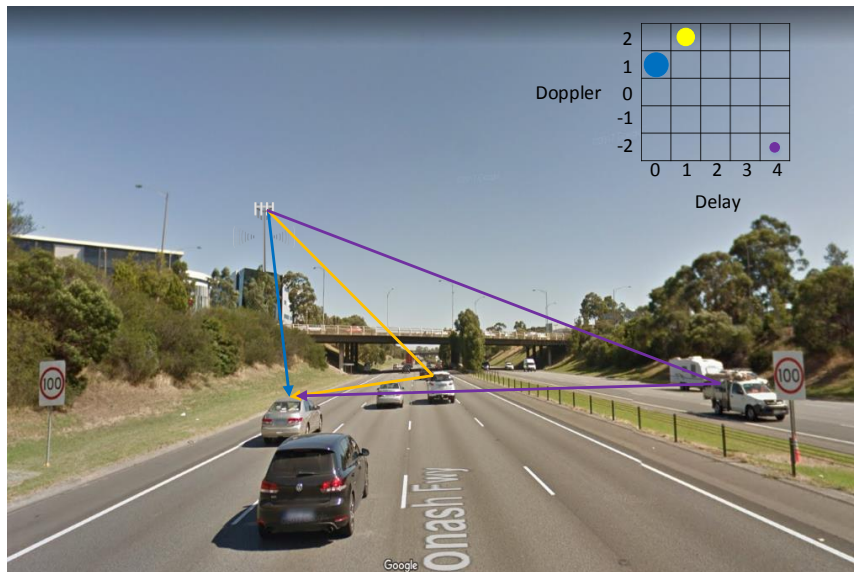
$$= \int \int \underbrace{h(\tau, \nu)}_{\text{Delay-Doppler spreading function}} s(t - \tau) e^{j2\pi\nu t} d\tau d\nu \rightarrow \text{Delay-Doppler Channel}$$

$$= \int \underbrace{H(t, f)}_{\text{time-frequency response}} S(f) e^{j2\pi f t} df \rightarrow \text{Time-Frequency Channel}$$

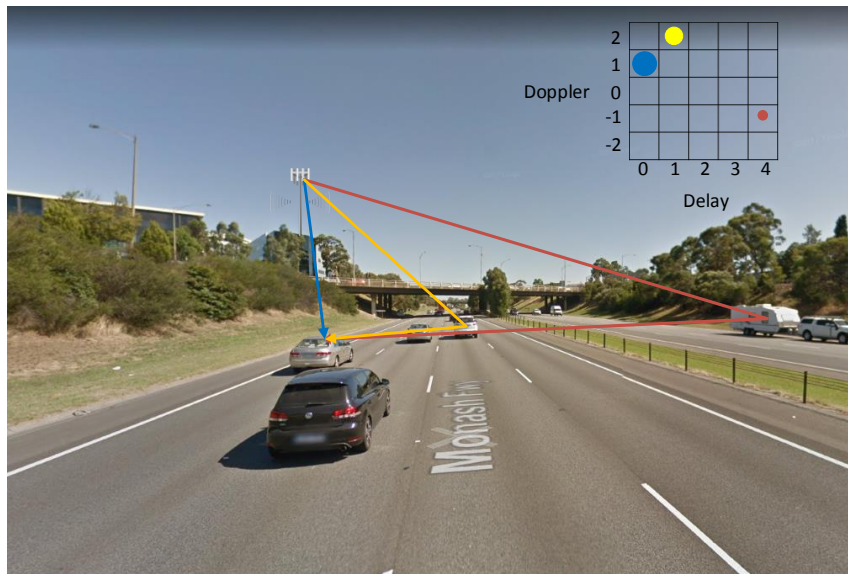
- Relation between $h(\tau, \nu)$ and $H(t, f)$

$$\left. \begin{aligned} h(\tau, \nu) &= \int \int H(t, f) e^{-j2\pi(\nu t - f\tau)} dt df \\ H(t, f) &= \int \int h(\tau, \nu) e^{j2\pi(\nu t - f\tau)} d\tau d\nu \end{aligned} \right\} \text{Pair of 2D symplectic FT}$$

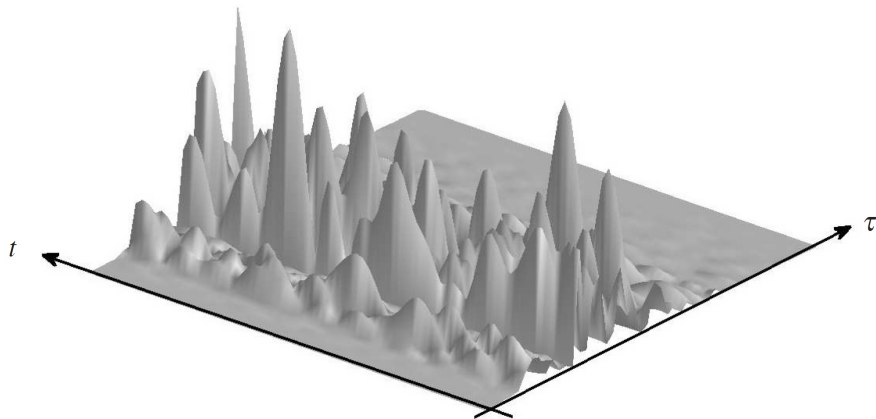
Wireless channel representation



Wireless channel representation

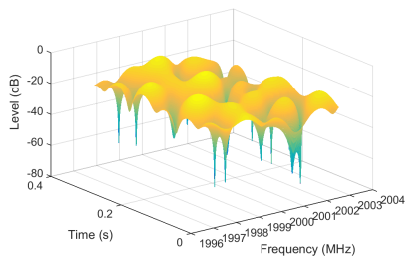


Time-variant impulse response $g(t, \tau)$

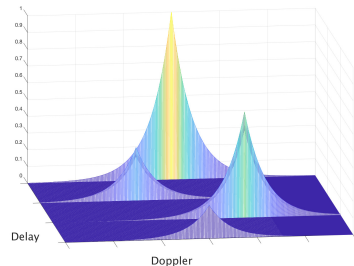


* G. Matz and F. Hlawatsch, *Chapter 1, Wireless Communications Over Rapidly Time-Varying Channels*. New York, NY, USA: Academic, 2011

Time-frequency and delay-Doppler responses



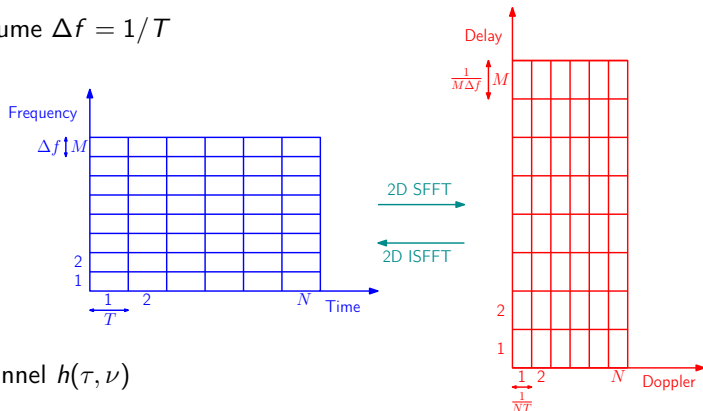
SFFT
← ISFFT



Channel in Time-frequency $H(t, f)$ and delay-Doppler $h(\tau, \nu)$

Time–Frequency and delay–Doppler grids

- Assume $\Delta f = 1/T$



- Channel $h(\tau, \nu)$

$$h(\tau, \nu) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i)$$

- Assume $\tau_i = l_{\tau_i} \left(\frac{1}{M\Delta f} \right)$ and $\nu_i = k_{\nu_i} \left(\frac{1}{NT} \right)$

OTFS Parameters

Subcarrier spacing (Δf)	M	Bandwidth ($W = M\Delta f$)	Symbol duration ($T_s = 1/W$)	delay spread	$l_{\tau_{\max}}$
15 KHz	1024	15 MHz	$0.067 \mu s$	$4.7 \mu s$	71 ($\approx 7\%$)

Carrier frequency (f_c)	N	Latency ($NMT_s = NT$)	Doppler resolution ($1/NT$)	UE speed (v)	Doppler frequency ($f_d = f_c \frac{v}{c}$)	$k_{\nu_{\max}}$
4 GHz	128	8.75 ms	114 Hz	30 Kmph	111 Hz	1 ($\approx 1.5\%$)
				120 Kmph	444 Hz	4 ($\approx 6\%$)
				500 Kmph	1850 Hz	16 ($\approx 25\%$)

OTFS modulation

OTFS modulation

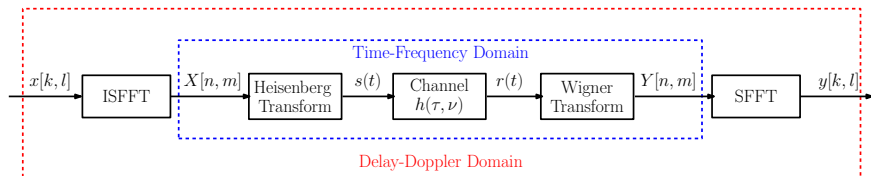


Figure: OTFS mod/demod

- Time-frequency domain is similar to an OFDM system with N symbols in a frame (Pulse-Shaped OFDM)

Time–frequency domain

- **Modulator** – Heisenberg transform

$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} X[n, m] g_{\text{tx}}(t - nT) e^{j2\pi m \Delta f (t - nT)}$$

- Simplifies to IFFT in the case of $N = 1$ and rectangular g_{tx}
- **Channel**

$$r(t) = \int H(t, f) S(f) e^{j2\pi f t} df$$

- **Matched filter** – Wigner transform

$$Y(t, f) = A_{g_{\text{rx}}, r}(t, f) \triangleq \int g_{\text{rx}}^*(t' - t) r(t') e^{-j2\pi f (t' - t)} dt'$$

$$Y[n, m] = Y(t, f)|_{t=nT, f=m\Delta f}$$

- Simplifies to FFT in the case of $N = 1$ and rectangular g_{rx}

Time–frequency domain – ideal pulses

- If g_{tx} and g_{rx} are perfectly localized in time and frequency then they satisfy the **bi-orthogonality condition** and

$$Y[n, m] = H[n, m]X[n, m]$$

where

$$H[n, m] = \int \int h(\tau, \nu) e^{j2\pi\nu nT} e^{-j2\pi m\Delta f\tau} d\tau d\nu$$

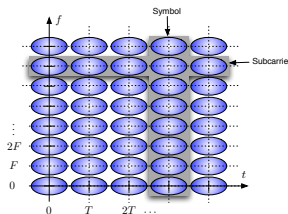


Figure: Time–frequency domain

* F. Hlawatsch and G. Matz, Eds., *Chapter 2, Wireless Communications Over Rapidly Time-Varying Channels*. New York, NY, USA: Academic, 2011 (PS-OFDM)

Signaling in the delay–Doppler domain

- Time–frequency input-output relation

$$Y[n, m] = H[n, m]X[n, m]$$

where

$$H[n, m] = \sum_k \sum_l h[k, l] e^{j2\pi \left(\frac{nk}{N} - \frac{ml}{M} \right)}$$

- ISFFT

$$X[n, m] = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} x[k, l] e^{j2\pi \left(\frac{nk}{N} - \frac{ml}{M} \right)}$$

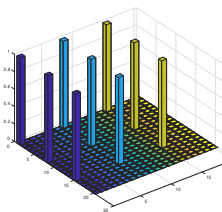
- SFFT

$$y[k, l] = \frac{1}{\sqrt{NM}} \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} Y[n, m] e^{-j2\pi \left(\frac{nk}{N} - \frac{ml}{M} \right)}$$

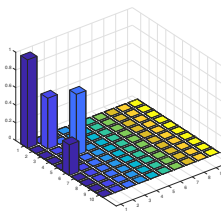
Delay–Doppler domain input-output relation

- Received signal in delay–Doppler domain

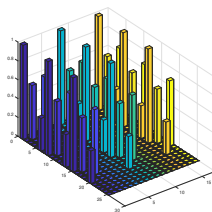
$$\begin{aligned}y[k, l] &= \sum_{i=1}^P h_i x[[k - k_{\nu_i}]_N, [l - l_{\tau_i}]_M] \\ &= h[k, l] \circledast x[k, l] \quad (\text{2D Circular Convolution})\end{aligned}$$



(a) Input signal, $x[k, l]$



(b) Channel, $h[k, l]$



(c) Output signal, $y[k, l]$

Figure: OTFS signals

Fractional doppler effect

- Actual Doppler may not be perfectly aligned with the grid

$$\nu_i = (k_{\nu_i} + \kappa_{\nu_i}) \left(\frac{1}{NT} \right), k_{\nu_i} \in \mathbb{Z}, -1/2 < \kappa_{\nu_i} < 1/2$$

- Induces interference from the neighbor points of k_{ν_i} in the Doppler grid due to non-orthogonality in channel relation – [Inter Doppler Interference \(IDI\)](#)
- Received signal equation becomes

$$y(k, l) = \sum_{i=1}^P \sum_{q=-N_i}^{N_i} h_i \left(\frac{e^{j2\pi(-q-\kappa_{\nu_i})} - 1}{Ne^{j\frac{2\pi}{N}(-q-\kappa_{\nu_i})} - N} \right) \times [[k - k_{\nu_i} + q]_N, [l - l_{\tau_i}]_M]$$

Compatibility with OFDM architecture

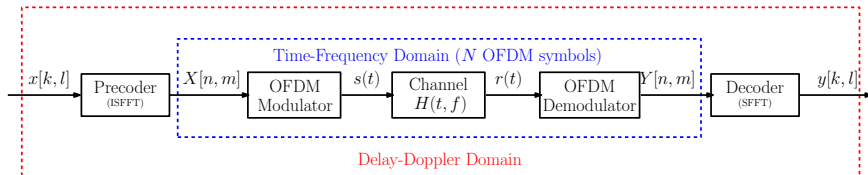


Figure: OTFS mod/demod

- OTFS is compatible with LTE system
- OTFS can be easily implemented by applying a precoding and decoding blocks on N consecutive OFDM symbols

OTFS with rectangular pulses – time–frequency domain

- Assume g_{tx} and g_{rx} to be rectangular pulses (same as OFDM) – don't follow bi-orthogonality condition
- Time–frequency input-output relation

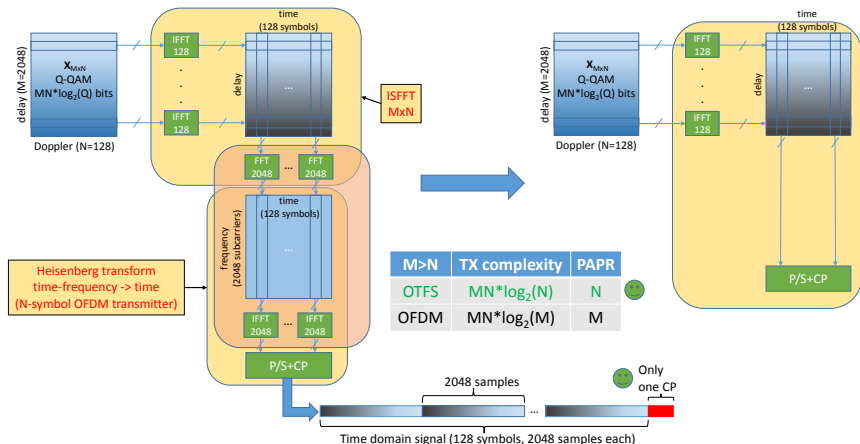
$$Y[n, m] = H[n, m]X[n, m] + \text{ICI} + \text{ISI}$$

- ICI – loss of orthogonality in frequency domain due to Dopplers
- ISI – loss of orthogonality in time domain due to delays

(*) P. Raviteja, K. T. Phan, Y. Hong, and E. Viterbo, “Interference cancellation and iterative detection for orthogonal time frequency space modulation,” *IEEE Trans. Wireless Commun.*, vol. 17, no. 10, pp. 6501-6515, Oct. 2018. Available on: <https://arxiv.org/abs/1802.05242>

OTFS Input-Output Relation in Matrix Form

OTFS transmitter implementation: $M = 2048, N = 128$



- OTFS transmitter implements **inverse ZAK** transform ($2D \rightarrow 1D$)

OTFS: Tx matrix representation

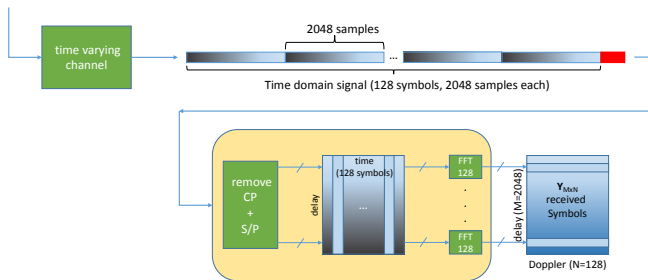
- Transmit signal at 2D time domain: ISFFT + Heisenberg + pulse shaping on delay-Doppler

$$\mathbf{S} = \mathbf{G}_{\text{tx}} \mathbf{F}_M^H \underbrace{(\mathbf{F}_M \mathbf{X} \mathbf{F}_N^H)}_{\text{ISFFT}} = \mathbf{G}_{\text{tx}} \mathbf{X} \mathbf{F}_N^H$$

- In vector form:

$$\mathbf{s} = \text{vec}(\mathbf{S}) = (\mathbf{F}_N^H \otimes \mathbf{G}_{\text{tx}}) \mathbf{x}$$

OTFS receiver implementation: $M = 2048$, $N = 128$



- Received signal at delay–Doppler domain: **pulse shaping** + **Wigner** + **SFFT** on time–frequency received signal

$$\mathbf{Y} = \mathbf{F}_M^H (\mathbf{F}_M \mathbf{G}_{\text{rx}} \mathbf{R}) \mathbf{F}_N = \mathbf{G}_{\text{rx}} \mathbf{R} \mathbf{F}_N$$

- In vector form:

$$\mathbf{y} = (\mathbf{F}_N \otimes \mathbf{G}_{\text{rx}}) \mathbf{r}$$

- OTFS receiver implements **ZAK** transform (1D→2D)

OTFS: matrix representation – channel

- Received signal in the time–frequency domain

$$r(t) = \int \int h(\tau, \nu) s(t - \tau) e^{j2\pi\nu(t-\tau)} d\tau d\nu + w(t)$$

- Channel

$$h(\tau, \nu) = \sum_{i=1}^P h_i \delta(\tau - \tau_i) \delta(\nu - \nu_i)$$

- Received signal in discrete form

$$r(n) = \sum_{i=1}^P h_i \underbrace{e^{\frac{j2\pi k_i(n-l_i)}{MN}}}_{\text{Doppler}} \underbrace{s([n - l_i]_{MN})}_{\text{Delay}} + w(n), 0 \leq n \leq MN - 1$$

OTFS: matrix representation – channel

- Received signal in vector form

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w}$$

- \mathbf{H} is an $MN \times MN$ matrix of the following form

$$\mathbf{H} = \sum_{i=1}^P h_i \mathbf{\Pi}^{l_i} \mathbf{\Delta}^{(k_i)},$$

where, $\mathbf{\Pi}$ is the permutation matrix (forward cyclic shift), and $\mathbf{\Delta}^{(k_i)}$ is the diagonal matrix

$$\underbrace{\mathbf{\Pi} = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & \ddots & & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}}_{\text{Delay (similar to OFDM)}}^{MN \times MN}, \underbrace{\mathbf{\Delta}^{(k_i)} = \begin{bmatrix} e^{\frac{j2\pi k_i(0)}{MN}} & 0 & \cdots & 0 \\ 0 & e^{\frac{j2\pi k_i(1)}{MN}} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & e^{\frac{j2\pi k_i(MN-1)}{MN}} \end{bmatrix}}_{\text{Doppler}}$$

OTFS: matrix representation – channel

- Received signal at delay–Doppler domain

$$\begin{aligned}\mathbf{y} &= [(\mathbf{F}_N \otimes \mathbf{G}_{\text{rx}}) \mathbf{H}(\mathbf{F}_N^H \otimes \mathbf{G}_{\text{tx}})] \mathbf{x} + (\mathbf{F}_N \otimes \mathbf{G}_{\text{rx}}) \mathbf{w} \\ &= \mathbf{H}_{\text{eff}} \mathbf{x} + \tilde{\mathbf{w}}\end{aligned}$$

- Effective channel for arbitrary pulses

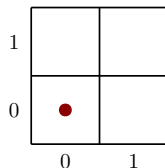
$$\begin{aligned}\mathbf{H}_{\text{eff}} &= (\mathbf{I}_N \otimes \mathbf{G}_{\text{rx}})(\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H}(\mathbf{F}_N^H \otimes \mathbf{I}_M)(\mathbf{I}_N \otimes \mathbf{G}_{\text{tx}}) \\ &= (\mathbf{I}_N \otimes \mathbf{G}_{\text{rx}}) \underbrace{\mathbf{H}_{\text{eff}}^{\text{rect}}}_{\text{Channel for rectangular pulses } (\mathbf{G}_{\text{tx}} = \mathbf{G}_{\text{rx}} = \mathbf{I}_M)} (\mathbf{I}_N \otimes \mathbf{G}_{\text{tx}})\end{aligned}$$

- Effective channel for rectangular pulses

$$\begin{aligned}\mathbf{H}_{\text{eff}}^{\text{rect}} &= \sum_{i=1}^P h_i \underbrace{[(\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{\Pi}^{(i)} (\mathbf{F}_N^H \otimes \mathbf{I}_M)]}_{\mathbf{P}^{(i)} \text{ (delay)}} \underbrace{[(\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{\Delta}^{(k_i)} (\mathbf{F}_N^H \otimes \mathbf{I}_M)]}_{\mathbf{Q}^{(i)} \text{ (Doppler)}} \\ &= \sum_{i=1}^P h_i \mathbf{P}^{(i)} \mathbf{Q}^{(i)} = \sum_{i=1}^P h_i \mathbf{T}^{(i)}\end{aligned}$$

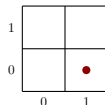
OTFS: Example for computing $\mathbf{H}_{\text{eff}}^{\text{rect}}$

- $M = 2, N = 2, MN = 4$
- $l_i = 0$ and $k_i = 0$ (no delay and Doppler)
 - $\Pi^{l_i=0} = \mathbf{I}_4 \Rightarrow \mathbf{P}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2)(\mathbf{F}_2^H \otimes \mathbf{I}_2) = \mathbf{I}_4$
 - $\Delta^{(k_i=0)} = \mathbf{I}_4 \Rightarrow \mathbf{Q}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2)(\mathbf{F}_2^H \otimes \mathbf{I}_2) = \mathbf{I}_4$
 - $\mathbf{T}^{(i)} = \mathbf{P}^{(i)}\mathbf{Q}^{(i)} = \mathbf{I}_4 \Rightarrow \text{Narrowband channel}$



OTFS: Example for computing $\mathbf{H}_{\text{eff}}^{\text{rect}}$

- $l_i = 1$ and $k_i = 0$ (delay but no Doppler)



- $\mathbf{\Pi}^{l_i=1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \Rightarrow$ block circulant matrix with 2×2 ($M \times M$) block size

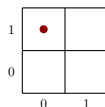
- $\mathbf{P}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2) \mathbf{\Pi} (\mathbf{F}_2^H \otimes \mathbf{I}_2) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-j2\pi \frac{1}{2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$

(using the block circulant matrix decomposition \rightarrow generalization of circulant matrix decomposition in OFDM)

- $\mathbf{\Delta}^{(k_i=0)} = \mathbf{I}_4 \Rightarrow \mathbf{Q}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2)(\mathbf{F}_2^H \otimes \mathbf{I}_2) = \mathbf{I}_4$
- $\mathbf{T}^{(i)} = \mathbf{P}^{(i)} \Rightarrow \mathbf{T}^{(i)} \mathbf{s} \rightarrow$ circularly shifts the elements in each block (size M) of \mathbf{s} by 1 (delay shift)

OTFS: Example for computing $\mathbf{H}_{\text{eff}}^{\text{rect}}$

- $l_i = 0$ and $k_i = 1$ (Doppler but no delay)



- $\mathbf{\Pi}^{l_i=0} = \mathbf{I}_4 \Rightarrow \mathbf{P}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2)(\mathbf{F}_2^H \otimes \mathbf{I}_2) = \mathbf{I}_4$

- $\mathbf{\Delta}^{(k_i=1)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{j2\pi\frac{1}{4}} & 0 & 0 \\ 0 & 0 & e^{j2\pi\frac{2}{4}} & 0 \\ 0 & 0 & 0 & e^{j2\pi\frac{3}{4}} \end{bmatrix} \Rightarrow \text{block diagonal matrix with } 2 \times 2 \text{ } (M \times M)$
block size

- $\mathbf{Q}^{(i)} = (\mathbf{F}_2 \otimes \mathbf{I}_2)\mathbf{\Delta}^{(1)}(\mathbf{F}_2^H \otimes \mathbf{I}_2) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{j2\pi\frac{1}{4}} \\ 1 & 0 & 0 & 0 \\ 0 & e^{j2\pi\frac{1}{4}} & 0 & 0 \end{bmatrix}$

(using the block circulant matrix decomposition in reverse direction)

- $\mathbf{T}^{(i)} = \mathbf{Q}^{(i)} \Rightarrow \mathbf{T}^{(i)} \mathbf{s} \rightarrow$ circularly shifts the blocks (size M) of \mathbf{s} by 1 (Doppler shift)

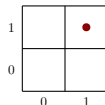
OTFS: Example for computing $\mathbf{H}_{\text{eff}}^{\text{rect}}$

- $l_i = 1$ and $k_i = 1$ (both delay and Doppler)

- $\mathbf{P}^{(i)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & e^{-j2\pi\frac{1}{2}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$

- $\mathbf{Q}^{(i)} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{j2\pi\frac{1}{4}} \\ 1 & 0 & 0 & 0 \\ 0 & e^{j2\pi\frac{1}{4}} & 0 & 0 \end{bmatrix}$

- $\mathbf{T}^{(i)} = \mathbf{P}^{(i)}\mathbf{Q}^{(i)} \Rightarrow \mathbf{T}^{(i)}\mathbf{s} \rightarrow$ circularly shifts both the blocks (size M) and the elements in each block of \mathbf{s} by 1 (delay and Doppler shifts)



OTFS: channel for rectangular pulses

- $\mathbf{T}^{(i)}$ has only **one non-zero element** in each row and the position and value of the non-zero element depends on the delay and Doppler values.

$$\mathbf{T}^{(i)}(p, q) = \begin{cases} e^{-j2\pi \frac{p}{N}} e^{j2\pi \frac{k_i([m-l_i]_M)}{MN}}, & \text{if } q = [m - l_i]_M + M[n - k_i]_N \text{ and } m < l_i \\ e^{j2\pi \frac{k_i([m-l_i]_M)}{MN}}, & \text{if } q = [m - l_i]_M + M[n - k_i]_N \text{ and } m \geq l_i \\ 0, & \text{otherwise.} \end{cases}$$

- Example: $l_i = 1$ and $k_i = 1$

$$\mathbf{T}^{(i)} = \begin{bmatrix} 0 & 0 & 0 & e^{j2\pi \frac{1}{4}} \\ 0 & 0 & 1 & 0 \\ 0 & e^{-j2\pi \frac{1}{4}} & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

OTFS Signal Detection

Vectorized formulation of the input-output relation

- The input-output relation in the delay–Doppler domain is a 2D convolution (with i.i.d. additive noise $w[k, l]$)

$$y[k, l] = \sum_{i=1}^P h_i x[[k - k_{\nu_i}]_N, [l - l_{\tau_i}]_M] + w[k, l] \quad k = 1 \dots N, l = 1 \dots M \quad (1)$$

- Detection of information symbols $x[k, l]$ requires a deconvolution operation i.e., the solution of the linear system of NM equations

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w} \quad (2)$$

where $\mathbf{x}, \mathbf{y}, \mathbf{w}$ are $x[k, l], y[k, l], w[k, l]$ in vectorized form and \mathbf{H} is the $NM \times NM$ coefficient matrix of (1).

- Given the sparse nature of \mathbf{H} we can solve (2) by using a message passing algorithm similar to (*)

(*) P. Som, T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, “Low-complexity detection in large-dimension MIMO-ISI channels using graphical models,” *IEEE J. Sel. Topics in Signal Processing*, vol. 5, no. 8, pp. 1497-1511, December 2011.

Message passing based detection

- Symbol-by-symbol MAP detection

$$\begin{aligned}\hat{x}[c] &= \arg \max_{a_j \in \mathbb{A}} \Pr(x[c] = a_j | \mathbf{y}, \mathbf{H}) \\ &= \arg \max_{a_j \in \mathbb{A}} \frac{1}{Q} \Pr(\mathbf{y} | x[c] = a_j, \mathbf{H}) \\ &\approx \arg \max_{a_j \in \mathbb{A}} \prod_{d \in \mathcal{J}_c} \Pr(y[d] | x[c] = a_j, \mathbf{H})\end{aligned}$$

- Received signal $y[d]$

$$y[d] = x[c]H[d, c] + \underbrace{\sum_{e \in \mathcal{I}_d, e \neq c} x[e]H[d, e] + z[d]}_{\zeta_{d,c}^{(i)} \rightarrow \text{assumed to be Gaussian}}$$

Messages in factor graph

Algorithm 1 MP algorithm for OTFS symbol detection

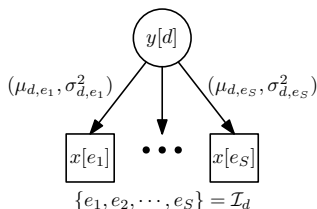
Input: Received signal \mathbf{y} , channel matrix \mathbf{H}

Initialization: pmf $\mathbf{p}_{c,d}^{(0)} = 1/Q$ **repeat**

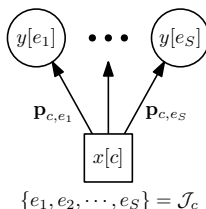
- Observation nodes send the mean and variance to variable nodes
- Variable nodes send the pmf to the observation nodes
- Update the decision

until *Stopping criteria*;

Output: The decision on transmitted symbols $\hat{x}[c]$



Observation node messages

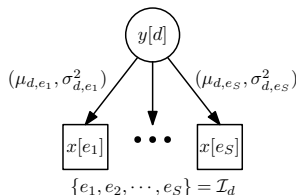


Variable node messages

Messages in factor graph – observation node messages

- Received signal

$$y[d] = x[c]H[d, c] + \underbrace{\sum_{e \in \mathcal{I}(d), e \neq c} x[e]H[d, e]}_{\zeta_{d,c}^{(i)} \rightarrow \text{assumed to be Gaussian}} + z[d]$$



- Mean and Variance

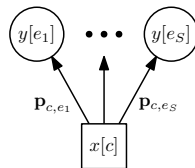
$$\mu_{d,c}^{(i)} = \sum_{e \in \mathcal{I}(d), e \neq c} \sum_{j=1}^Q p_{e,d}^{(i-1)}(a_j) a_j H[d, e]$$

$$(\sigma_{d,c}^{(i)})^2 = \sum_{e \in \mathcal{I}(d), e \neq c} \left(\sum_{j=1}^Q p_{e,d}^{(i-1)}(a_j) |a_j|^2 |H[d, e]|^2 - \left| \sum_{j=1}^Q p_{e,d}^{(i-1)}(a_j) a_j H[d, e] \right|^2 \right) + \sigma^2$$

Messages in factor graph – variable node messages

- Probability update with damping factor Δ

$$p_{c,d}^{(i)}(a_j) = \Delta \cdot \tilde{p}_{c,d}^{(i)}(a_j) + (1 - \Delta) \cdot p_{c,d}^{(i-1)}(a_j), a_j \in \mathbb{A}$$



where

$$\begin{aligned} \tilde{p}_{c,d}^{(i)}(a_j) &\propto \prod_{e \in \mathcal{J}(c), e \neq d} \Pr(y[e] | x[c] = a_j, \mathbf{H}) \\ &= \prod_{e \in \mathcal{J}(c), e \neq d} \frac{\xi^{(i)}(e, c, j)}{\sum_{k=1}^Q \xi^{(i)}(e, c, k)} \\ \xi^{(i)}(e, c, k) &= \exp \left(\frac{-|y[e] - \mu_{e,c}^{(i)} - H_{e,c} a_k|^2}{(\sigma_{e,c}^{(i)})^2} \right) \end{aligned}$$

Final update and stopping criterion

- Final update

$$p_c^{(i)}(a_j) = \prod_{e \in \mathcal{J}(c)} \frac{\xi^{(i)}(e, c, j)}{\sum_{k=1}^Q \xi^{(i)}(e, c, k)}$$
$$\hat{x}[c] = \arg \max_{a_j \in \mathbb{A}} p_c^{(i)}(a_j), \quad c = 1, \dots, NM.$$

- Stopping Criterion

- Convergence Indicator $\eta^{(i)} = 1$

$$\eta^{(i)} = \frac{1}{NM} \sum_{c=1}^{NM} \mathbb{I} \left(\max_{a_j \in \mathbb{A}} p_c^{(i)}(a_j) \geq 0.99 \right)$$

- Maximum number of Iterations
- **Complexity (linear)** – $\mathcal{O}(n_{iter} SQ)$ per symbol which is much less even compared to a linear MMSE detector $\mathcal{O}((NM)^2)$

Simulation results – damping factor Δ

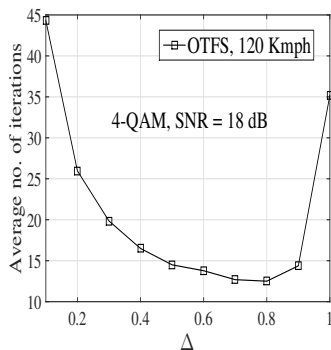
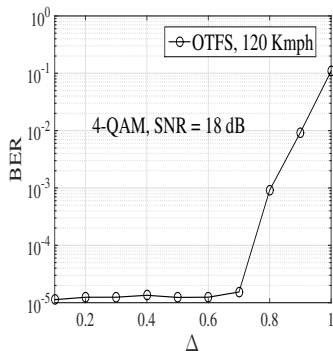


Figure: Variation of BER and average iterations no. with Δ . Optimal for $\Delta = 0.7$

Simulation results – OTFS vs OFDM with ideal pulses

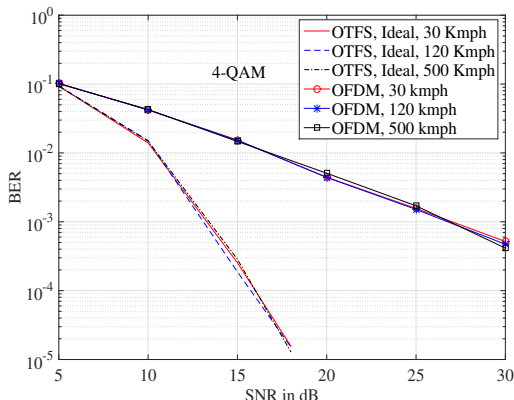


Figure: The BER performance comparison between OTFS with ideal pulses and OFDM systems at different Doppler frequencies.

Simulation results – IDI effect

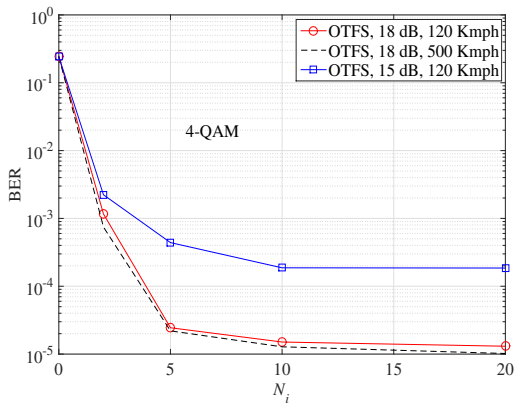


Figure: The BER performance of OTFS for different number of interference terms N_i with 4-QAM.

Simulation results – Ideal and Rectangular pulses

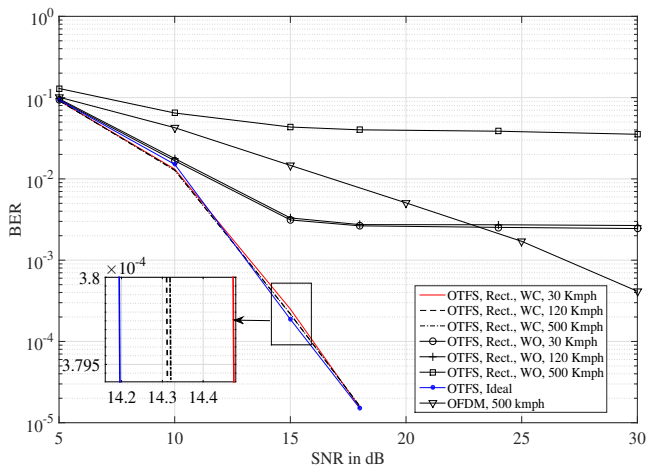


Figure: The BER performance of OTFS with rectangular and ideal pulses at different Doppler frequencies for 4-QAM.

Simulation results – Ideal and Rect. pulses - 16-QAM

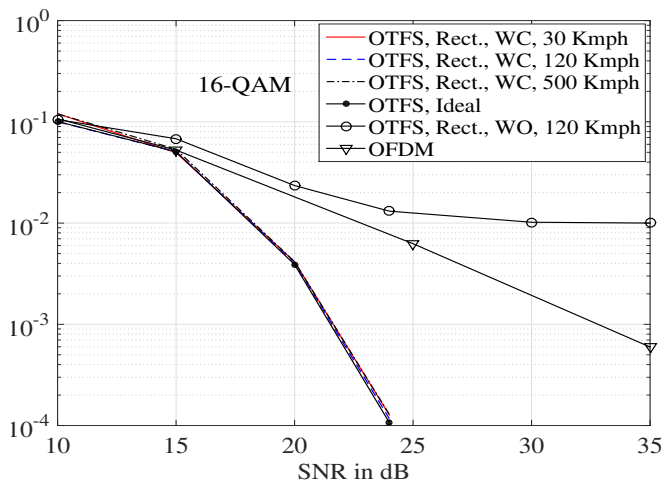


Figure: The BER performance of OTFS with rectangular and ideal pulses at different Doppler frequencies for 16-QAM.

Simulation results – Low latency

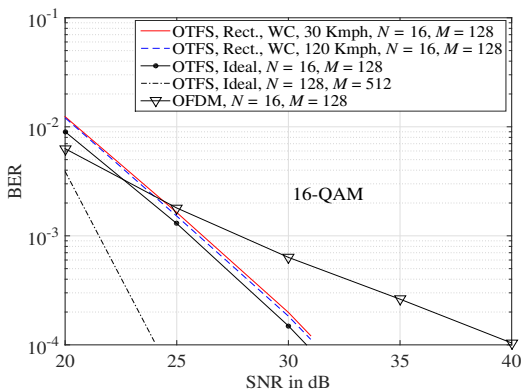


Figure: The BER performance of OTFS with rectangular pulses and low latency ($N = 16$, $T_f \approx 1.1$ ms).

- OTFS_sample_code.m

→ OTFS_modulation – 1. ISFFT, 2. Heisenberg transform

```
X = fft(fft(x)).'/sqrt(M/N); % ISFFT  
s_mat = ifft(X.').*sqrt(M); % Heisenberg transform  
s = s_mat(:);
```

→ OTFS_channel_gen – generates wireless channel
output: (delay_taps,Doppler_taps,chan_coef)

→ OTFS_channel_output – wireless channel and noise

```
L = max(delay_taps);  
s = [s(N*M-L+1:N*M);s];% add one cp  
s_chan = 0;  
for itao = 1:taps  
    s_chan = s_chan+chan_coef(itao)*circshift([s.*exp(1j*2*pi/M...  
        *(-L:-L+length(s)-1)*Doppler_taps(itao)/N).';zeros(L,1)],delay_taps(itao));  
end  
noise = sqrt(sigma_2/2)*(randn(size(s_chan)) + 1i*randn(size(s_chan)));  
r = s_chan + noise;  
r = r(L+1:L+(N*M));% discard cp
```

→ OTFS_demodulation – 1. Wiegner transform, 2. SFFT

```
r_mat = reshape(r,M,N);  
Y = fft(r_mat)/sqrt(M); % Wigner transform  
Y = Y.';  
y = ifft(fft(Y).').'/sqrt(N/M); % SFFT
```

→ OTFS_mp_detector – message passing detector

Other detection methods

- We will present a new low-complexity detection method at WCNC2020 on Tuesday in the Session T1-S7: Waveform and modulation

- Output OTFS signal: $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$

- ① MMSE detection:

$$\hat{\mathbf{x}} = (\mathbf{H}^H \mathbf{H} + \lambda \mathbf{I})^{-1} \mathbf{H}^H \mathbf{y}$$

- Provides diversity but **high complex** $\mathcal{O}((NM)^3)$

- ② OTFS FDE (frequency domain equalization) in [1]

- Equalization in time–frequency domain (one-tap) and apply the SFFT
- Low complexity equalizer
- Phase shifts can't be applied and **bad performance at high Dopplers**
- Small improvement on OFDM

[1]. Li Li, H. Wei, Y. Huang, Y. Yao, W. Ling, G. Chen, P. Li, and Y. Cai, “A simple two-stage equalizer With simplified orthogonal time frequency space modulation over rapidly time-varying channels,” available online: <https://arxiv.org/abs/1709.02505>.

Other detection methods

3 OTFS MMSE-PIC (parallel ISI cancellation) in [2]

- First applies the equalization in time–frequency domain (one-tap) and then applies successive cancellation with coding
- Successive cancellation

$$\hat{\mathbf{y}}(i)_{j+1} = \mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_j + \mathbf{H}(:, i)\hat{\mathbf{x}}(i)_j$$
$$\hat{\mathbf{x}}(i)_{j+1} = \arg \min_{\mathbf{a} \in \mathcal{A}} \left(\hat{\mathbf{y}}(i)_{j+1} - \mathbf{H}(:, i)\mathbf{a} \right)$$

- Moderate complexity
- Better performance than [1] but still **struggles with the high Doppler**

4 MCMC sampling [3]

- Approximate ML solution using Gibbs sampling based MCMC technique
- **High complexity $\mathcal{O}(n_{iter}NM)$** compared to message passing ($\mathcal{O}(n_{iter}SQ)$)
(Does not take advantage of sparsity of the channel matrix)

[2]. T. Zemen, M. Hofer, and D. Loeschenbrand, “Low-complexity equalization for orthogonal time and frequency signaling (OTFS),” available online: <https://arxiv.org/pdf/1710.09916.pdf>.

[3]. K. R. Murali and A. Chockalingam, “On OTFS modulation for high-Doppler fading channels,” in *Proc. ITA'2018*, San Diego, Feb. 2018.

OTFS channel estimation

Channel estimation in time–frequency domain

- (l_{τ_i}, k_{ν_i}) ((delay,Doppler)) values are obtained from the baseband **time domain** signal equation

$$y(t) = \sum_{i=1}^P h_i x(t - \tau_i) e^{j2\pi\nu_i(t-\tau_i)}$$

- PN based pilots and 2D matched filter matrix is used to determine (l_{τ_i}, k_{ν_i})
- **Highly complex**

-
- ④ A. Fish, S. Gurevich, R. Hadani, A. M. Sayeed, and O. Schwartz, “Delay-Doppler channel estimation in almost linear complexity,” *IEEE Trans. Inf. Theory*, vol. 59, no. 11, pp. 7632-7644, Nov. 2013.
 - ② K. R. Murali, and A. Chockalingam, “On OTFS modulation for high-Doppler fading channels,” in *Proc. ITA'2018*, San Diego, Feb. 2018.

Channel estimation using impulses in the delay-Doppler domain

- Each transmit and receive antenna pair sees a different channel having a finite support in the delay-Doppler domain
- The support is determined by the delay and Doppler spread of the channel
- The OTFS input-output relation for p th transmit antenna and q th receive antenna pair can be written as

$$\hat{x}_q[k, l] = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x_p[n, m] \frac{1}{MN} h_{w_{qp}} \left(\frac{k-n}{NT}, \frac{l-m}{M\Delta f} \right) + v_q[k, l].$$

-
- 1 P. Raviteja, K.T. Phan, and Y. Hong, "Embedded Pilot-Aided Channel Estimation for OTFS in Delay-Doppler Channels", *IEEE Trans. on Veh. Tech.*, March 2019 (Early Access).
 - 2 M. K. Ramachandran and A. Chockalingam, "MIMO-OTFS in high-Doppler fading channels: Signal detection and channel estimation," available online: <https://arxiv.org/abs/1805.02209>.
 - 3 R. Hadani and S. Rakib, "OTFS methods of data channel characterization and uses thereof." U.S. Patent 9 444 514 B2, Sept. 13, 2016.

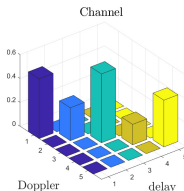
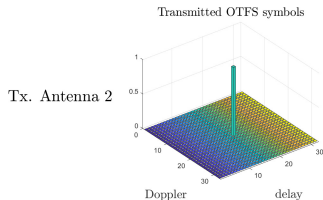
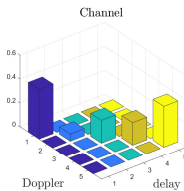
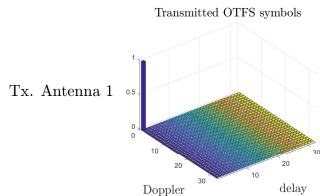
- If we transmit

$$\begin{aligned} x_p[n, m] &= 1 \text{ if } (n, m) = (n_p, m_p) \\ &= 0 \quad \forall (n, m) \neq (n_p, m_p), \end{aligned}$$

as pilot from the p th antenna, the received signal at the q th antenna will be

$$\hat{x}_q[k, l] = \frac{1}{MN} h_{w_{qp}} \left(\frac{k - n_p}{NT}, \frac{l - m_p}{M\Delta f} \right) + v_q[k, l].$$

- $\frac{1}{MN} h_{w_{qp}} \left(\frac{k}{NT}, \frac{l}{M\Delta f} \right)$ and thus $\hat{\mathbf{H}}_{qp}$ can be estimated, since n_p and m_p are known at the receiver a priori
- Impulse at $(n, m) = (n_p, m_p)$ spreads only to the extent of the support of the channel in the delay-Doppler domain (2D convolution)
- If the pilot impulses have sufficient spacing in the delay-Doppler domain, they will be received without overlap



2×1 system , SNR= 4 dB

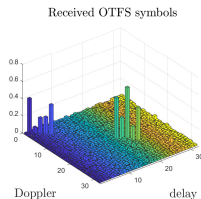
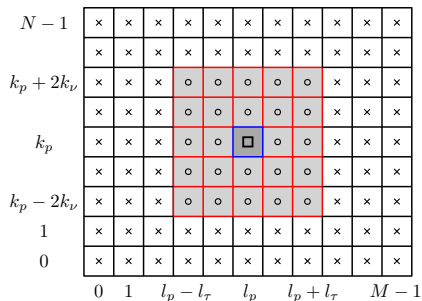
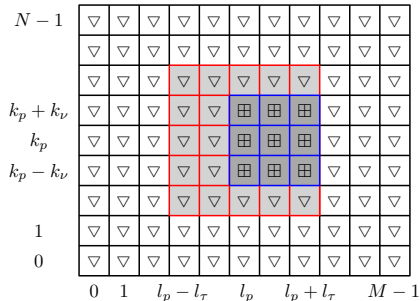


Figure: Illustration of pilots and channel response in delay-Doppler domain in a 2×1 MIMO-OTFS system

SISO OTFS system with integer Doppler



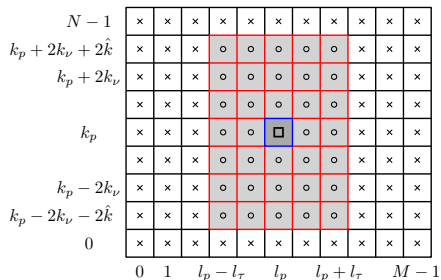
(a) Tx symbol arrangement (\square : pilot; \circ : guard symbols; \times : data symbols)



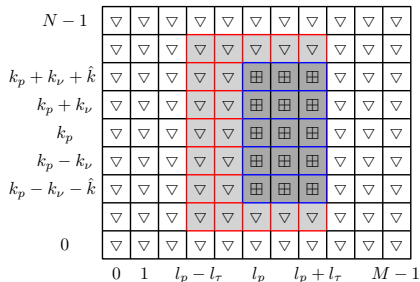
(b) Rx symbol pattern (∇ : data detection, \boxplus : channel estimation)

Figure: Tx pilot, guard, and data symbols and Rx received symbols

SISO OTFS system with fractional Doppler



(a) Tx symbol arrangement (\square : pilot; \circ : guard symbols; \times : data symbols)



(b) Rx symbol pattern (∇ : data detection, \boxplus : channel estimation)

Figure: Tx pilot, guard, and data symbols and Rx received symbols

MIMO OTFS system with fractional Doppler

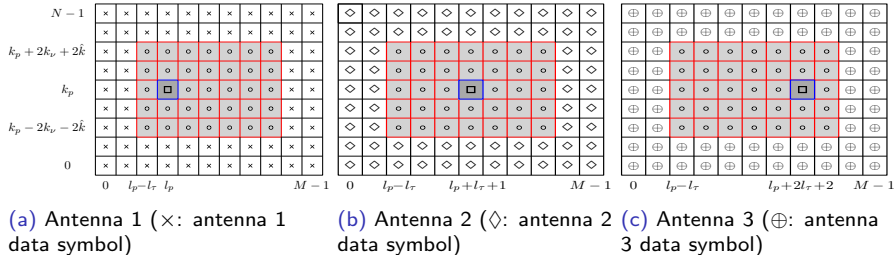


Figure: Tx pilot, guard, and data symbols for MIMO OTFS system (\square : pilot; \circ : guard)

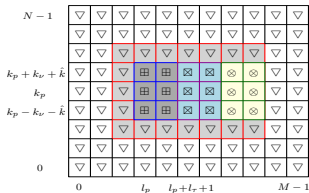


Figure: Rx symbol pattern at antenna 1 of MIMO OTFS system (∇ : data detection, \boxplus , \boxtimes , \otimes : channel estimation for Tx antenna 1, 2, and 3, respectively)

Multiuser OTFS system – uplink

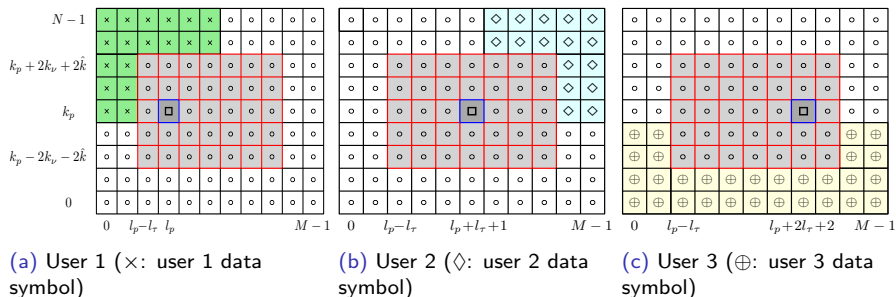
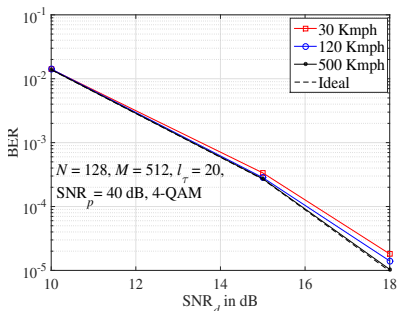


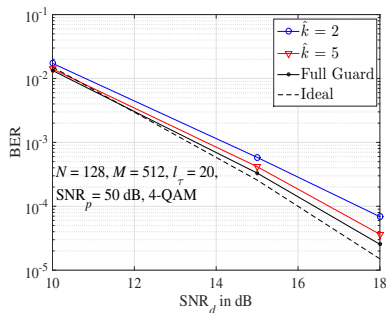
Figure: Tx pilot, guard, and data symbols for multiuser uplink OTFS system (\square : pilot; \circ : guard symbols)

SISO-OTFS performance with the estimated channel

- Simulation parameters: Carrier frequency of 4GHz, sub-carrier spacing of 15KHz, $M = 512$, $N = 128$, 4-QAM signaling, LTE EVA channel model
- Let SNR_p and SNR_d denote the average pilot and data SNRs
- Channel estimation threshold is $3\sigma_p$, where $\sigma_p^2 = 1/SNR_p$ is effective noise power of the pilot signal



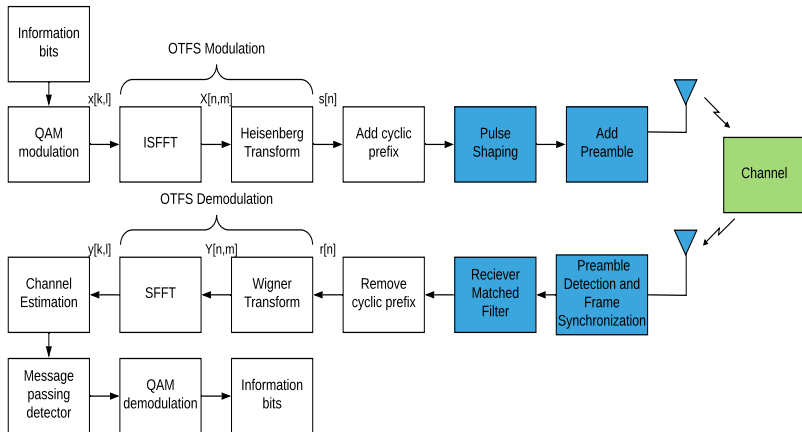
(a) BER for estimated channels of different Integer Dopplers



(b) BER for estimated channels of Fractional Doppler

OTFS applications

OTFS modem SDR implementation block diagram



Experiment setup and parameters

- The wireless propagation channel can be observed in real time using LabView GUI at the RX while receiving the OTFS frames.

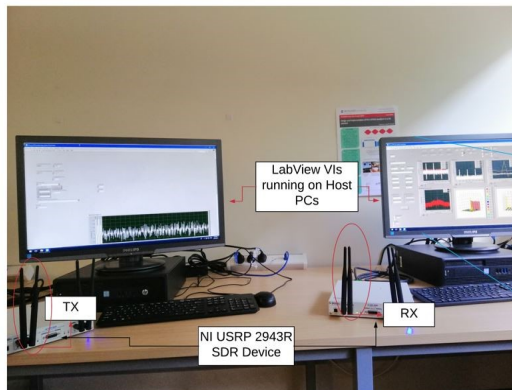
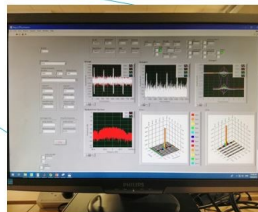


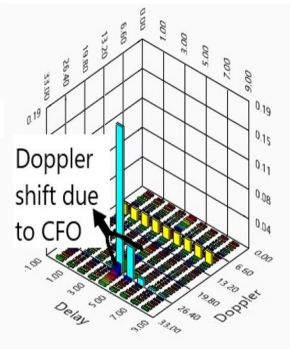
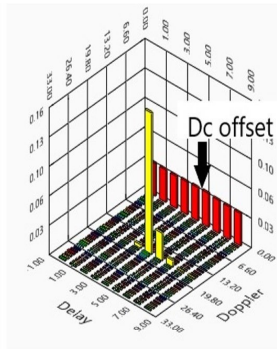
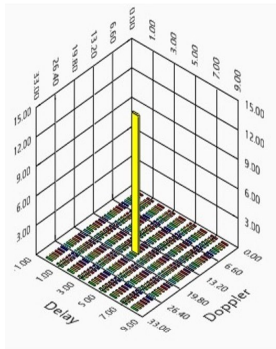
TABLE I
EXPERIMENT PARAMETERS

Symbol	Parameter	Value
f_c	Carrier frequency	4 GHz
M	Number of subcarriers	32
N	Number of symbols	32
Q	Modulation alphabet size	4,16
T	Symbol Time	1.28 micro secs
Δf	Subcarrier spacing	781.25 KHz
$1/M\Delta f$	delay resolution	40 nano secs
$1/NT$	Doppler resolution	24.4 KHz
F_{max}	Maximum Doppler spread	400KHz
d	Tx-Rx Distance	1.5 meters



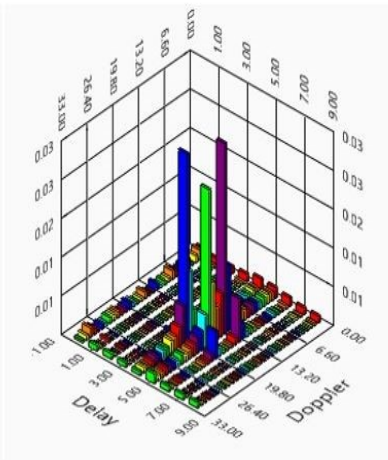
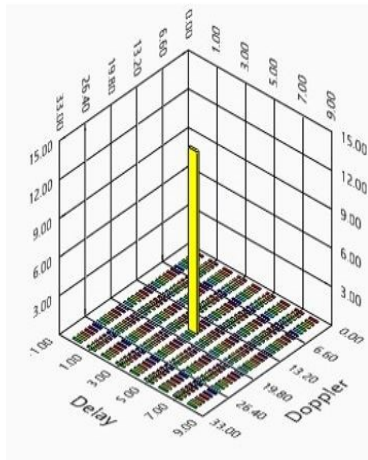
OTFS received pilot in a real indoor wireless channel

- DC Offset manifests itself as a constant signal in the delay-Doppler plane shifted by Doppler equal to CFO.

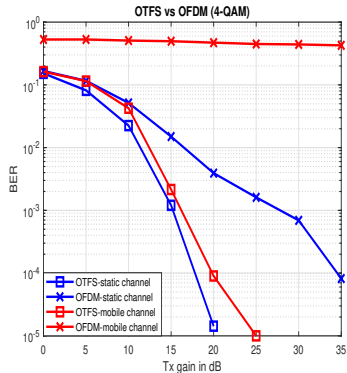
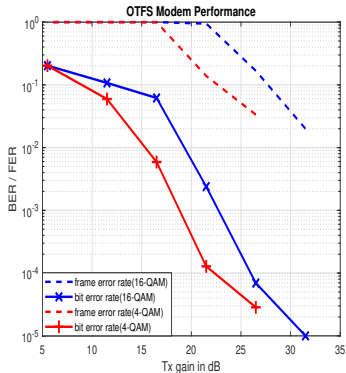


OTFS received pilot in a partially emulated indoor mobile channel

- Doppler paths were added to the TX OTFS waveform and transmitted it into a real indoor wireless channel for a time selective channel.



Error performance



OTFS with static multipath channels (zero Doppler)

- Received signal

$$(\text{size } MN \times 1) \mathbf{y} = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H} (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{x} + \tilde{\mathbf{w}}$$

↓ zero Doppler

$$(\text{size } M \times 1) \mathbf{y}_n = \check{\mathbf{H}}_n \mathbf{x}_n + \tilde{\mathbf{w}}_n, \text{ for } n = 0, \dots, N-1$$

- Equivalent to A-OFDM (asymmetric OFDM) in (*)

- $\check{\mathbf{H}}_n$ structure for $M \geq L$

$$\check{\mathbf{H}}_n = \begin{bmatrix} h_0 & 0 & \dots & h_1 e^{-j2\pi \frac{n}{N}} \\ h_1 & h_0 & \dots & h_2 e^{-j2\pi \frac{n}{N}} \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & h_1 & h_0 \end{bmatrix}_{M \times M}$$

- Achieves maximum diversity when $M \geq L$ (max. delay)
 $\iff N$ parallel CPSC transmissions each of length M

(*) J. Zhang, A. D. S. Jayalath, and Y. Chen, "Asymmetric OFDM systems based on layered FFT structure," *IEEE Signal Proces. Lett.*, vol. 14, no. 11, pp. 812-815, Nov. 2007.

OTFS with static multipath channels (zero Doppler)

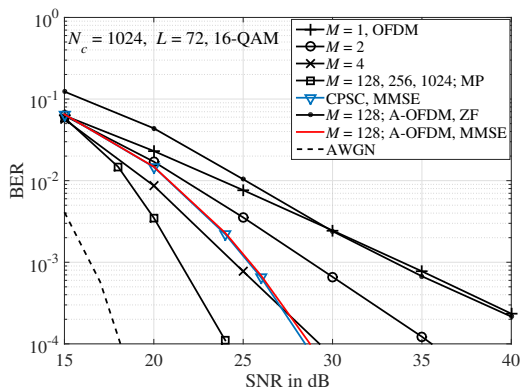


Figure: BER of OTFS for different M with $MN = N_c = 1024$, $L = 72$, and 16-QAM

(*) P. Raviteja, Y. Hong, and E. Viterbo, "OTFS performance on static multipath channels," *IEEE Wireless Commun. Lett.*, Jan. 2019, doi: 10.1109/LWC.2018.2890643.

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Thank you!!!

Questions?