Extinction Times of Epidemics with State-dependent Infectiousness

Avhishek Chatterjee (IIT Madras)

Joint work with Akhil Bhimaraju (IIT Madras, UIUC) and Lav R. Varshney (UIUC)

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+ Teenage-midlife problem

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What is this work about (and not about)?

A stylized mathematical model for obtaining structural insights

- impact of lockdown and precautionary measures

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This work is NOT about accurate forecasting

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Background

Study of epidemics has a long history: contagious disease, computer virus, opinion, . . .

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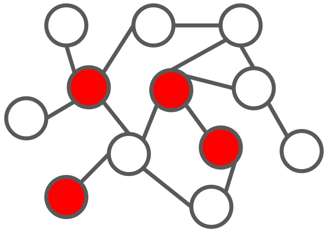

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Susceptible becomes infected at a rate (exp) β #(infected neighbors)

Infected becomes susceptible at a rate (exp) δ

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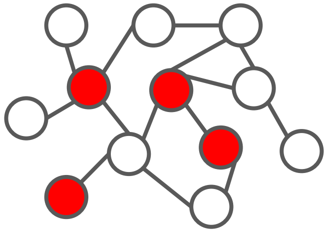

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The quantity of interest

- extinction time: time to hit 0

$$\beta \times \{\text{spectral radius}(A)\} < \delta : \mathbb{E}[T^{(0)}] = O(\log|V|)$$

$$\beta \times \{\text{expansion const}(G)\} > \delta : \mathbb{E}[T^{(0)}] = \Omega(\exp(|V|^{\gamma}), 0 < \gamma < 1$$

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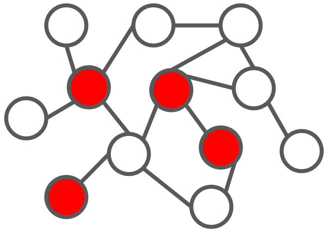

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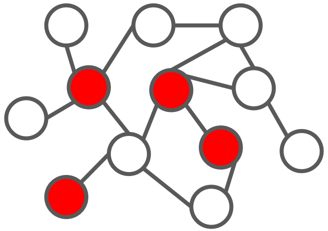

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Study epidemic on a **coarser graph** capturing interactions **between population centers**

Our model

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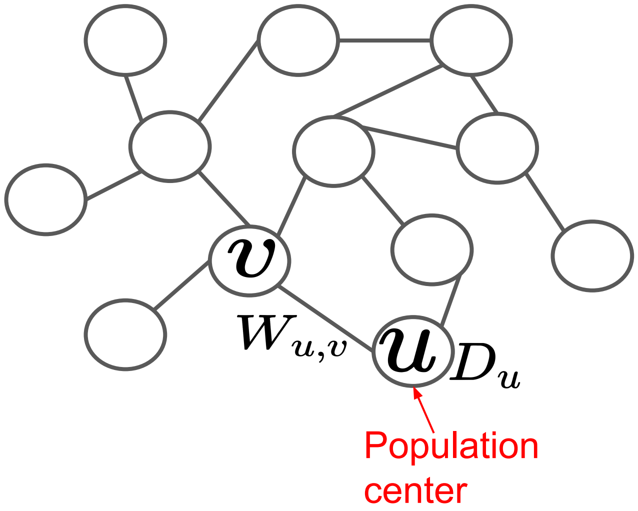

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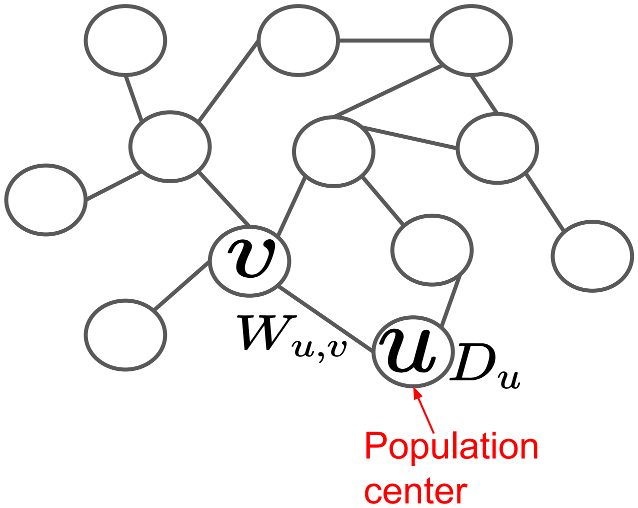

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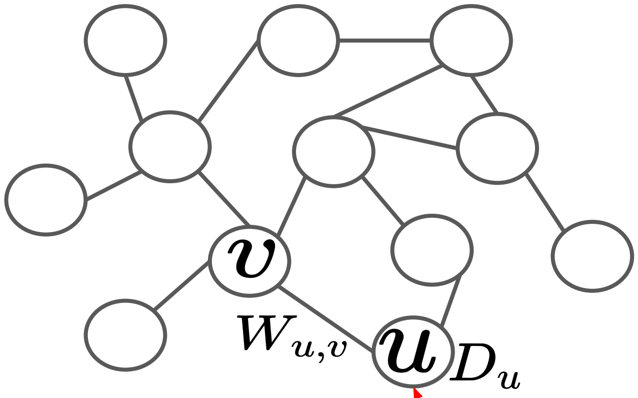

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#infection in that center

$$t \to t + dt : X_u(t) \to X_u(t) + 1 \text{ at rate}$$

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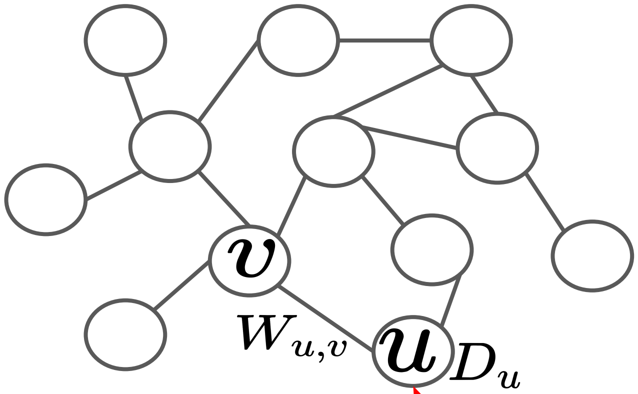

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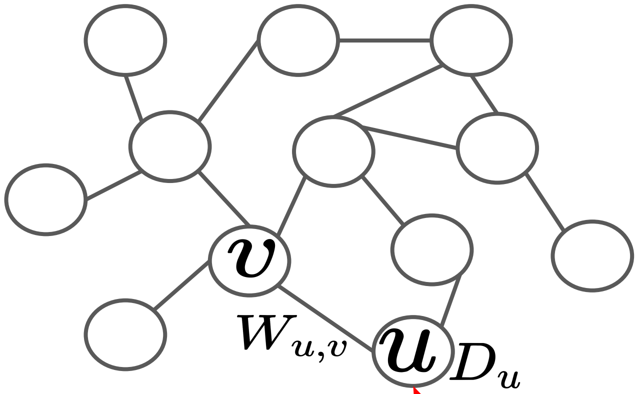

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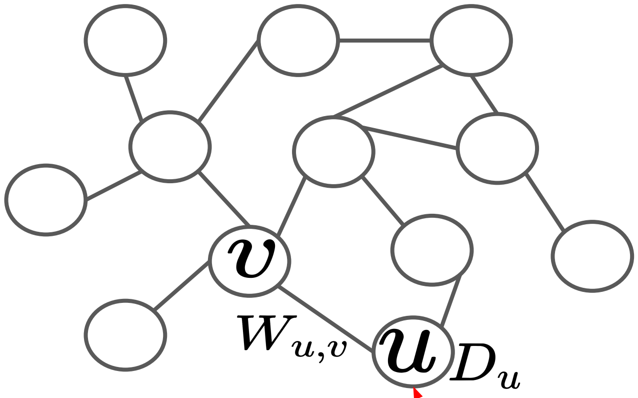

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Related work

Structural similarity with metapopulation models

- mean field dynamics (ODE involving $\mathbb{E}[X_u(t)]$) of different metapopulation models studied by Colizza et al. 2007, 2008
    - assume a sharp phase transition threshold and finds that

    *- naturally emerges from our **stochastic** analysis*
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Sharp threshold from stochastic analysis of epidemics and . . . .
- population-center based model with state-dependent infectiousness

Phase transition

$$\beta_\infty = \lim_{n\to\infty} \beta(n)$$

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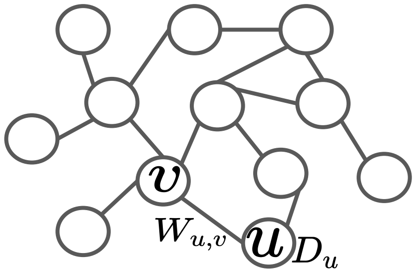

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First part: builds on Ganesh et al. 2005

- analyze decay of $\mathbb{E}[\mathbf{q}^T\mathbf{X}(t)]$, state-dependence

Second part: show transitivity of the CTMC

- embedded chain using Lyapunov $\mathbf{q}^T\mathbf{X}(t)$
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Some important questions emerge!

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What is the intuition behind the result?

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- Insights into lockdown strategy: within an area $\mathbf{D}$, across areas $\mathbf{W}$ and precautions β

-optimize economy/ease-of-living vs disease spread tradeoff (online oracle)

What is the intuition behind the result?

$$\text{For large } \sum_u X_u(t) : \beta(\cdot) \approx \beta^{\infty}. \text{ Defining } \mu_u(t) := \mathbb{E}[X_u(t)]$$

$$\frac{d}{dt}\mu(t) \approx \left(\beta_{\infty}^{\text{EXT}}\,\mathbf{W} + \beta_{\infty}^{\text{INT}}\,\text{diag}(\mathbf{D}) - \delta\,\mathbf{I}\right)\mu(t)$$

$$\text{Solving: } \mu(t) \approx \exp\left(\left(\beta_{\infty}^{\text{EXT}}\,\mathbf{W} + \beta_{\infty}^{\text{INT}}\,\text{diag}(\mathbf{D}) - \delta\,\mathbf{I}\right)t\right)\mu(0)$$

$$\Rightarrow ||\mu(t)||_2 \approx \exp\left(\left(\left\{\text{spectral radius}\left(\beta_{\infty}^{\text{EXT}}\,\mathbf{W} + \beta_{\infty}^{\text{INT}}\,\text{diag}(\mathbf{D})\right)\right\} - \delta\right)t\right)||\mu(0)||_2$$

$t \to t + dt : X_u(t) \to X_u(t) + 1$ at rate

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$t \to t + dt : X_u(t) \to X_u(t) - 1$ at rate $\delta \times X_u(t)$

Does the non-asymptotic behavior of $\beta^{\text{EXT}}, \beta^{\text{INT}}$ matter?

- Yes and no, depending on the quantity of interest

$$\left\{\text{spectral radius}\left(\beta_{\infty}^{\text{EXT}}\,\mathbf{W} + \beta_{\infty}^{\text{INT}}\,\text{diag}(\mathbf{D})\right)\right\} < \delta :$$

$$\mathbb{E}[T^{(0)} | \{X_u(0)\}] \le C \log\left(\sum_u X_u(0)\right) + o\left(\log \sum_u X_u(0)\right)$$

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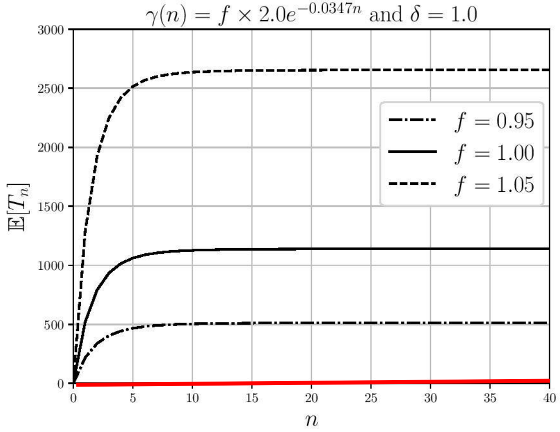

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Behavior of the lower order term

Define $S_n := \mathbb{E}[T_n] - \mathbb{E}[T_{n-1}]$

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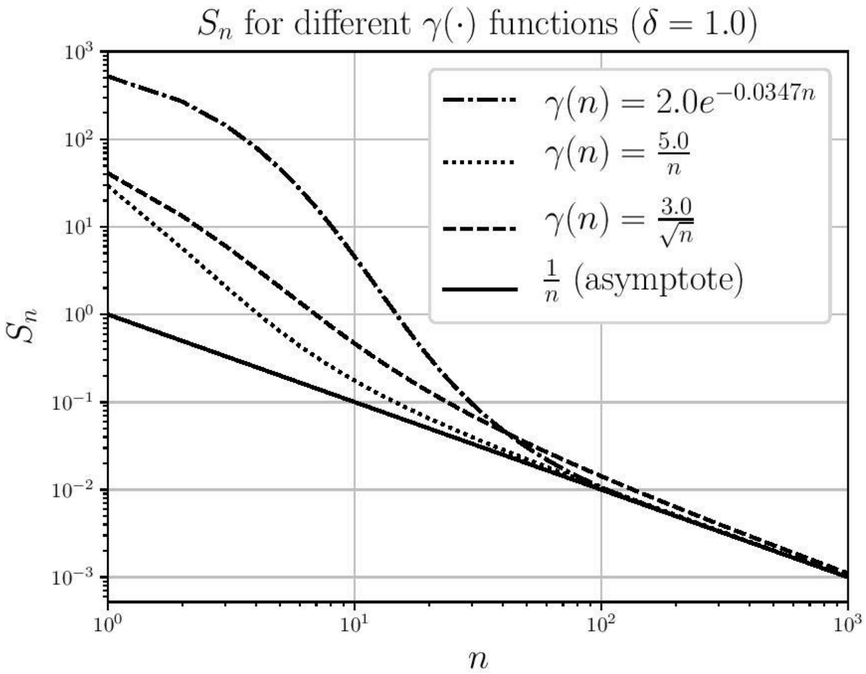

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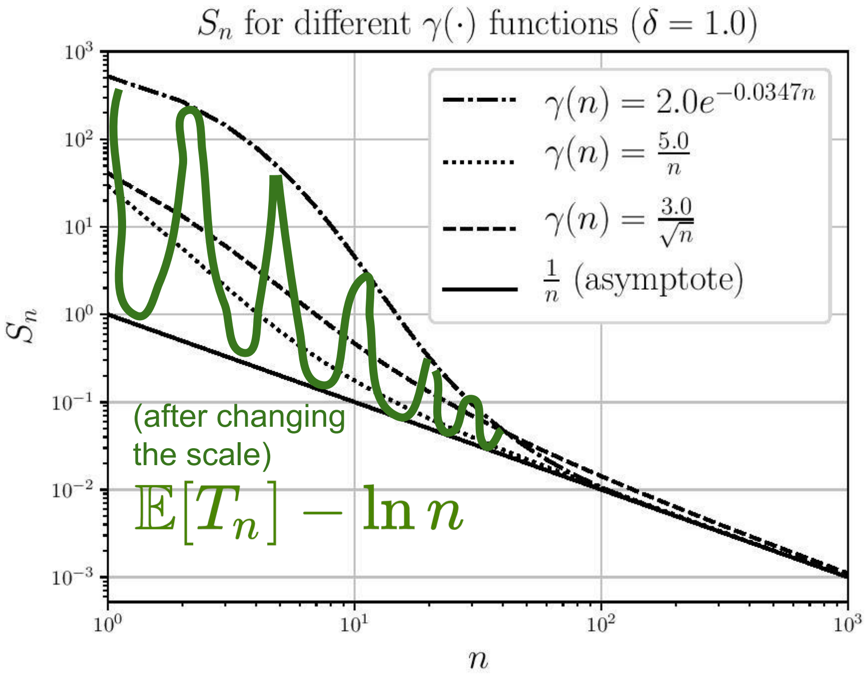

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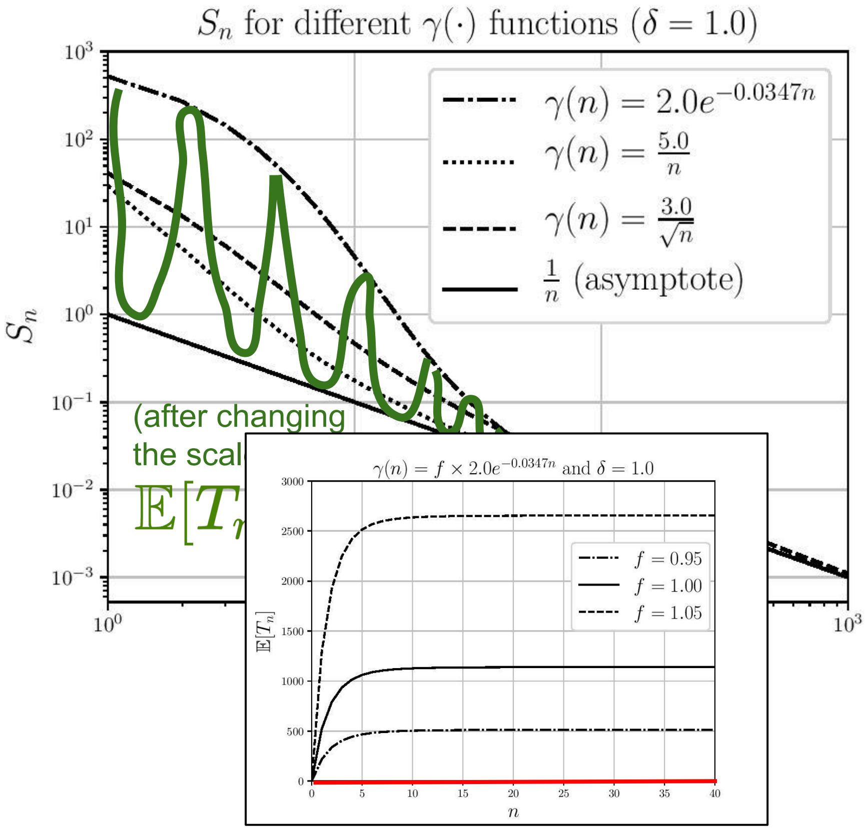

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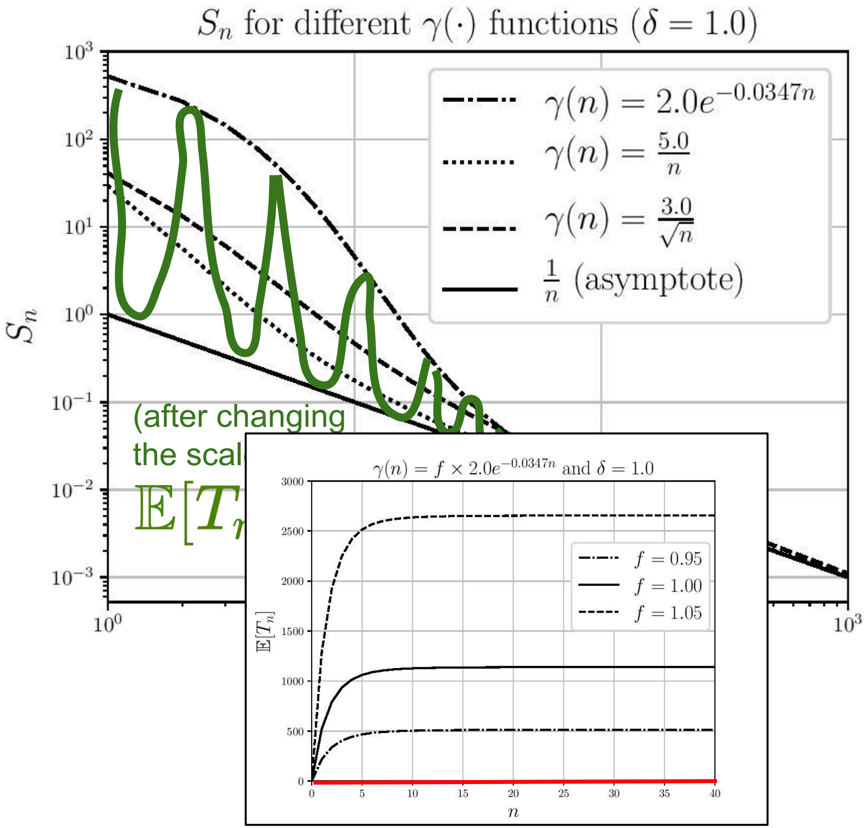

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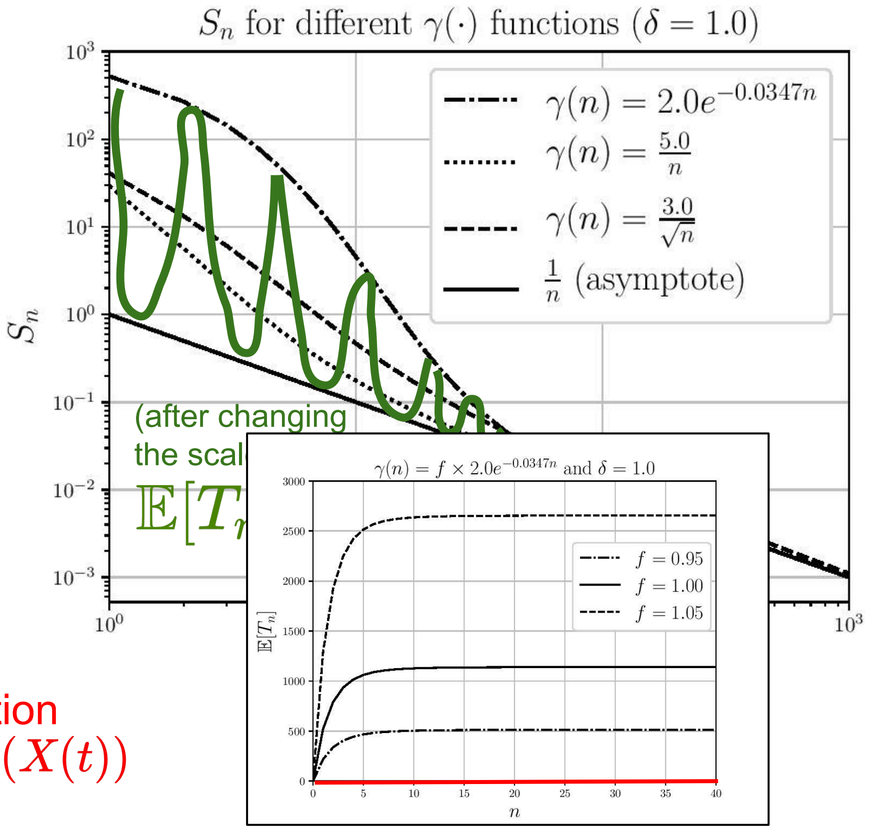

Lower order terms

- significant for low/moderate initial infection
- sensitive to non-asymptotic values of $\gamma(X(t))$

Some takeaways

Epidemic with “precautions” and “lockdowns” using “coarse location” graph: stochastic analysis

- structural insights can help in first order planning
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. . . quickly through **teenage-midlife problem**

Teenage-midlife problem

Teenage years: 13, 14, 15, . . .

Midlife years: 37, 38, 39, . . .

(global life expectancy ~ 73)

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Today
13-7-22

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$$\begin{aligned} 2\, p_2 - 1 &= p_1 \\ 3\, p_3 - 2 &= p_1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} 2\, n_2 - 1 &= p_1 \\ 3\, n_3 - 2 &= p_1 \end{aligned}$$

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$$2\,p_2 - 1 = p_1 \qquad\qquad 2\,n_2 - 1 = p_1 \qquad\qquad 2\,p_2 - 1 = n \qquad \frac{n+1}{2} = p_2$$

$$3\,p_3 - 2 = p_1 \quad\Rightarrow\quad 3\,n_3 - 2 = p_1 \quad\Rightarrow\quad 3\,p_3 - 2 = n \quad\Leftrightarrow\quad \frac{n+2}{3} = p_3$$

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This seems to have a connection with Corollary 1.9 in Green and Tao 2008.
(Can be true!)

Ending note

Please do NOT take my simplifications and proof ideas seriously

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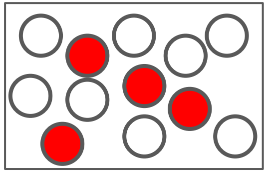
ThanQ

Background

Study of epidemics has a long history: contagious disease, computer virus, opinion, . . .

SIS: susceptible-infected-susceptible



infectiousness

Susceptible becomes infected at a rate (exp) $\beta \times \#\text{infected}$

Infected becomes susceptible at a rate (exp) δ

The continuous time stochastic process of interest: $\#\text{infected}(t)$

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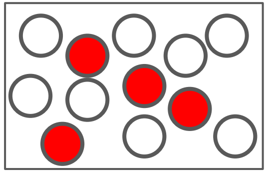

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This model does not capture the effect of social/contact graph